ap

Asian Journal of Physics
Vol. 26, No 2 (2017) 117-124
Available on: www.asianjournalofphysics.in

Holographic display based on complex wavefront modulation

Dezhao Kong[1,2], Xueju Shen[2], Liangcai Cao[1,*], Hao Zhang[1], and Guofan Jin[1]

[1]State Key Laboratory of Precision Measurement Technology and Instruments, Tsinghua University, Beijing, China

[2]Department of Electronics and Optics, Mechanical Engineering College, Shijiazhuang, China

A holographic display method based on complex wavefront modulation is proposed. Complex wavefront modulation is realized by double phases and Fourier filter. The modulated beam by single phase-only spatial light modulator passes through a 4f optical system to synthesize the expected complex modulated wavefront on the output plane, with an appropriate spatial filter in the Fourier plane. Then holographic display is achieved due to complex wavefront modulation. The proposed spatial filter structure is efficient for the realization of complex wavefront modulation with a phase-only modulator. The performance of holographic display is also improved by complex wavefront modulation, compared with the holographic display based on phase-only wavefront modulation. The proposed encoding and display technique is theoretically demonstrated, as well as validated in numerical simulations.

Keywords: holography; computer-generated hologram; complex amplitude modulation.

1 Introduction

Holographic display is a promising and ideal display technique since it can reconstruct the whole optical wave field of a 3-D scene and provide all depth cues. With the development of spatial light modulators (SLMs) and computing technology, holographic display based on computer generated holograms (CGHs) can be capable of a high-resolution display [1] and dynamic 3-D displays [2-4]. A hologram with a complex distribution could yield accurate predictions of the whole diffracted field without paraxial approximation.

An ideal support device for holographic display is a spatial light modulator (SLM) that provides complex wavefront modulation, because any restricted coding domain or operating curve will result in reduced efficiency, noise terms, and complementary diffraction orders. However, the phase and amplitude at each pixel of the SLM cannot be controlled independently and simultaneously. Currently, most holographic display approaches based on CGH are using phase-only liquid crystal SLM, meaning that the original complex amplitude distribution is hard to be reproduced perfectly [5-6]. In order to relieve the display defects, a random phase is employed to be superimposed on the object in encoding process of phase-only CGH, for the reason that random phase effectively distributes the information content over the entire region occupied by the hologram just like a diffuser. Although good quality reconstructions can be still achieved without considering the amplitude information, the perfect holographic display needs the complex wavefront modulation.

So far, in order to realize complex wavefront modulation by single SLM, both device methods [7-10] and encoding methods with a phase-only or amplitude-only SLM [11–21] have been proposed. The device methods, in order to get phase and amplitude modulated interference patterns, include the cascade coupled phase and amplitude modulators [8], two interfered modulated wavefront [9], and common path interferometry [10]. These solutions provide high spatial resolution, but the coupling of the two displays with subpixel accuracy is a technical challenge [11]. In order to avoid the pixel matching problem, the

Corresponding author :
e-mail: clc@tsinghua.edu.cn (Liangcai Cao)

encoding techniques are proposed and optimized. Among the encoding techniques, a representative encoding method is the technique for encoding amplitude information onto a phase-only filter with a single liquid-crystal spatial light modulator [12-13]. Another representative encoding technique is based on double-phase hologram (DPH), which decomposes a complex amplitude element into two pure phase values with constant magnitude. One kind of DPH encoding technique to modulate an arbitrary complex wavefront is based on pixelated phase holograms [14-16]. As the other kind of DPH encoding technique, complex modulation using a phase modulating spatial light modulator and a low-pass spatial filter are proposed and optimized [11, 17-21]. Compared with other encoding techniques, the spatial filter has a better performance due to an enlarged space-bandwidth product (SPB) or a more efficient utilization of SPB.

Therefore, to realize the holographic display based on the complex wavefront modulation, we adopt the encoding technique of spatial filter. In this work, to improve the performance of complex wavefront modulation, the structure of spatial filter is redesigned and optimized. Furthermore, higher quality reconstructed object in arbitrary space is obtained, compared with the reconstruction by SLM directly. The modified encoding method is free of pixel alignment and suppresses the noise. The feasibility of this method is verified and analyzed by numerical simulations.

2 The complex wavefront modulation model

(*1*) *The complex wavefront decomposition*

Commonly, any complex amplitude value $A\exp(j\theta)$ inside the unit circle ($0 < A < 1$) can be decomposed into the sum of two constant magnitude vectors as expressed by

$$P = A\exp(j\theta) = \frac{P_1 + P_2}{2} = \frac{\exp[j(\theta+\alpha) + \exp[j(\theta-\alpha)]}{2} \tag{1}$$

where $\alpha = \cos^{-1} A$, $(0 \le \alpha \le 2\pi)$. Similarly, the two-dimensional amplitude $C(x, y)$ and phase $\varphi(x,y)$ of a complex amplitude $U(x,y)$ can be conveniently rewritten in the form

$$U(x,y) = C(x,y)\exp\{i\varphi(x,y)\} = B\exp\{i\phi_1(x,y)\} + B\exp\{i\phi_2(x,y)\} \tag{2}$$

where $B = A_{max} / 2$ is a constant, A_{max} is the maximum value of $A(x,y)$,

$$\phi_1(x,y) = \theta + \alpha = \varphi(x,y) + \cos^{-1}[A(x,y) / A_{max}] \tag{3}$$

$$\phi_1(x,y) = \theta - \alpha = \varphi(x,y) - \cos^{-1}[A(x,y) / A_{max}] \tag{4}$$

Consequently, the complex field $U(x,y)$ is mainly determined by the phase functions $\phi_1(x,y)$ and $\phi_2(x,y)$, because the constant term B has little influence on the distribution of the field. Therefore, it is apparent that $U(x,y)$ can be retrieved from the coherent superposition of two uniform waves having constant amplitude.

In conventional DPH encoding using low-pass filtering and 4-f system, one data pixel of the output complex wavefront is usually synthesized by two or more neighboring pixels of the input phase modulator. In order to utilize the SBP more efficiently, we adopt the encoding method proposed by Omel Mendoza-Yero *et al* [21], of which complex wavefront can be synthesized from a phase-only hologram with the same size and resolution of complex wavefront. In the previous proposed method, a two-dimensional binary grating (chessboard pattern) and a low-pass filter are also able to obtain superposition of two pure phase elements.

The complex amplitude $U(x,y)$ can be encoded into a phase-only CGH $u(x,y) = \exp\{i\alpha(x,y)\}$ with the same size as $U(x,y)$,

$$u(x,y) = \varphi_1(x,y)\, M_1(x,y) + \varphi_2(x,y)\, M_2(x,y) \tag{5}$$

where $M_1(x,y)$ and $M_2(x,y)$ denote the binary gratings, the discrete form can be expressed as:

$$M_1\,(r\,\Delta x\,,\,t\,\Delta y\,) = \begin{cases} 1 & \mathrm{mod}(r+t\,,\,2)=1 \\ 0 & \mathrm{mod}(r+t\,,\,2)=0 \end{cases} \qquad (0<r\le m, 0<t\le n) \tag{6}$$

$$M_2\,(r\,\Delta x\,,\,t\,\Delta y\,) = \begin{cases} 0 & \mathrm{mod}(r+t\,,\,2)=1 \\ 1 & \mathrm{mod}(r+t\,,\,2)=0 \end{cases} \qquad (0<r\le m, 0<t\le n) \tag{7}$$

where m, n denote the size of the encoded complex amplitude $U(x,y)$, Δx , Δy denote the pixel intervals of the filed $U(x,y)$, mod( ) denotes the operation of taking the remainder. It is apparent that the functions M_1 (x,y) and M_2 (x, y) hold for the transmittance of complementary two-dimensional binary gratings (chessboard patterns) taken at the Nyquist limit, such as $M_1\,(x,y) + M_2\,(x,y) = 1$. When a low-pass filter $P(u,v)$ is applied to block all diffraction orders except the zero one, the spectrum in the frequency plane is reduced to the expression [21],

$$H(u,v)\;P(u,v) = \frac{1}{2}\,F\;\{U(x\;,y)\} \tag{8}$$

where $H\,(u,v)$ is Fourier transform of u(x, y) , $F\{\ \}$ denotes the Fourier transform. Thus the complex amplitude $U(x,y)$ is successfully expressed by the a phase-only CGH $u(x, y)$.

(*2*) *Optical realization of complex wavefront modulation*

A 4-f optical system with a low-pass filter placed in the Fourier plane is employed to realize the complex amplitude modulation, combining two phase holograms in Fourier plane and reconstructing complex amplitude in the output plane, shown as Fig. 1. The 4-f imaging system is made up of a couple of identical refractive lenses with the same focal length. In the Fourier plane, the beam is transmitted through a low-pass spatial filter. Complex wavefront is obtained in the output plane of the system.

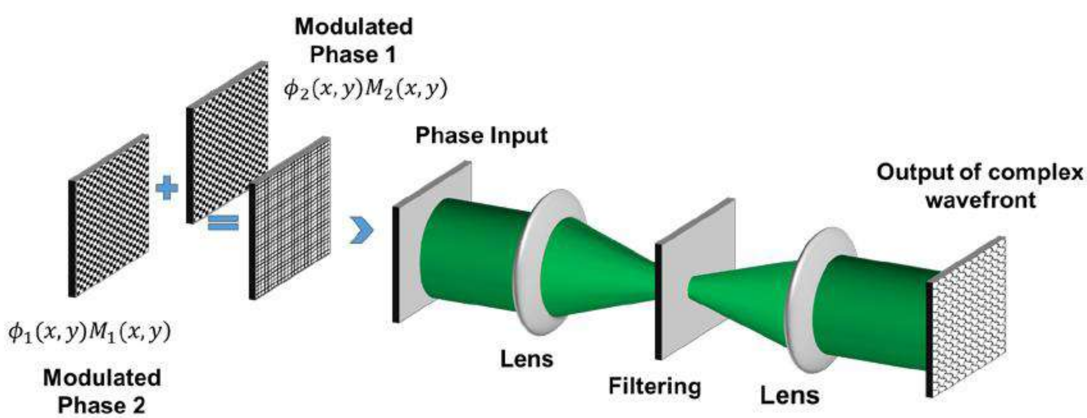


Fig 1. Optical system of complex modulation

(*3*) *Filter Design*

The low-pass spatial filters reported for generating complex wavefront are commonly rectangle aperture [17-21]. The selection of appropriate filter has a great influence on the complex wavefront. In order to achieve more accurate complex wavefront and suppress the noise, a new shape of filter is designed for the system.

The spatial filter in the Fourier plane is applied to shape spectral density functions of phase-only CGH to be similar to the spectral density of the complex modulation. The filter can be designed by the similarity of the spectra distribution. Therefore, the redesigned spatial filter $H_{\mathrm{SF}}(u, v)$ is defined by the ratio of two spectral density functions,

$$H_{\mathrm{SF}}(u,v) = \begin{cases} 1 & p(u,v) > \rho \\ 0 & p(u,v) < \rho \end{cases} \qquad (0 < \rho < 1) \tag{9}$$

$$P(u,v) = \frac{|F\{U(x,y)\}|}{|F\{u(x,y)\}|} \tag{10}$$

where ρ denotes threshold value, transform. | | denotes the operation of absolute value, $F\{\ \}$ denotes the Fourier transform. Since the filter $p(u,v)$ is generated by the ratio of the ideal and the realistic spectral density functions, the spectral density of the phase modulation is converted to the ideal spectral density function in the case where the filter is used in the Fourier plane [11]. We simplify the filter $p(u,v)$ to make it easy to be used in practice. Therefore, it is convenient to transfer spectral density functions of phase modulation into the spectral density of the complex modulation by the spatial filter $H_{\mathrm{SF}}(u,v)$.

3 Verification of complex wavefront modulation

In order to demonstrate the performance of complex modulation of the proposed method, the complex wavefront $U(x,y)$ is proposed with amplitude of encoded image "elaine" (Fig 2(a)) and phase of image "Lena" (Fig. 2(b)). The sizes of two images are both 1024× 1024 pixels. The two binary gratings $M_1(x,y)$ and $M_2(x, y)$ holding for the transmittance of complementary are chessboard patterns, shown as Fig. 2(c) and Fig. 2(d). The phase-only CGH $u(x,y)$ encoded by the proposed method is shown as Fig. 2(e).

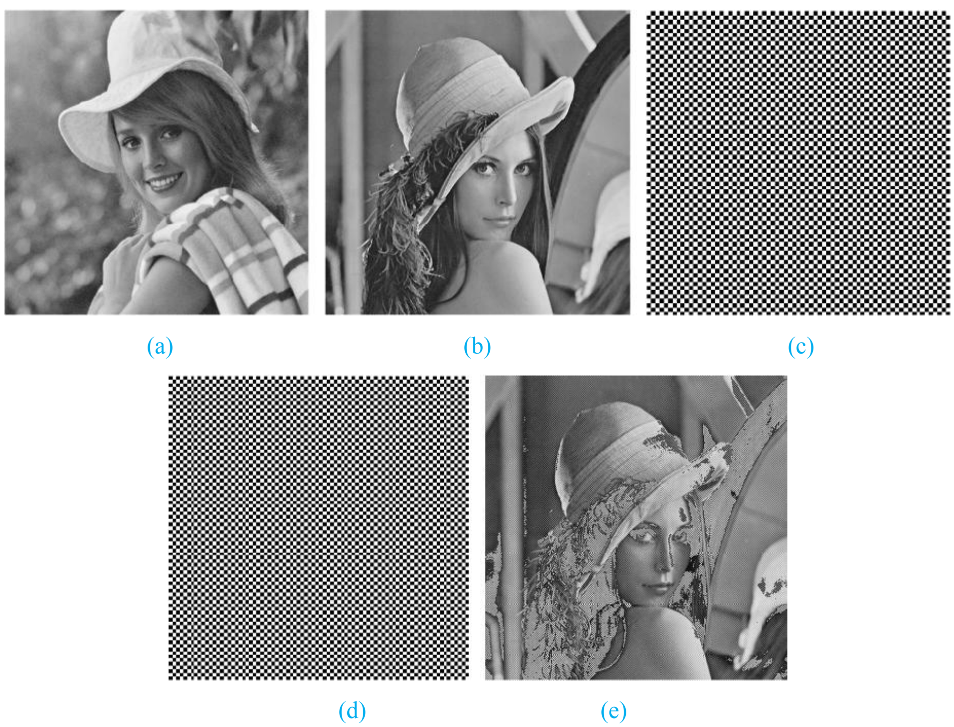
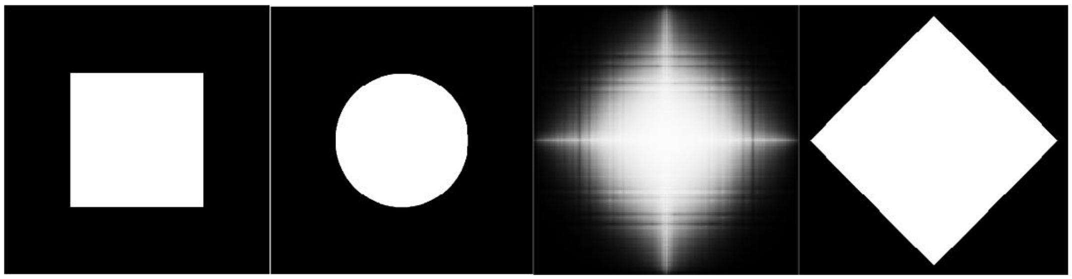


Fig 2. Simulation results (a) the amplitude of complex wavefront $U(x,y)$, (b) the phase of complex wavefront $U(x, y)$, (c) binary gratings $M_1(x, y)$, (d) binary gratings $M_2(x,y)$, (e) phase-only CGH $u(x,y)$ encoded

The common low-pass spatial filters mainly include the rectangle aperture and circular aperture, shown in Fig 3(a) and Fig 3(b). According to our method, the new filter $p(u,v)$ and the more practical filter $H_{SF}(u,v)$ are shown as in Fig 3(c) and Fig 3(d).



(a) (b) (c) (d)

Fig 3. Filters (a) rectangle aperture (b) circular aperture (c) filter $p(u,v)$ (d) filter $H_{SF}(u,v)$

By the modulation of binary gratings $M_1(x,y)$ and $M_2(x,y)$ and low-pass spatial filter $H_{SF}(u,v)$, the reconstructed complex wavefront produced from the phase-only CGH $u(x,y)$ is shown in Fig 4. Figure 4 (a) represents the reconstructed amplitude and Fig 4 (b) represents the reconstructed phase. The numerical results demonstrate that the amplitude and the phase of the optical field can be reconstructed successfully with high image quality. Thus, independent modulation of amplitude and phase can be achieved simultaneously by encoded phase-only hologram.

(a) (b)

Fig 4. Reconstructed complex wavefront, (a) amplitude, (b) phase

In order to assess qualities of reconstructed complex wavefront and characters of the different filters, the root-mean-square error between the recovered information $A'(x,y)$ and the original information $A(x,y)$ is introduced as

$$RMSE = \sqrt{\frac{\sum_{m=1}^{M} \sum_{n=1}^{N} [|A'(x,y)| - |A(x,y)|]^2}{M \times N}} \tag{10}$$

where $M \times N$ is the size of the image. The RMSEs of amplitude in complex wavefront with circular aperture filter, rectangle aperture and our filter are 9.4731×10^{-4}, 9.0551×10^{-4} and 6.3338×10^{-4}, respectively. The RMSEs of phase in complex wavefront with circular aperture filter, rectangle aperture and our filter are 3.7917×10^{-4}, 5.9985×10^{-4} and 2.9314×10^{-4}, respectively. It can be seen that our proposed filter has better performance for complex modulation.

4 The holographic display by complex wavefront modulation

The holographic display system by complex wavefront modulation is shown in Fig 5. The encoded CGH is reconstructed with a phase-only SLM. The pixel pitch of the SLM is set as 6.4 μm, and the working wavelength of laser is set as 532 nm. To demonstrate the feasibility of the proposed approach, wavefront $U(x, y)$ with amplitude of encoded image “elaine” (Fig 2(a)) and phase of image “Lena” (Fig 2(b)) is also used as the object of holographic display. The transmission wavefront $G(x,y)$ (shown as Fig 6(a)) is calculated by the angular spectrum propagation algorithm with the propagation distance z. Then the encoded phase-only CGH $g(x,y)$ (shown as Fig 6(b)) generated from the $G(x,y)$ can be calculated. Based on the holographic system, the amplitude and phase of reconstructed complex wavefront $U'(x,y)$ are shown as Fig 6(c) and Fig 6(d), respectively. The RMSEs of amplitude and phase between the recovered wavefront $U(x,y)$ and the original wavefront $U'(x,y)$ are 1.0576 and 0.7095, respectively. It can be seen that the reconstructed information is similar with the original wavefront. The reconstructed quality can satisfy the needs of holographic display.

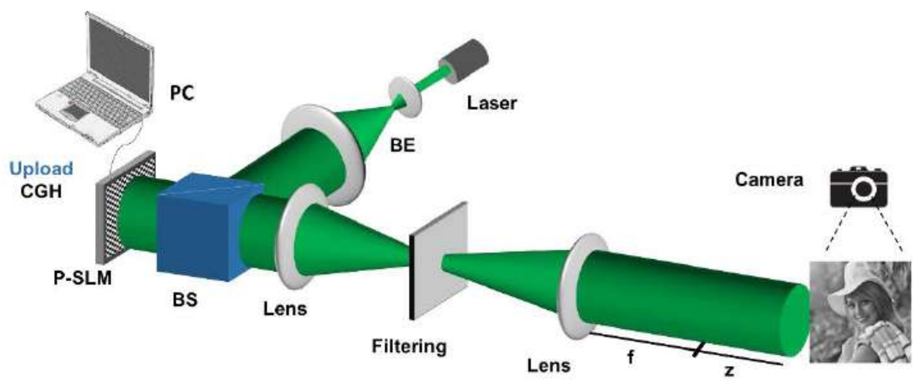


Fig 5. holographic display system by complex wavefront modulation

(a) (b) (c) (d)

Fig 6. Simulation results (a) The transmission wavefront $G(x,y)$, (b) encoded phase-only CGH $g(x,y)$, (c) amplitude of reconstructed complex wavefront $U'(x,y)$, and (d) phase of reconstructed complex wavefront $U'(x,y)$.

5 Conclusion

In this paper, a complex wavefront modulation technique is introduced into holographic display to improve display performance. An optimized filter is proposed to suppress the noise and improve the accuracy of the complex modulation. The object image caculated by the angular spectrum propagation algorithm can be reconstructed successfully at arbitrary distance behind the 4-f system. Given single-pixel mathematical expression of the wavefront, instead of using an array of subpixels to express each pixel of the input plane, the proposed method employs SBP more efficiently. Besides, for the sake of its analyticity, the proposed encoding method, avoiding the iteration calculation, can be more suitable for dynamic holography display. The holographic display by complex wavefront modulation can also avoid the problems caused by the phase-modulation directly and offer improved visual effects.

Acknowledgements

This work is supported by the National Basic Research Program of China (No. 2013CB328801), the National Natural Science Foundation of China (No. 61275013, 61505095, 61361160418, 61327902), and the Innovation Method Fund of China (No. 2015IM020500).

Reference

1. Wakunami K, Yamaguchi M, Javidi B, "High-resolution threedimensional holographic display using dense ray sampling from integral imaging," *Opt Lett*, 37(2012)5103-5105.
2. Paturzo M, Memmolo P, Finizio A, Nasanen R, Naughton T J, Ferraro P, "Synthesis and display of dynamic holographic 3D scenes with real-world objects," *Opt Express*, 18 (2010)8806-8815.
3. Onural L, Yaras F, Kang H, "Digital holographic three dimensional video displays," *Proc IEEE*, 99(2011)576-589.
4. Ichihashi Y, Nakayama H, Ito T, Masuda N, Shimobaba T, Shiraki A, Sugie T, "HORN-6 special-purpose clustered computing system for electro-holography," *Opt Express*, 17(2009)13895-13903.
5. Shiraki A, Takada N, Niwa M, Ichihashi Y, Shimobaba T, Masuda N, Ito T, "Simplified electroholographic color reconstruction system using graphics processing unit and liquid crystal display projector," *Opt Express*, 17 (2009) 16038-16045.
6. Kim H, Hahn J, Lee B, "Mathematical modeling of triangle mesh-modeled three-dimensional surface objects for digital holography," *Appl Opt*, 47(2008)D117-D127.
7. Gregory D A, Kirsch J C, Tam E C, "Full complex modulation using liquid-crystal televisions," *Appl Opt*, 31(1992)163-165.
8. Neto L G, Roberge D, Sheng Y. "Full-range, continuous, complex modulation by the use of two coupled-mode liquid-crystal televisions," *Appl Opt*, 35(1996)4567-4576.
9. Tudela R, Labastida I, Vallmitjana S, Juvells I, Carnicer A, "Full complex Fresnel holograms displayed on liquid crystal devices," *J Opt A: Pure and Appl Opt*, 5(2003)189-194.
10. Reichelt S, Häussler R, Fütterer G, Leister N, Kato H, Usukura N, Kanbayashi Y, Full-range, complex spatial light modulator for real-time holography, *Opt Lett*, 37(2012)1955-1957.
11. Sarkadi T, Kettinger A, Koppa P, "Spatial filters for complex wavefront modulation", *Appl Opt*, 52(2013)5449-5454.
12. Davis J A, Cottrell D M, Campos J, Yzuel M J, Moreno I, "Encoding amplitude information onto phase-only filters," *Appl Opt*, 38(1999)5004-5013.
13. Arrizón V, Ruiz U, Carrada R, González L A, "Pixelated phase computer holograms for the accurate encoding of scalar complex fields," *J Opt Soc Am A*, 24(2007)3500-3507.
14. Hsueh C K, Sawchuk A A, Computer-generated double-phase holograms, *Appl Opt*, 17(1978)3874-3883.
15. Arrizón V, "Complex modulation with a twisted-nematic liquid-crystal spatial light modulator: double-pixel approach," *Opt Lett*, 28(2003)1359-1361.
16. Arrizón V, "Optimum on-axis computer-generated hologram encoded into low-resolution phase-modulation devices," *Opt Lett*, 28(2003)2521-2523.
17. Cohn R W, Liang M, "Approximating fully complex spatial modulation with pseudorandom phase-only modulation," *Appl. Opt*, 33(1994)4406-4415.
18. Birch P, Young R, Budgett D, Chatwin C, "Dynamic complex wave-front modulation with an analog spatial light modulator," *Opt Lett*, 26(2001)920-922.

19. Florence J M, Juday R D, "Full complex spatial filtering with a phase mostly DMD," *Proc SPIE*, 1558(1991)487-498.
20. Göröcs Z, Erdei G, Sarkadi T, Ujhelyi F, Reményi J, Koppa P, Lőrincz E, "Hybrid multinary modulation using a phase modulating spatial light modulator and a low-pass spatial filter," *Opt Lett*, 32(2007)2336-2338.
21. Mendoza-Yero O, Mínguez-Vega G, Lancis J, Encoding complex fields by using a phase-only optical element, *Opt Lett*, 39(2014)1740-1743.

[*Received*: 16.1.2017; *accepted*: 29.1.2017]