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Generation of Stokes vortices in three, four and six circularly polarized beam interference

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This article is dedicated to Prof Kehar Singh for his significant contributions to Optics and Photonics

In this article we have shown generation of Stokes fields vortices from the interference of three, four, and six polarization engineered circularly polarized vector beams. In addition to this, the interference method is extended to phase and polarization engineered six circularly polarized beams for realizing two interesting lattice fields embedded with all three Stokes vortices simultaneously. We believe that such polarization lattice fields may bring up novel concept of structured polarization illumination methods in super resolution microscopy.

Keywords: Interference, Polarization, Optical Vortices, Polarization Singularity.

1 Introduction

Paraxial optical fields with slowly and spatially varying polarization distributions may host polarization singularities in their cross sections [1-5]. These isolated point singularities of polarization can be broadly classified into three types, namely, C-points, L-lines, and V-points. Vector point singularities (V-points) are isolated singular points in the spatially varying linearly polarized field distribution at which the azimuth of linear polarization is indeterminate. These singularities are characterized by Poincare–Hopf index (η). Although, it is possible to generate V-points with arbitrary positive or negative integral values of (η), the V-point singularities with $\eta = \pm 1$ are generic in nature. The isolated singular points of spatially varying ellipse field distributions are called elliptic point singularities. At C-point the state of polarization (SOP) is circular, where the azimuth of polarization ellipse is indeterminate. There is also L-line, on which the SOP is linear, and hence handedness is undefined, whereas the orientation of the major axis of the ellipse is indeterminate at C-point. L-lines separate left- and right-handed regions of an ellipse field distribution. The topological index I_c is used to characterize C-points. Unlike V-points, the C-points can have both half integral and integral I_c values. The generic C-points lemon and monstar correspond to $I_c = 1/2$ and when $I_c = -1/2$, the C-point is called the star.

The SOP of any optical field can be described by four measurable quantities known as Stokes parameters [6, 7]. The normalized Stokes parameters in terms of transverse electric field components E_x and E_y are expressed by

$$s_0 = |E_x|^2 + |E_y|^2, \quad S_1 = s_0^{-1} (|E_x|^2 - |E_y|^2)$$

$$S_2 = 2s_0^{-1} Re(E_x^* E_y), S_3 = 2s_0^{-1} Im(E_x^* E_y) \tag{1}$$

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The Stokes parameter s_0 indicates the total intensity of the optical field. The state of polarization of the optical field can be explained by S_1, S_2 and S_3 Stokes parameters. In other words, the polarization singularities can be analyzed by using Stokes parameters. A description of polarization singularities in terms of the phase singularities of complex Stokes scalar fields, i.e., Stokes singularities were given by ref [8-10]. Three complex Stokes fields namely $S_{12} = S_1 + iS_2$, $S_{23} = S_2 + iS_3$ and $S_{31} = S_3 + iS_1$ are constructed by using normalized Stokes parameters. Since each of these Stokes fields are complex, so they will have both amplitude and phase distributions. The general expression for all three Stokes fields and their corresponding amplitudes and phase distributions are given by

$$S_{ij} = S_i + iS_j = A_{ij}\, e^{i\phi_{ij}},\ \text{i,j} = 1,2,3$$

$$A_{ij} = \sqrt{\left(|S_i|^2 + |S_j|^2\right)} \qquad (2)$$

$$\phi_{ij} = \tan^{-1}(S_j/S_i)$$

The phase singularities of these three complex Stokes fields are called Stokes vortices. Similar to normal phase singularities, these vortices too obey sign rules. These vortices lie at the intersections of zero crossings of S_i and S_j. Around the Stokes vortices $\oint \nabla\phi_{ij} \cdot dl \neq 0$. Since the azimuth of a polarization ellipse is half of the ϕ_{12} Stokes phase, the polarization singularities of the electric field distribution appear as phase vortices in the S_{12} Stokes field phase distribution. The polarization singularities in vector wave fields have attracted much interest and have been extensively studied numerically, theoretically, and as well experimentally. The singularities of the either of the transverse field components can be located in the phase distributions of S_{23} or S_{31} Stokes fields depending on the choice of coordinate system. The $S_{23}(S_{31})$ Stokes field is used for xy-coordinate system (45° rotated - xy coordinate system). The S_{23} and S_{31} Stokes field vortices are also called as Poincare vortices in the literature. At the S_{12} Stokes phase vortices both S_1 and S_2 are zero and $S_3 = \pm 1$. From Eq (1) it can be seen that $S_3 = \pm 1$ correspond to left or right circular components. Similarly at the S_{23} Stokes field phase vortices both S_2 and S_3 are zero and $S_1 = \pm 1$. From Eq (1), $S_1 = \pm 1$ implies that either $E_x = 0$ or $E_y = 0$. Similarly the phase vortices of S_{31} Stokes field correspond to $S_2 = \pm 1$, where S_3 and S_1 are zero. This article is devoted to the generation of Stokes vortices from the interference of three, four and six phase and polarization engineered circularly polarized vector beams.

2 Results and Discussion

Optical lattices are periodic structures of electromagnetic fields generated by interference. These periodic structures can be of intensity, phase, polarization or coherence. There are many phase singularity lattice realizations reported in literature [11-18]. The phase, amplitude and polarization of the interfering beams can be engineered to achieve polarization singularity lattices. In one proposal [19], based on two -beam interference simultaneous modulation of amplitude and phase (using SLM) of each of the two orthogonally polarized beams for polarization lattice generation is suggested. Using Wollaston prism in modified Mach-Zehnder type interferometer [20,21], polarization singularity lattice is experimentally realized. In the literature superposition of three beams with different states of polarization is also used to generate polarization singularity lattice fields [22-24]. More recently periodic array of polarization singularities such as C-points and V-points has been realized by the interference of three or more linearly polarized plane waves with appropriate plane of polarizations [25-33]. This is experimentally achieved by using an S-wave plate or a q-plate, which is made up of segmented half wave plates [34], in which the fast axis in each segment is oriented in different predetermined directions. In all these cases the polarization of the interfering beams are homogeneously linearly polarized. It is desirable to understand their new features. Nevertheless, there is a need to develop new methods, as different methods offer different advantages.

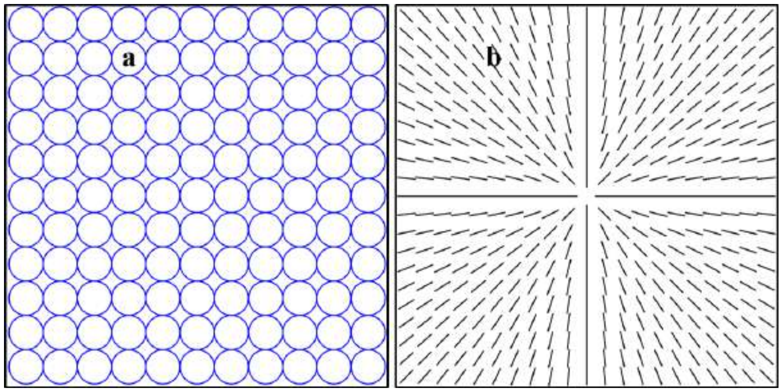

Fig 1. Generation of radially polarized light from right circularly polarized light (RCP) by using a SVQWP: (a) Right circular polarization distribution; (b) Radial polarization distribution.

However, recently it is shown that a spatially varying quarter wave plate (SVQWP) can be used to generate radially or azimuthally polarized light [35] from circularly polarized light. Figure 1 and Fig 2 show the generation of radially and azimuhally polarized light by using SVQWP from right and left circularly polarized light, respectively. The Jones matrix of an m^{th} order spatially varying quarter wave plate can be written as

$$\begin{pmatrix} \cos^2 m\theta + i\sin^2 m\theta & (1-i)\sin m\theta\cos m\theta \\ (1-i)\sin m\theta\cos m\theta & \sin^2 m\theta + i\cos^2 m\theta \end{pmatrix} \tag{3}$$

where, θ is the azimuthal angle. In this work, we show that interference of three, four and six circularly polarized beams can be used to generate lattice fields embedded with arrays of (a) both C-points and V-points, (b) only V-points, and (c) only C-points. First we consider, three circularly polarized beams with appropriate plane of polarizations for the generation of lattice field embedded with both C-points and V-points. The wave vectors corresponding to these interfering beams are as given by Eq (1) of ref 25. All the interfering beams are at same angle from the direction of propagation (here it is z-axis). The interfering circularly polarized beams pass through a spatially varying quarter wave plate, which converts these circularly polarized beams into plane polarized beams. The polarization distribution and resultant intensity distribution corresponding to the three circularly polarized beam interference is shown in Fig 3. The regions of right and left handed polarizations are indicated by blue and red colours, respectively. From Fig 3 it can be seen that the polarization distribution is embedded with both C-points as well as V-points. Interestingly the polarization distribution is found to be embedded with both right and left handed C-points (stars). Figure 4 shows phase distributions of three Stokes fields corresponding to the resultant field as shown in Fig 3. From the Stokes phase distributions it can be seen that only the S_{12} Stokes phase distribution (Fig 4a) is embedded with phase singularities. Phase singularities of charge (–1) and (+2) correspond to C-points (stars) and V-points (type III), respectively.

Next we consider interference of four circularly polarized beams. The propagation vectors of the four plane waves are as given in ref [27]. The interfering circularly polarized beams pass through a SVQWP, which converts these circularly polarized beams into plane polarized beams. The polarization and intensity distribution corresponding to the resultant field of four circularly polarized beam-interference is shown in Fig 5. The resultant intensity distribution is shown as inset in Fig 5. From Fig 5 it can be seen that the polarization distribution is embedded with only V-points. Interestingly the polarization distribution is found to be embedded with both V-points of type III and type IV. The V-points are arranged periodically along the vertices of a square. Stokes phase distributions of three Stokes fields corresponding to the resultant field of Fig 5 are shown in Fig 6. From the Stokes field phase distributions it can be seen that only the S_{12} Stokes field phase distribution (Fig 6 (a)) is infested with phase singularities. Phase singularities of charge (– 2) correspond to V-point of type IV polarization singularity and phase singularity of charge (+2) correspond to V-point polarization singularity of type III, respectively.

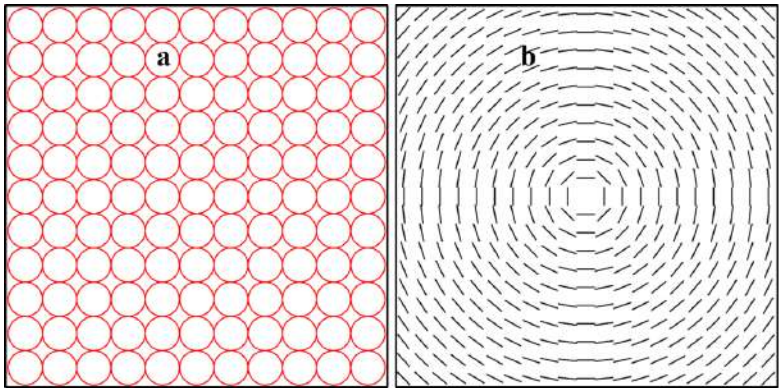

Fig 2. Generation of azimuthally polarized light from left circularly polarized light (LCP) by using a SVQWP: (a) Left circular polarization distribution; (b) Azimuthal polarization distribution.

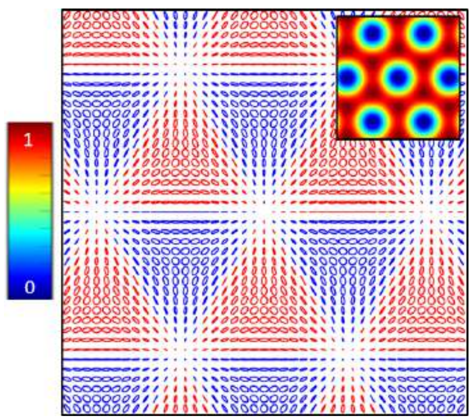

Fig 3. Simulated polarization distribution corresponding to three circularly polarized beam-interference after passing through a SVQWP. Inset shows resultant intensity distribution.

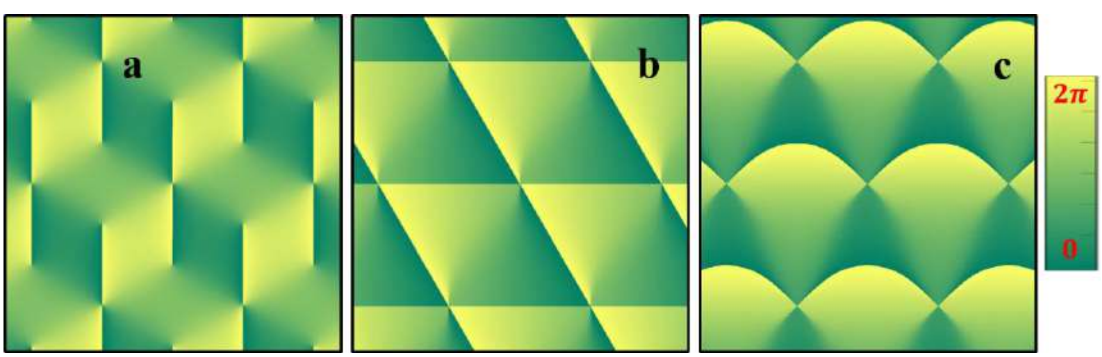

Fig 4. Simulated Stokes phase distributions of resultant field as shown in Fig 3: (a) S_{12}-Stokes field, (b) S_{23}-Stokes field and (c) S_{31}-Stokes field.

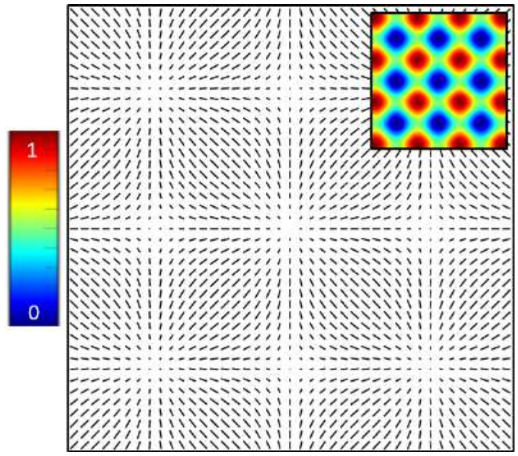

Fig 5. Simulated polarization distribution corresponding to four circularly polarized beam-interference after passing through a SVQWP. Inset shows resultant intensity distribution.

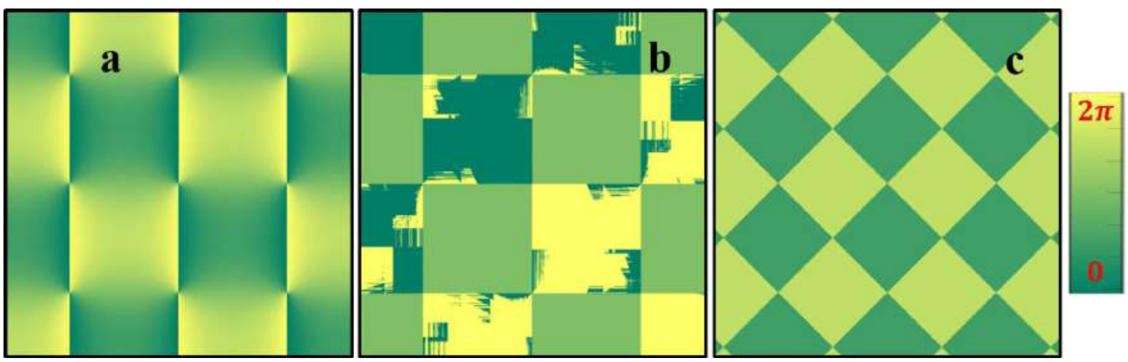

Fig 6. Simulated Stokes phase distributions of resultant field as shown in Fig 5. (a) S_{12} -Stokes field, (b) S_{23}-Stokes field and (c) S_{31} -Stokes field.

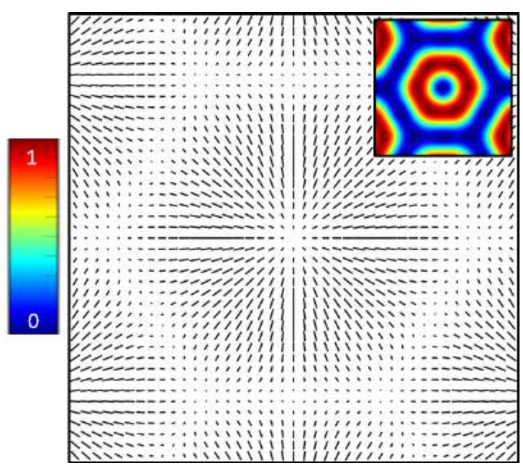

Fig 7. Simulated polarization distribution corresponding to six circularly polarized beam-interference after passing through a SVQWP. Inset shows resultant intensity distribution.

The resultant polarization and intensity distribution corresponding to the interference of six circularly polarized beams after passing through a SVQWP is shown in Fig 7. The propagation vectors of the six plane waves are as given in ref. 27. The polarization distribution in Fig 7 is embedded with only V-points. Similar to four- beam interference in this case also the resultant field is found to be embedded with both positive and negative V-points of same Poincare Hopf index. The three Stokes field phase distributions corresponding to the resultant field of Fig 7 are shown in Fig 8. Only the S_{12} Stokes field phase distribution (Fig 8(a)) is embedded with negative and positive phase singularities of charge (–2) and (+2), respectively. Unlike four beam- interference, in six beam- interference the V-points are arranged periodically at the center as well as vertices of a hexagon.

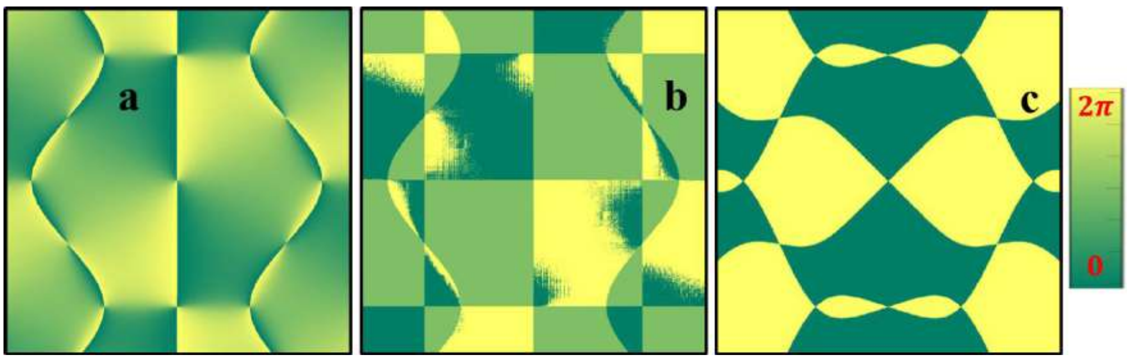

Fig 8. Simulated Stokes phase distributions of resultant field as shown in Fig 7: (a) S_{12}-Stokes field, (b) S_{23}-Stokes field and (c) S_{31}-Stokes field.

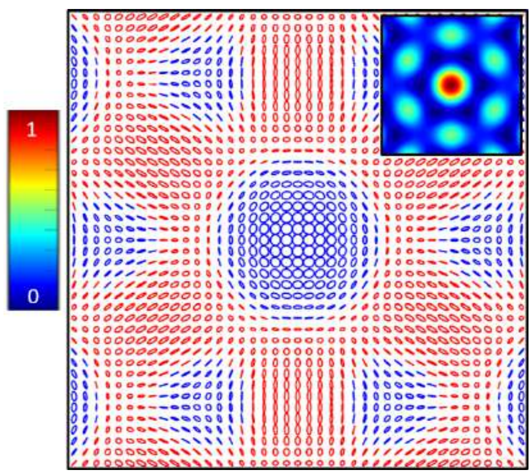

Fig 9. Simulated polarization distribution corresponding to six phase engineered circularly polarized beam interference after passing through a SVQWP. Inset shows resultant intensity distribution. Note that the total initial phase offsets between the interfering beams is 2π.

Next we invoke both phase and polarization engineering technique simultaneously to six circularly polarized beam- interference to generate lattice fields embedded with only C-points. The total phase offsets between the interfering beams is 2π. When the phase offset to the jth interfering beam is ($j2\pi/6$), where $j = 1$ to 6, the polarization distribution of the resultant field happens to be embedded with only C-points as shown in Fig 9. In this lattice an integral C-point ($I_c = +1$) is surrounded by six half integral C-points ($I_c = -1/2$). Interestingly both the integral as well as half integral C-points are of same handedness. The resultant intensity distribution is shown as an inset. The lattice field is very interesting as it contains both integral and half integral C-points. Stokes phase distributions of various Stokes fields for the resultant field as shown in Fig 9 are depicted in Fig 10. Interestingly all the three Stokes fields are embedded with singularities. The S_{12} Stokes field phase distribution (as shown in Fig 10(a)) is embedded with both charge (–1) and (+2) phase vortices, whereas S_{23} and S_{31} Stokes field phase distributions (Fig 10(b) and 10(c)) are populated with vortices of charge (±1).

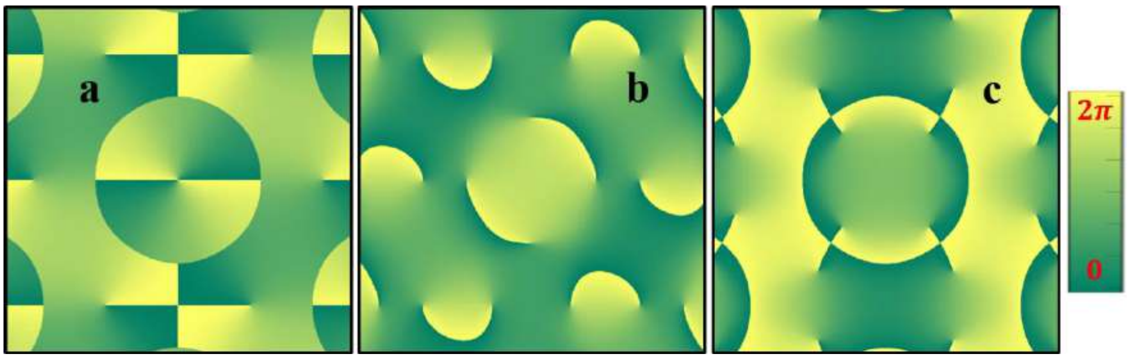

Fig 10. Simulated Stokes phase distributions of resultant field as shown in Fig 9: (a) S_{12}-Stokes field, (b) S_{23}-Stokes field and (c) S_{31}-Stokes field.

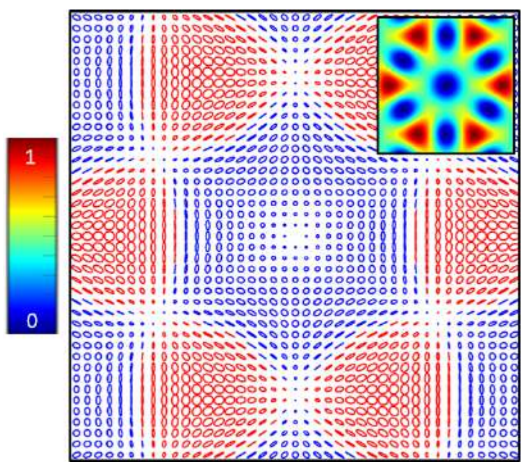

Fig 11. Simulated polarization distribution corresponding to six phase engineered circularly polarized beams interference after passing through a SVQWP. Inset shows resultant intensity distribution. Note that the total initial phase offsets between the interfering beams is 4π.

Figure 11 shows the polarization distribution of the resultant field, when the total initial phase offset between the interfering beams is 4π. Here, the phase offset associated with the jth interfering beam is given by ($j4\pi/6$), where j = 1 to 6. Surprisingly, in this case the polarization distribution of the resultant field is embedded with only half integral C-points. All the half integral C-points are of same handedness. The resultant intensity distribution is shown as an inset. Stokes phase distributions of various Stokes fields for the resultant field as shown in Fig. 11 are depicted in Fig 12. Interestingly all the three Stokes fields are embedded with phase vortices. The S_{12} Stokes field phase distribution (as shown in Fig 12a) is embedded with phase vortices of charge (–1), whereas S_{23} and S_{31} Stokes field phase distributions (Figs 12b and 12c) are populated with phase vortices of charge (±2). These vortices seen in stokes phase distributions are stokes vortices.

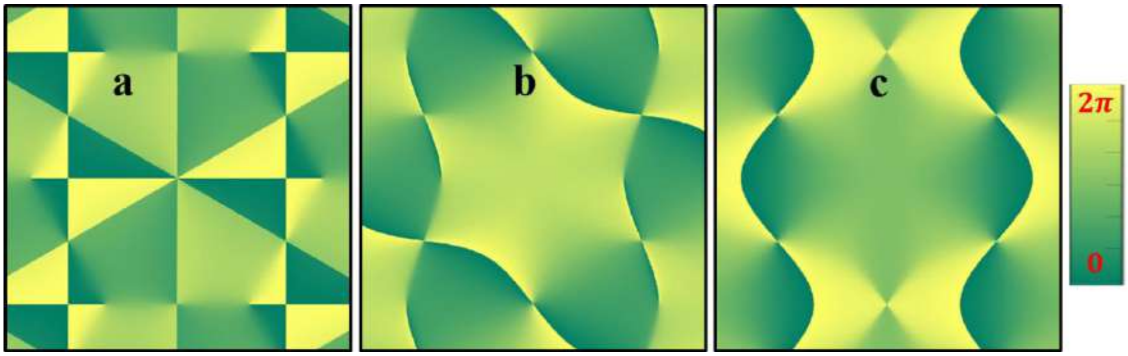

Fig 12. Simulated Stokes phase distributions of resultant field as shown in Fig 11: (a) S_{12} -Stokes field, (b) S_{23}-Stokes field and (c) S_{31}-Stokes field.

3 Conclusion

In conclusion, we have demonstrated the effect of phase and polarization engineering to realize various lattice structures in three, four and six circularly polarized beam- interference. A spatially varying quarter wave plate is used to control the state of polarizations of each of the interfering circularly polarized beams. Lattice fields populated with (a) only V-points, (b) only C-points and (c) both C-points and V-points are generated. The salient features of these lattice fields are discussed. Lattice field that contains all three Stokes vortices is generated by adopting phase-engineering methods.

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