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Dispersion in twisted PMC clad optical fibers

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The paper reports dispersion characteristics of twisted clad optical fibers under the situation when a sheath helix, composed of perfect magnetic conductor (PMC), is introduced in the core-clad interface. For this purpose, the allowed values of wave propagation constants corresponding to the sustained modes are deduced by the use of eigenvalue relation, as developed by implementing suitable field continuity conditions. Moreover, modes corresponding to the slow- and the fast-waves in the guide are analyzed by investigating the dispersion characteristics.

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1 Introduction

Since the last decade, studies of metamaterials have been of great interest as these find many useful applications, viz. perfect lensing, biomedical sensing, cloaking, slow- and fast-wave structures, broad- and narrow-band antennas etc. [1]. Apart from these, unusual phenomena, such as negative reflection/refraction, reversal of Doppler's effect, backward wave propagation etc., can also be achieved [2–5] by the use of these. Metamaterials owe properties due to the shape, size and geometry of their structure rather than their compositions. Moreover, these are the periodic assemblies of unit cells. However, the size of each unit cell must be lower than the operating wavelength, in order to invoke metamaterial properties.

Interestingly, more recently plasmonic based metamaterials have attracted the R & D community. Control over the manipulation of electromagnetic waves in plasmonic metamaterial structures leads towards the development of novel devices [6-13]. These are found in varieties of applications such as sensing, energy harvesting and slow-wave propagation. These are used in ultrasensitive biomedical devices [1]. It has been reported that plasmonic based artificial mediums enhance the efficiency of biomedical sensors and solar cells [14,15].

Optical guides composed of different forms of microstructures have been reported in the literature [16-23]. Baqir *et al* investigated varieties of twisted clad optical fiber structures [16-18] implementing the concept of travelling tubes [24]. Within the context, refs [25-28] demonstrate guided structures that support both slow and fast propagation modes.

In the present communication, we investigate a specially designed microstructured optical fiber wherein a sheath helix structure, composed of perfect magnetic conductor (PMC), is introduced in the core-clad interface of fiber with varying orientations of helix, as defined by the helix pitch angle. Essentially, the PMC is treated as the kind of perfect conductor in which the tangential components of magnetic field vanish [29], and defined as, $\hat{n} \times \vec{\mathrm{H}} = 0$, with $\hat{n}$ as the normal to the surface of conductor. The dispersion behavior of guide is investigated in this paper. It has been found that the guide supports both the slow- and the fast-modes corresponding to different pitch angles of twisted sheath helix structure.

2 Analytical treatment

Figure 1 shows the schematic of twisted clad optical fiber structure taken into account. The core and the clad regions with the parametric values $0 < \rho \leq a$ and $a < \rho \leq \infty\, a$, respectively, are assumed to be composed of nonlinear, reciprocal, lossless, non-dispersive, non-magnetic and isotropic dielectric mediums

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with the respective refractive indices (RIs) as n_{co} and n_{cl}. A PMC sheath helix wrap is introduced in the core-clad interface with certain orientation defined by the helix pitch angle ψ. In order to simplify the problem, the outer clad is considered to be infinitely extended along the z-direction. Further, we assume time t-harmonic and axis z-harmonic electromagnetic waves to propagate, which ultimately yields the axial and the transverse components of the electric and the magnetic fields upon the usage of Maxwell's field equations [30].

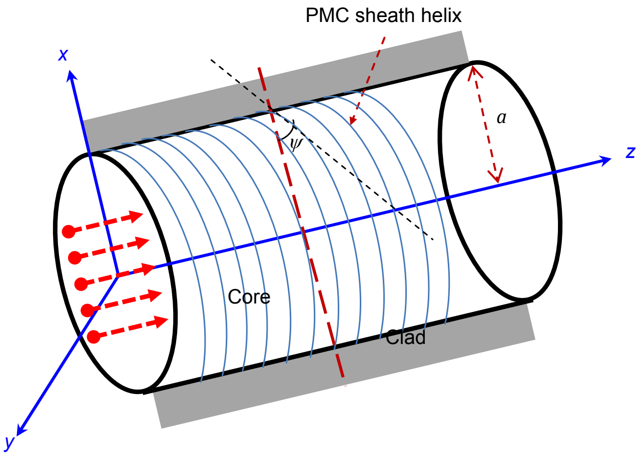

Fig 1. Schematic of the twisted PMC sheath helix loaded fiber.

The axial components of electromagnetic field in the core and the clad regions of cylindrical optical fiber can be written as

$$E_{z1} = AJ_\nu(ur)\, e^{j(\omega t - \beta z + \nu\varphi)} \tag{1a}$$

$$H_{z1} = BJ_\nu(ur)\, e^{j(\omega t - \beta z + \nu\varphi)} \tag{1b}$$

and

$$H_{z2} = CK_\nu(wr)\, e^{j(\omega t - \beta z + \nu\varphi)} \tag{2a}$$

$$E_{z2} = DK_\nu(wr)\, e^{j(\omega t - \beta z + \nu\varphi)} \tag{2b}$$

In the above Eqs (1) and (2), the subscripts 1 and 2 correspond to the situations in the core and the clad regions of fiber, respectively and A, B, C and D are the unknown constants which can be determined by applying the boundary condition at the core-clad interface. Also, $J(\cdot)$ and $K(\cdot)$ are Bessel and the modified Bessel functions, respectively, and ν, ω and β are the mode designating parameter, angular frequency in the unbounded medium and the modal propagation constant, respectively. Apart from these, the values of parameters u and w are defined through the equations

$$u^2 = \{(n_{co}k_o)^2 - \beta^2\} \tag{3a}$$

$$w^2 = \{\beta^2 - (n_{cl}k_o)^2\} \tag{3b}$$

with k_0 as the free-space wavenumber. The transverse field components are derived by substituting the axial components of electromagnetic field in the source free Maxwell's equations.

In order to study the dispersion behavior, we develop the eigenvalue equation to determine the allowed values of modal propagation constant β. This is done by implementing the conditions of continuity, defined as the tangential components of the magnetic field to vanish at the interface of PMC twists. Furthermore, the spacing between two adjacent PMC windings is composed of dielectric medium, and therefore, the tangential components of the electric/magnetic fields will be continuous at the interface.

Using these boundary conditions, one can finally obtain the following dispersion relation:

$$[-2\{K_\nu(wa)\}^2\chi - \omega^2\,\varepsilon_{cl}\,\mu_{cl}\,a^2w^3\,M\sin\psi] \times \{J_\nu(ua)\,\Theta + ju^2\,\mu_{co}\,\omega\,L\,K_\nu(wa)\cos\psi\} = 0 \tag{4}$$

where

$$M = J_{\nu-1}(ua) - J_{\nu+1}(ua) \tag{5a}$$

$$L = K_{\nu-1}(wa) + K_{\nu+1}(wa) \tag{5b}$$

$$\chi = 2aw^2\,\beta\nu\cos 2\psi + (a^2\,w^4 - \beta\nu^2)\sin 2\psi \tag{5c}$$

$$\Theta = j\,\frac{\omega\mu_{cl}w^2}{a}\,M\cos\ \psi + 2(u^2 - w^2)\,K_\nu(wa)\sin\psi \tag{5d}$$

3 Results and Discussion

We now investigate the dispersion behavior of fiber for which the operating wavelength λ is kept as 1.55 µm, the core radius as 5 µm, and the core/clad RI values as 1.50 (crown glass) and 1.34 (CYTOP), respectively. Using these parametric values, the allowed values of propagation constant β are numerically deduced by solving the eigenvalue relation, as obtained in Eq (4). Furthermore, we take into account three different values of helix pitch angle ψ, viz. 0°, 45° and 90°.

Figures 2a, 2b and 2c, respectively, demonstrate the dispersion features of fiber corresponding to the pitch angles of sheath helix as 0°, 45° and 90°. The effective RI $ne_{ff} = \beta/k_0$ is plotted against the normalized frequency, $V = (2\pi a/\lambda)\,(n^2_{co} - n^2_{cl})^2$ taking into account the two low-order hybrid modes H_{01} (shown by solid lines) and H_{11} (shown by dashed lines).

It becomes obvious from Fig 2a that, for the H_{01} mode, the value of n_{eff} initially decreases until the normalized frequency V reaches 3.7, corresponding to which $n_{eff} \approx 1.37$. Further, the increase in V causes increase in the n_{eff} value with a little fluctuation in the case of H_{01} mode. However, for the H_{11} mode, ripples in the value of n_{eff} are greatly reduced. Moreover, these ripples correspond to propagating modes combined with the slow- and the fast-waves through the guide. Corresponding to the low-order H_{01} mode, however, the slow- and the fast-wave modes remain prominent. In the case of H_{01} mode, as $2 \le V < 4$, the $\frac{dn_{eff}}{dV} < 0$ – the situation with corresponds to the existence of fast-wave modes [28]. Corresponding to further increase in V, in the span $4 < V \le 5.5$, we observe $\frac{dn_{eff}}{dV} > 0$ – the situation which corresponds to the existence of slow-waves [28]. Further increase in the normalized frequency parameter V also shows mixed behavior of the existence of slow- and fast-waves in the guide. However, in the case of H_{11} mode, the presence of slow- and fast-waves are less prominent.

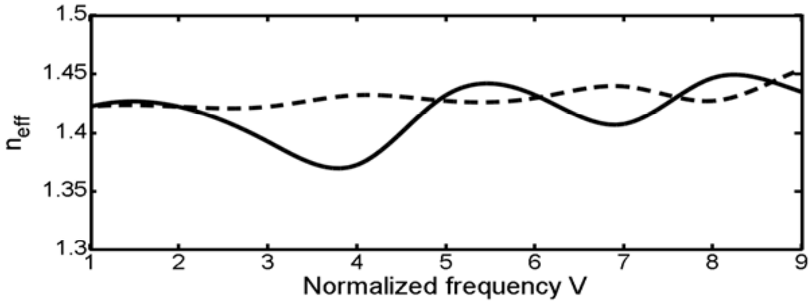

Fig 2a. Plots of n_{eff} vs V corresponding to the helix pitch angle 0°.

Now, taking into account the fiber structure having helix orientation with a pitch angle as $\psi = 45°$, the obtained results are illustrated in Fig 2b. It is noticed that n_{eff} values for both the H_{01} and the H_{11} modes remain almost unchanged corresponding to $1 \le V \le 5$, whereas n_{eff} for the low-order H_{01} mode gets decreased upon further increase in V. It is also observed that, in the case of H_{11} mode, n_{eff} increases when the value of V is above 7, and again it gets lowered as $V \geq 8$ (Fig 2b). In order to study the propagation speed of electromagnetic waves, the $V - n_{eff}$ slope remains greatly important. In the present case, the incident wave propagates with the normal speed as $V \leq 5$, i.e. $\frac{dn_{eff}}{dV} = 0$, corresponding to both the H_{01} and the H_{11} modes. As $V > 5$, for the H_{01} mode, we observe that, $\frac{dn_{eff}}{dV} < 0$, which corresponds to the fast-waves. Corresponding to the higher-order H_{11} mode, the $V - n_{eff}$ slope becomes positive for $V > 7$, and then it becomes negative as $V > 8$, which essentially states the existence of both the slow- and the fast modes (Fig 2b).

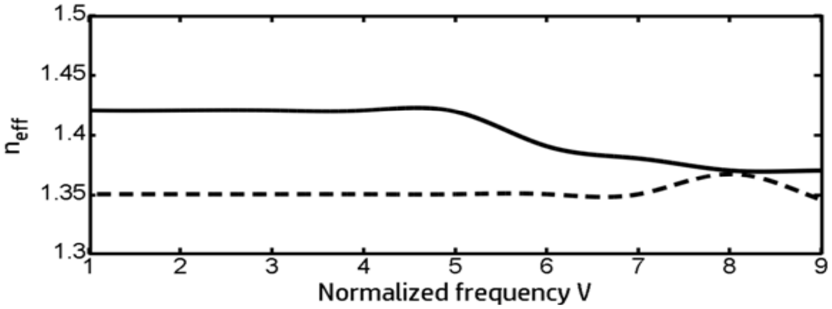

Fig 2b. Plots of n_{eff} vs V corresponding to the helix pitch angle 45°.

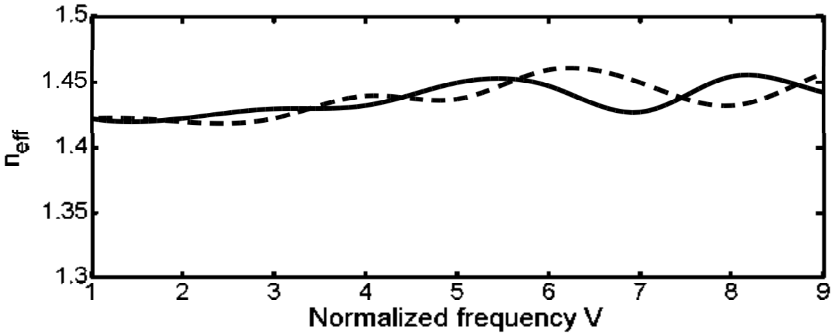

Fig 2c. Plots of n_{eff} vs V corresponding to the helix pitch angle 90°.

Considering Fig 2c, we observe that the values of n_{eff} corresponding to both the modes (i.e. H_{01} and H_{11}) remain fairly stable with the increase in normalized frequency V. The plots, however, get a little oscillatory nature corresponding to the larger values of V. Therefore, the case of 90° pitch angle exhibits less ripples in the n_{eff} values than those observed in the case of 0° pitch angle. In the present case, both the H_{01} and H_{11} modes contribute to the slow- and the fast-waves corresponding to the higher values of V, as is obvious from Fig 2c. Furthermore, both the H_{01} and the H_{11} modes exhibit the oscillatory nature in the n_{eff} values, which indicates that fiber supports both the slow- and the fast-wave modes in the present case.

4 Conclusion

From the above discussion, conclusion can be drawn that the dispersion behavior of optical fiber under PMC twists depends on the pitch of twists and the normalized frequency. Further, the oscillatory nature of effective RI in respect of both the low-order H_{01} and H_{11} modes, as obtained in the dispersion plots, indicate some interesting optical applications as both the slow- and the fast-wave modes propagate through optical fiber. It is also inferred that, by suitably tailoring the constituents of the proposed fiber structure, the required dispersion behavior can be attained.

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