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Coma and astigmatism in a two-mirror system

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Dedicated to Prof Joseph Shamir

A system of two identical spherical mirrors has many practical applications. They are used, for example in systems to image the retina of the eye using adaptive optics, in lithography and in many optics experiments. The image of a small object has to be as good as possible. The image is either small or just a point. However, the necessary tilts of the mirrors make the point objects and image off-axis points. Here, we describe the coma and astigmatism in these systems.

Keywords: Coma and astigmatism, Lithography, Spherical aberration, Coma aberration, Optical design.

1 Introduction

A system with by two identical lenses or spherical mirrors has many practical applications, for example in systems to image the retina of the eye using adaptive optics and in many other uses. The image of a small object light source has to be as good as possible. The image is either small or just a point. The main aberrations to be corrected in order to form a good image with a small off-axis monochromatic light source, are spherical aberration, chromatic aberrations, coma and astigmatism. In a symmetric system like in Fig 1, the coma aberration as well as all other anti-symmetric aberrations, are automatically corrected for the whole system.

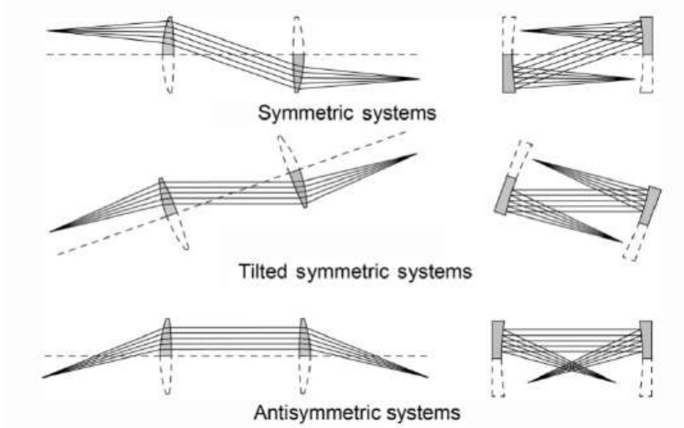

Fig 1. Symmetric and antisymmetric systems with two lenses and with two mirrors. The image and object are point sources or very small, but located off-axis.

Frequently, the light sources being used are infrared, but unfortunately glass is not transparent to infrared. Lenses not only have the disadvantage that they are not transparent to the infrared, but also that they have chromatic aberrations. So, a system of two mirrors is a better option. Spherical mirrors have spherical aberration if either the object or the image is at an infinite distance from the mirror, but it is negligible if the radius of curvature is very long compared with its diameter, as it is usually the case in many practical applications. However, the coma and astigmatism are present in spherical mirrors, as described

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by Malacara-Hernández *et al* [1]. Generally, the spherical aberration is not a problem since the focal ratios frequently used are long f/10 or more. We only need to worry about coma and astigmatism. Field curvature and distortion are not a problem since the object is either a point source or very small.

The system of two spherical mirrors cannot be used on-axis because the object and point image would not be easily accessible. This means that the mirrors have to be tilted off-axis. These systems have been referred to, as oblique incidence systems or tilted-decentered systems. Nevertheless, it is simpler to consider the system as symmetric or anti-symmetric systems, as we will describe next, and it is shown in Fig 1. Since the off-axis object is a small light source we do not need the complete lenses, as illustrated in this figure, we may imagine the two lenses of the system as two small lenses with a prismatic component introduced by a wedge. In the case of the prisms, the prismatic component is replaced by a tilt (rotation) of the mirrors. Another manner to visualize these systems is by tilting the whole system until the rays between the two elements are parallel to the x axis. When the two mirrors are rotated in the same direction, the object and image will be in opposite sides of the system. This is a symmetric system.

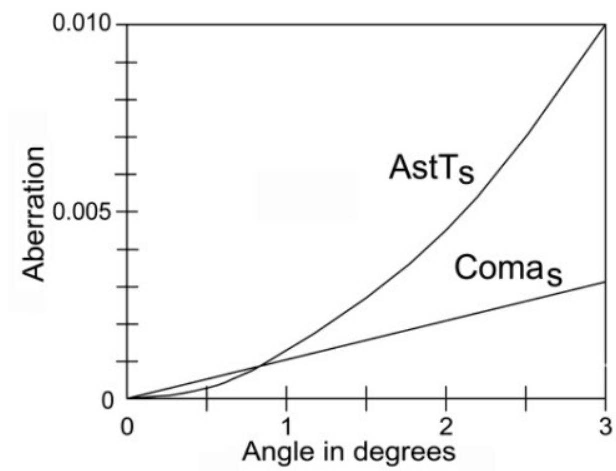

Fig 2. Sagittal coma and transverse sagittal astigmatism variation with the tilting of a spherical mirror. The diameter is 30 mm and the radius of curvature is 1000 mm.

As we know, coma and astigmatism vary with the image height as in Fig 2. So, for very small off axis angles coma is more important than astigmatism, but coma is anyway zero in the symmetric system, and thus the only remaining aberration limiting the image quality is astigmatism. Unfortunately, the off-axis angle cannot be made very small due to the size of the mirrors. For the mirror in Fig 2, if the off-axis angle is greater than only one degree, the astigmatism becomes the most important aberration.

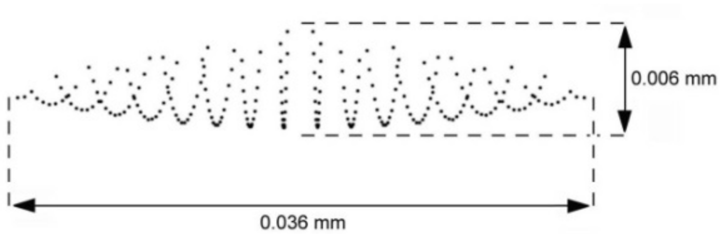

Fig 3. Image with coma and astigmatism produced by one of the mirrors at an off-axis angle equal to two degrees. The mirror diameter is 30 mm and the focal length is 1000 mm.

If the off-axis angle is only two degrees for a mirror with a diameter equal to 30 mm and a focal length equal to 1000 mm, the astigmatism and the coma aberration produce an image as illustrated in Fig. 3, where we can see that the astigmatism is more important than the coma.

Theoretical Analysis

Let us examine with some detail these concepts. The wavefront aberration introduced by each of the spherical mirrors can be expressed by:

$$W(x, y) = a_1 x + a_2 y + a_3 (x^2 + y^2) + a_4 y (x^2 + y^2) + a_5 (y^2 - x^2) \quad (1)$$

where the coefficients a_i are

Coefficient	*Aberration*
a_1	Tilt about the y axis
a_2	Tilt about the x axis
a_3	Defocusing
a_4	Coma along the y axis
a_5	Astigmatism at 0° or 90°

The wavefront distortion due to the comawin the system of two mirrors can be written as the sum of the contribution of the two mirrors as follows:

$$Coma_w = a_4 y (x^2 + y^2) + a_4 (-y)(x^2 - y^2) = 0 \quad (2)$$

where the coma coefficient a_4 is the same for both mirrors. The sign of y has been made negative for the second mirror because the tilts in this axis are in opposite directions. The astigmatism component of the wavefront distortion Ast_w for the system is:

$$Ast_w = a_5 (y^2 - x^2) + a_5\left((-y)^2 - x^2\right) = 2a_5 (y^2 - x^2) \quad (3)$$

As we see, that in a symmetrical system the astigmatism is not canceled, but duplicated, as illustrated in Fig 4. This is the main limitation of this system. As shown in the same figure, in the antisymmetric system, both the coma and the astigmatism are duplicated.

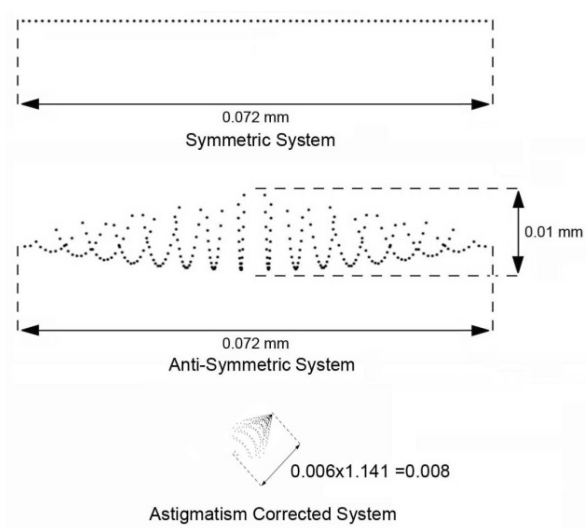

Fig 4. Images produced by symmetric and antisymmetric systems with two lenses.

As described by Gomez-Vieyra and Malacara-Hernández [3], the astigmatism can be eliminated if the second mirror is not tilted in the opposite direction to the first mirror, but in an orthogonal direction. The lens version of this system is in Fig 5, where the second lens is displaced not in the opposite direction to the displacement of the first lens, but in the orthogonal direction. In other words, the optical system is not in a single plane, as in the case of the symmetric and the antisymmetric systems. In this case the wavefront aberration distortion due to the coma and astigmatism components is:

$$\begin{aligned} W(x,y) &= a_4 x(x^2+y^2) + a_4 y(x^2+y^2) + a_5(y^2-x^2) + a_5(x^2-y^2) \\ &= a_4\left(x+y\right)(x^2+y^2) \end{aligned} \tag{4}$$

where the variables x and y have been interchanged in the second mirror, with respect to those variables in the first mirror. As we can notice, the astigmatism has been canceled, but the coma is now greater by a factor of 1.41, with a different orientation, at 45º, as illustrated at the bottom of Fig 4.

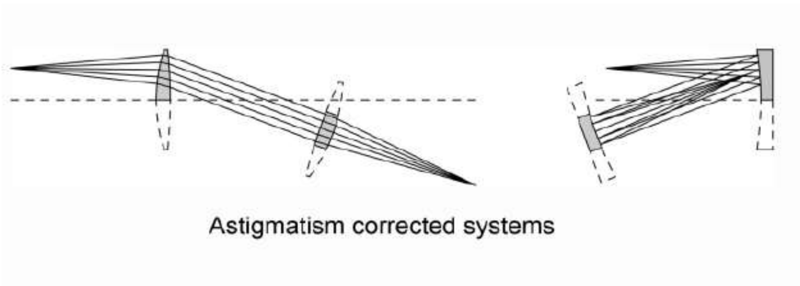

Fig 5. Coma corrected system by tilting the two mirrors in orthogonal directions.

2 Conclusions

In conclusion, by tilting the two mirrors in two orthogonal directions the astigmatism has been eliminated at the price of introducing coma, equal to 1.4 times the coma of a single mirror. However, since in general the tilt angle for the mirrors is not very small, the magnitude of the coma is much smaller than the magnitude of the astigmatism for each of the two mirrors. For this reason, the image quality is greatly improved.

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