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Introduction to optical signal processing

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Dedicated to Prof Kehar Singh

This paper presents an overview of recent developments in the area of linear optical signal processing (OSP) under the paraxial approximation. The availability of high quality spatial light modulators (SLM) and high resolution digital cameras make it more feasible to implement many OSP functions. Some fundamental concepts concerning basic Fourier analysis and functions are described for such systems. Then the Fourier transform (FT), the optical Fourier transform (OFT), the discrete Fourier transform (DFT), the fractional Fourier transform (FRT), the Fresnel transform (FST), and the general case of the linear canonical transform (LCT) are discussed. In particular two significant applications of OSP, i.e. optical encryption systems and digital watermarking techniques, are reviewed including basic optical systems and numerical algorithms. Some publications indicating how these concepts have been successfully applied over many years by Kehar Singh *et al* are referenced.

1 Overview of the Fourier optical signal processing

Optical signal processing is a broad engineering discipline that is as rich in theoretical physics as it is in application. During the last decade, much progress in Fourier optical processing (OSP) techniques has been reported with particular focus on information security systems. This has received increased interest as a result of new optical formulations that can be implemented using spatial light modulators (SLM) and digital cameras (CCD). In this paper a general overview of the basic concepts and underpinning of Fourier optical signal processing and its significance for information security applications are provided. Typically it is assumed that the paraxial approximation holds. It is assumed only rays or plane waves traveling at small angles relative to the principle axis of the optical system are retained in the analysis, i.e. only they contain significant power and information [1–4].

Generally, by Fourier optical signal processing (OSP), we imply "the intentional manipulations of a two dimensional signal, i.e. an image, to modify that image, or some process based on that image (e.g. image reconstruction), or recognition of that image by an autonomous system" [5]. Fourier OSP goes back at least to the work of Abbe's theory in 1893 [2, 6]. Image formation in the microscope was described and an investigation of the resulting implications was verified. However, OSP as distinct from imaging truly emerged in the 1960s. In 1964, VanderLugt introduced an optical scheme for convolution/correlation based on two optical systems performing the Fourier transform with a Fourier plane filter mask between them [7]. This architecture is referred to as a *frequency plane correlator*, which realizes the most important shift-invariant operations in signal/image processing, used for filtering and pattern recognition. A correlation between two functions, $f_1(x)$ and $f_2(x)$, is performed based on the auto correlation theorem and the property of a lens using coherent light. The correlation function, $C(x)$, can be represented as follows:

$$C(x) = FT^{-1}\{\widehat{f_1}(k_x) \times \widehat{f_2}^{*}(k_x)\}, \tag{1}$$

where $FT^{-1}\{–\}$ denotes the inverse Fourier transformation (FT) and '*' is the complex conjugation operation. $\widehat{f_1}(k_x)$ and $\widehat{f_2}(k_x)$ are the Fourier transforms of the functions $f_1(x)$ and $f_2(x)$, respectively, and the operator $\widehat{\ }$ indicates a signal after transformation throughout this paper. For brevity we frequently limit the representation of such systems to the 1D case, with the space domain coordinates x replacing (x, y) and the spatial frequency domain coordinates x instead of (k_x, k_y). A two dimensional (2D) *frequency plane correlator* is illustrated in Fig 1. The correlation between the input signal $f_1(x, y)$ and reference function $f_2(x, y)$ produces $C(x, y)$

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in the output plane. The complex conjugate of the FT of $f_2(x, y)$ i.e. the reference function, is positioned in the "filter plane" and can be expressed as a complex function having an amplitude and phase:

$$\widehat{f_2} = |\widehat{f_2}| \times \exp\{i\varphi\}. \tag{2}$$

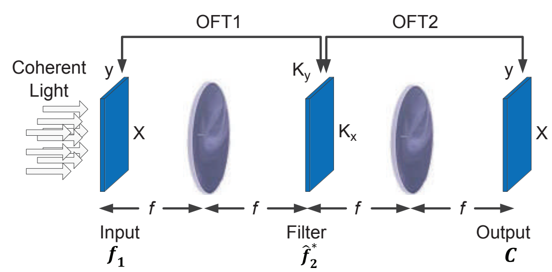

Fig 1. The basic Fourier optical processor setup, i.e. *frequency plane optical correlator*

The holography method [8, 9] can be used in order to create such a complex function on a photographic medium. Complex field data can also be captured digitally using a Charge-coupled device (CCD) [10–13]. This involves interferencing the object beam with a reference beam and capturing the patterns. In this setup, the input illumination must be coherent light, i.e. from a laser, and the processing is realized fully optically in parallel. We note that this paraxial optical system contains two analog coherent Fourier optical signal processing implementations, OFT1 and OFT2 [1]. Each is capable of performing a two-dimensional Fourier transform (FT) on an input signal using a thin convergent lens. We also note that if $\widehat{f_2}$ were replaced by a simple pupil function, i.e. an aperture, this OSP system would operate as an imaging system. Since both lenses are identical there is no magnification in the system.

The availability of high quality spatial light modulators (SLM) and high resolution CCD cameras have made it technologically more feasible to perform many optical signal processing functions and created new possible avenues of research in this field [14]. Under the paraxial approximation of scalar direction theory, the effects on coherent light propagating through every linear lossless shift invariant (discussed later in Section 2.1) OSP system can be described using the linear canonical transform (LCT) [15]. The LCT can be interpreted by examining the evolution of the Wigner Distribution Function (WDF) [16, 17] of the signal as it propagates through the optical system. The WDF represents a 1D complex field as a 2D real function in phase space, where the power of the signal is defined simultaneously at each position and spatial frequency coordinate [18]. As the FT transforms a 1D signal in the space domain into a 1D signal in the spatial frequency domain, so the signal is formed into different mixed domains using the LCT [19]. This generalization opens new perspectives for optical signal processing. We note that the Fourier transform (FT), the fractional Fourier transform (FRT), and the Fresnel transform (FST) are all special cases of the linear canonical transform (LCT). Thus, Fourier OSP has been expanded with more sophisticated signal processing tools and applications using different optical LCT implementations [20].

Fast algorithms to compute the LCT have been developed in order to simulate paraxial optical systems [21]. In such systems, a SLM, i.e. a liquid crystal display, is often incorporated which may be used to modulate the input digital data onto a coherent wave field. In particular, two OSP applications, i.e. optical encryption systems and digital watermarking techniques, offer the possibility of high speed parallel encryption and security of two-dimensional image data. SLMs can be used in such security systems in order to encode the input signals and provide the encryption key during both the encryption and decryption processes. The complex field (amplitude and phase) at the output plane can be obtained by recording the intensity pattern using a CCD camera and applying digital holographic techniques which allow the processing of the data by computer. In order to facilitate an understanding of the contents, some basic theoretical OSP analysis and its applications are provided. A general introduction to both optical encryption and watermarking techniques are presented. This paper is organized as follows: In Section 2, we present some fundamental concepts of (i) the linear lossless shift invariant systems; (ii) the Fourier transform (FT), both its optical and numerical implementations; (iii) the fractional Fourier transform (FRT); (iv) the Fresnel transform (FST) which describes the effects of paraxial propagation; and (v) the linear canonical transform (LCT). Section 3 and 4 introduce optical encryption systems and digital watermarking techniques in general.

2 Theoretical Analysis

Fourier optics is a wave (physical optics) theory of optical systems involving thin lenses separated by sections of free space. Such systems are typically described using the Fourier transform (FT) or Fresnel transform (FST) [1]. The paraxial wave assumption and the thin lens approximations serve as the two underlying operational assumptions, allowing the description of the coherent field distribution propagation between two planes separated by homogeneous free space, using the FST, or between the front and back focal planes of a thin lens using the FT. As noted the optical field distributions at the input and output planes of any optical system consisting of such lenses separated by sections of free space can be described using the LCT [15]. In the context of OSP applications in optical encryption and digital watermarking, it is crucially important to introduce the reader to some fundamental concepts concerning basic Fourier analysis tools and other transformations. Later the fractional Fourier transform (FRT), the Fresnel transform (FST), and the linear canonical transform (LCT) are briefly introduced.

2.1. Linear Systems

Generally, the system discussed may be described as some physical process that maps an input signal to an output signal or as physical implementations of mathematical transformations [15]. In this paper we note that all optical systems are approximated using ideal lossless linear transforms. As noted this class of systems can be referred to as *Linear Shift Invariant Systems* (*LSI*) [22]. In this section the properties of such systems are briefly discussed.

Linear systems satisfy the properties of superposition and scaling. For instance, two signals, if $f_1(x)$ and $f_2(x)$, are input into a linear system H, then:

$$H\{\alpha f_1(x) + \beta f_2(x)\} = \alpha H\{f_1(x)\} + \beta H\{f_2(x)\}, \tag{3}$$

where the operator $H\{-\}$ denotes the effect of the system and α, β are two scalar values. More generally, linearity ensures the following property for multiple input signals:

$$H\left\{\sum_i \alpha_i f_i(x_{in})\right\}(x_{out}) = \sum_i \alpha_i H\{f_i(x_{in})\}(x_{out}). \tag{4}$$

$H\{f(x_{in})\}(x_{out})$ is the output of the system H in domain x_{out} for an input signal f in domain x_{in}. This type of operator notation will be applied throughout this paper to denote the operation of systems or transformations. Therefore, the output of a linear system can be treated as being the superimposed responses to a set of independent inputs.

A system is shift invariant if and only if a shift (translation) in the input domain of the input signal causes the same shift in the resulting output signal. In other words, some movement of the input signal will not bring about a change in the output signal other than a corresponding shift and/or the addition of a well defined phase term to the output wavefield. This property is used in the following sections when discussing the operation of various transformations.

Linear Shift Invariant Systems (*LSIs*) are often characterized by their "impulse response" function. This is the output of the system for an input impulse, and such an input impulse is usually described using a Dirac delta functional $\delta(x)$. Mathematically, any value of a function $f(x)$ can be extracted (sifted or sampled) as follows:

$$f(x) = \int_{-\infty}^{\infty} \delta(x - x') f(x') dx', \tag{5}$$

Application of a periodic series of shifted deltas, i.e. a comb function, can be used to sample a continuous function. $\delta(x)$ has several important properties. It can be viewed as a function on the real line which is zero everywhere except at the origin:

$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & x \neq 0 \end{cases} \tag{6}$$

and it has the property that:

$$\int_{-\infty}^{\infty} \delta(x)dx = 1. \tag{7}$$

Using $\delta(x)$ we can discretely approximate any input as being made up of an infinite number of approximately shifted and modulated impulses. The impulse response function, typically denoted by $h(x)$, is the output of the system for an input $\delta(x)$. Adding together all the output responses, the output signal $f_{out}(x)$ can be determined for a given input $f_{in}(x')$ as follows:

$$\begin{aligned} f_{out}(x) &= \int_{-\infty}^{\infty} h(x - x') f_{in}(x')dx' \\ &= f_{in}(x) * h(x), \end{aligned} \tag{8}$$

where $*$ denotes the convolution operation [1]. The impulse 2D response $h(x, y)$ is called the *point spread function* of the optical systems. A *Linear Shift Invariant System* is completely defined given its impulse response function.

2.2. Fourier Transform (*FT*)

Generally, transformations are mappings from one representation or domain to another, i.e. involve changing the coordinate system or the basis set. According to Parseval's theorem for the FT, energy is conserved and the transformation is unitary. Therefore the total energy of the signal remains constant regardless of the representation (space or spatial frequency) [23]. Fourier transforms (FT) provide an important mathematical tool for describing optical signal processing (OSP) using linear systems. We now discuss the FT and its properties as a means with which to introduce and compare some less well known transformations that are used frequently throughout the analysis. We recall that in general we will present a 1D analysis. However, in this section a full 2D analysis is presented to emphasis that we are dealing with 2D image data processing.

The normalized FT of a function f of two independent spatial variables x and y (i.e. in the 2D space domain) may be defined as follows:

$$\widehat{f}(k_x, k_y) = \mathrm{FT}\{f(x, y)\}(k_x, k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\exp\{-j2\pi(k_x x + k_y y)\}dxdy, \tag{9}$$

where FT{–} denotes the FT operation [1]. $\widehat{f}(k_x, k_y)$ is a complex-valued function of two independent spatial frequency k_x and k_y (i.e. in the spatial frequency or Fourier domain). Similarly, the inverse Fourier transform (IFT) of a function $\widehat{f}(k_x, k_y)$ completely recovers $f(x, y)$:

$$\begin{aligned} f(x, y) &= \mathrm{FT}^{-1}\{\widehat{f}(k_x, k_y)\}(x, y) \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \widehat{f}(k_x, k_y) \exp\{j2\pi(k_x x + k_y y)\}dk_x dk_y, \end{aligned} \tag{10}$$

where FT^{-1} {–} denotes the IFT operator.

The FT has a number of properties that allow us to characterize its behavior. We present eight of the basic mathematical properties of the FT [1]. These properties include:

(i) *Linearity property:*

Since the FT is linear,

$$\mathrm{FT}\left\{\sum_i \alpha_i f_i(x)\right\}(k_x) = \sum_i \alpha_i\, \mathrm{FT}\{f_i(x)\}(k_x) = \sum_i \alpha_i \hat{f}_i(k_x) \tag{11}$$

the FT of a sum of two (or more) functions is equal to the sum of the individual FTs

(ii) *Scaling* (*similarity*) *property:*

Stretching the coordinates in the space domain x results in a contraction of the coordinates in the corresponding spatial frequency domain k_x, and a scaling by a constant factor of the amplitude of the spectrum. Likewise, compression or squeezing in the space domain produces stretching (scaling) in the spatial frequency domain.

$$\mathrm{FT}\{f(ax)\}(k_x) = \frac{1}{|a|}\,\mathrm{FT}\{f(x)\}\left(\frac{k_x}{a}\right) = \frac{1}{|a|}\hat{f}\left(\frac{k_x}{a}\right) \tag{12}$$

This property is important when relating the FT to the optical Fourier transform (OFT), and is frequently applied to waveforms and spectra.

(iii) *Shift property:*

This relates the translation in the space domain to a linear phase shift in the spatial frequency domain and vice versa.

$$\mathrm{FT}\{f(x - x_0)\}(k_x) = e^{-j2\pi x_0 k_x}\hat{f}(k_x)$$

$$\mathrm{FT}\{e^{j2\pi x k_{x_0}} f(x)\}(k_x) = \hat{f}(k_x - k_{x_0}) \tag{13}$$

Clearly the FT is shift invariant.

(iv) *Conjugation property:*

This describes the Fourier transform of the complex conjugate of a signal:

$$\mathrm{FT}\{f^*(x)\}(k_x) = \hat{f}^*(-k_x) \tag{14}$$

(v) *Correlation Theorem:*

This states that the autocorrelation and the energy density function of a signal are a Fourier transform pair, i.e. the autocorrelation function of a signal is the FT of its power spectrum.

$$\text{Cross-correlation: FT}\left\{\int_{-\infty}^{\infty} f^*(y)\, g(x+y)dy\right\}(k_x) = \hat{f}^*(k_x) \times \hat{g}(k_x)$$

$$\Downarrow f = g$$

$$\text{Auto-correlation: FT}\left\{\int_{-\infty}^{\infty} f^*(y)\, f(x+y)dy\right\}(k_x) = |\hat{f}(k_x)|^2 \tag{15}$$

(vi) *Convolution Theorem:*

It relates the FT of the convolution of two functions to the product of their individual FTs. Correspondingly, multiplying two functions in the space domain is equivalent to convolving their FTs.

$$\mathrm{FT}\{f(x)*g(x)\}(k_x) = \mathrm{FT}\{f(x)\}(k_x) \times \mathrm{FT}\{g(x)\}(k_x)$$

$$\mathrm{FT}\{f(x) \times g(x)\}(k_x) = \frac{1}{2\pi}\,\mathrm{FT}\{f(x)\}(k_x) * \mathrm{FT}\{g(x)\}(k_x) \tag{16}$$

Significantly the resulting spatial frequency distribution will therefore have a bandwidth equal to the sum of the bandwidths of the two signals.

(vii) *Duality* (*Cyclic behavior*) *property:*

Two successive applications of the FT are equivalent to flipping the coordinates of the original signal, while four successive applications of the FT is equivalent to the identity operation.

$$\mathrm{FT}\{\mathrm{FT}\{f(x)\}(k_x)\}(x) = f(-x)$$
$$\mathrm{FT}\{\mathrm{FT}\{\mathrm{FT}\{\mathrm{FT}\{f(x)\}(k_x)\}(x)\}(k_x)\}(x) = f(x) \tag{17}$$

(viii) *Parseval's Theorem:*

This states that application of the FT preserves the energy of the original quantity.

$$\int_{-\infty}^{+\infty} |f(x)|^2\, dx = \int_{-\infty}^{+\infty} |\widehat{f}(k_x)|^2\, dk_x. \tag{18}$$

In Table 1 we summarize these properties and theorems (Note that the table contain the following two Fourier transform pairs: $\mathrm{FT}\{f(x)\} = \widehat{f}(k_x)$ and $\mathrm{FT}\{g(x)\} = \widehat{g}(k_x)$. We limited ourselves to the 1D case for the sake of brevity). The FT transforms a mathematical function of space (or time) into a new function of spatial frequency (or frequency). The FT can also be interpreted in phase space [1]. It has been shown that application of the FT has the effect of rotating a signal's Wigner distribution function through $\pi/2$ radians. Therefore, four applications of the FT are equivalent to a 2π rotation or the identity operation, see Eq (17). The FT has become a widely used tool with many engineering applications, especially in the area of signal processing, in part due to the existence of the fast Fourier transform (FFT) algorithm [24]. The FFT can be used to rapidly compute a signal's discrete Fourier transform (DFT) with the number of calculations reduced from N^2 to $N\log N$, where N is the number of samples of the input signals. This will be discussed later in Section 2.4.

Table 1. Some important properties of the Fourier transform (FT).

Property	**Fourier Transform (FT)**
Linearity	$\mathrm{FT}\{\alpha f(x) + \beta g(x)\} = \alpha\mathrm{FT}\{f(x)\} + \beta\mathrm{FT}\{g(x)\}$
Scaling (Similarity)	$\mathrm{FT}\{f(ax)\} = \frac{1}{\lvert a\rvert} \lvert \widehat{f}\left(\frac{k_x}{a}\right)$
Shift	$\mathrm{FT}\{f(x - x_0)\} = e^{-j2\pi x_0 k_x}\widehat{f}(k_x)$ $\mathrm{FT}\{e^{j2\pi x k_{x_0}} f(x)\} = \widehat{f}(k_x - k_{x_0})$
Conjugation	$\mathrm{FT}\{f^*(x)\} = \widehat{f^*}(-k_x)$
Correlation	$\mathrm{FT}\left\{\int_{-\infty}^{\infty} f^*(y)g(x+y)dy\right\} = \widehat{f^*}(k_x)\cdot\widehat{g}(k_x)$ $\mathrm{FT}\left\{\int_{-\infty}^{\infty} f^*(y)f(x+y)dy\right\} = \lvert\widehat{f}(k_x)\rvert^2$
Convolution	$\mathrm{FT}\{f(x)*g(x)\} = \mathrm{FT}\{f(x)\} \times \mathrm{FT}\{g(x)\}$ $\mathrm{FT}\{f(x) \times g(x)\} = \frac{1}{2\pi}\mathrm{FT}\{f(x)\}*\mathrm{FT}\{g(x)\}$
Duality (Cyclic)	$\mathrm{FT}\{\mathrm{FT}\{f(x)\}(k_x)\}(x) = f(-x)$ $\mathrm{FT}\{\mathrm{FT}\{\mathrm{FT}\{\mathrm{FT}\{f(x)\}(k_x)\}(x)\}(k_x)\}(x) = f(x)$
Parseval	$\int_{-\infty}^{+\infty} \lvert f(x)\rvert^2\, dx = \int_{-\infty}^{+\infty} \lvert\widehat{f}(k_x)\rvert^2\, dk_x$

2.3. Optical Fourier Transform (OFT)

As noted the FT has been used to describe many optical applications such as filtering and correlation [3, 4]. In this section we briefly present the optical Fourier transform (OFT). A single convex lens optical system with input and output planes located symmetrically on either side of the lens at distances equal to the lens focal length q performs a 2D OFT [1]. The relationship between the input and output fields can be defined as follows:

$$\begin{aligned}\widehat{f}(k_x, k_y) &= \mathrm{OFT}\{f(x, y)\}(k_x, k_y)\\ &= \frac{1}{\sqrt{\lambda q}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\exp\left\{-j\frac{2\pi}{\lambda q}(k_x x + k_y y)\right\}dxdy,\end{aligned} \tag{19}$$

where x and y denote the spatial coordinates (input space domain) and k_x and k_y denote the corresponding spatial frequencies (output spatial frequency domain). f denotes the input field and $\widehat{f}$ denotes the output field. λ is the wavelength of the light used. As discussed in Section 2.2, since the IFT operates almost identically to the FT, thus the single lens system described above can be used to implement both a forward and an inverse OFT. we illustrate the operation of the OFT in Fig 2. At the input front focal plane of the lens, two plane waves, A and B, interfere to form a wavefield that varies cosinusoidally in x. The spatial frequencies of the plane waves are determined by the wavelength of the light λ, and the angle at which the plane waves propagate with respect to the principal or optical axis of the system. The lens does not act to form an image. Instead in the output plane two bright spots appear with all the energy in the plane wave concentrated as indicated. The lens has acted to convert the uniform plane waves in space into delta functions (spots) in the frequency domain.

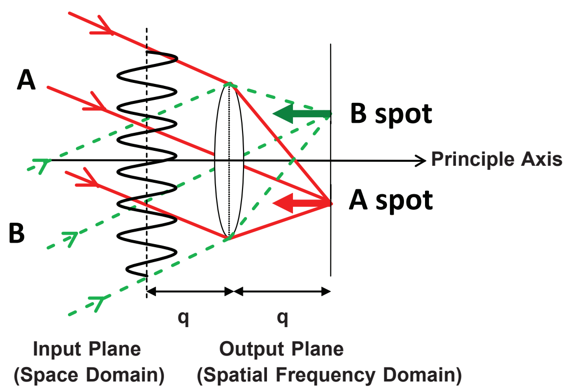

Fig. 2. One possible setup of optical Fourier transform (OFT). Plane waves: A (solid red lines) and B (dashed green lines).

In fact several optical systems can be used to implement the FT [1, 10, 11].

2.4. Discrete Fourier Transform (DFT)

2.4.1. Derivation and Definition of the DFT

In the discrete case, a sampled signal, $f_s(x, y)$, is generated by multiplying the continuous signal, $f(x, y)$, by a comb function, i.e. an infinite train of Dirac delta functionals: $\sum_{n_x=-\infty}^{\infty}\sum_{n_y=-\infty}^{\infty}\delta(x - n_xT_x, y - n_yT_y)$, and integrating, see Eq. (20). The sampled signal is defined to be zero everywhere except at those values of x and y given by $x = n_xT_x$, $y = n_yT_y$ (i.e. $f_s(x, y) = f(n_xT_x, n_yT_y)$), where T_x and T_y are the sampling intervals in the space domain, and n_x and n_y are integer indices. A completely equivalent representation is also given in Eq. (20) using the Fourier series of the comb functionals [25], where m_x and m_y are the integer indices in the spatial frequency domain. Following the notations discussed in Goodman's book [1], the sampled signal has the form:

$$\begin{aligned}f_s(x, y) &= f(x, y)\sum_{n_x=-\infty}^{\infty}\sum_{n_y=-\infty}^{\infty}\delta(x - n_xT_x, y - n_yT_y)\\ &= f(x, y)\sum_{m_x=-\infty}^{\infty}\sum_{m_y=-\infty}^{\infty}\frac{1}{T_xT_y}\exp\left\{j2\pi\left(\frac{m_x x}{T_x} + \frac{m_y y}{T_y}\right)\right\}.\end{aligned} \tag{20}$$

The discrete-time Fourier transform (DTFT) is given by taking the FT of the sampled signal, see Eq (21) below. The DTFT provides an approximation to the continuous-time (analogue) Fourier transform (FT) [24].

$$\begin{aligned}\widehat{f_s}(k_x, k_y) &= \mathrm{FT}\{f_s(x, y)\}(k_x, k_y)\\ &= \sum_{n_x=-\infty}^{\infty}\sum_{n_y=-\infty}^{\infty} f(n_xT_x, n_yT_y)\exp\{-j2\pi(k_xn_xT_x + k_yn_yT_y)\}\\ &= \sum_{m_x=-\infty}^{\infty}\sum_{m_y=-\infty}^{\infty} \frac{1}{T_xT_y}\widehat{f}\left(k_x - \frac{m_x}{T_x}, k_y - \frac{m_y}{T_y}\right).\end{aligned} \tag{21}$$

In Eq (21), the signal $\widehat{f_s}(k_x, k_y)$ can be defined in two different ways based on applying the FT to the two different expressions for $f_s(x, y)$ in Eq (20), respectively. The second expression of the DTFT in Eq (21) can be derived using the shift property of the FT, which indicates that the DTFT is infinitely periodic with periods of $1/T_x$ and $1/T_y$ in k_x and k_y , respectively. Sampling $f(x, y)$ in the space domain causes the continuous spectrum $\widehat{f}(k_x, k_y)$ to be periodically repeated in the spatial frequency domain. Therefore, the DTFT is composed of shifted copies of the FT of the continuous signal added together. These copies are shifted by $1/T_x$ and $1/T_y$ in k_x and k_y and they repeat infinitely. If these replicas are well separated (extracting the central example when $m_x = m_y = 0$ using a low pass filter) and applying an inverse Fourier transform, $f(x, y)$ can be accurately reconstructed. However, if the extent of $\widehat{f_s}(k_x,k_y)$ is greater than the separation distances $(1/T_x, 1/T_y)$, overlapping occurs. This is referred to as *aliasing* and it makes accurately recovering the continuous input signal $f(x, y)$ from its samples extremely difficult.

When moving from the continuous optical system to the discrete one, we must consider carefully the conditions under which such aliasing may occur. Aliasing can be avoided if we sample at a rate which is greater than or equal to the Nyquist rate, i.e., $1/T_x \geq B_x$ and $1/T_y \geq B_y$, where B_x and B_y denote the spatial frequency band widths of the original signal $\widehat{f}(k_x, k_y)$ in k_x and k_y, respectively.

So far we have described how to calculate the infinite periodic continuous spectrum of a sampled continuous input signal. However, numerically a sampled spectrum of discrete values is required. The DFT computes such discrete samples of the continuous DTFT. We sample in k_x and k_y with sampling intervals of $1/(T_xN_x)$ and $1/(T_yN_y)$ at discrete values $k_x \to m_x/(T_xN_x)$ and $k_y \to m_y/(T_yN_y)$. This causes the original signal $f_s(x, y)$ in the space domain to become infinitely periodic with periods of T_xN_x and T_yN_y in x and y, respectively, where N_x and N_y are the numbers of samples in x and y. This follows from using the shift and scaling properties presented in Table 1.

Therefore, the DFT is a mapping from one discrete infinitely periodic signal (input) to another (output). We note that aliasing can also take place in the space domain if we do not sample appropriately in the frequency domain. Therefore, particular care must be taken when multiplying two signals together in the Fourier domain. As noted one result of convolving functions is enlarged extent [24], see Table 1. We define $\widehat{f}(m_x, m_y)$ to be the DFT of a function $f(n_x, n_y)$ as follows, and we note that we have dropped T_x and T_y from $f(n_xT_x, n_y, T_y)$ for brevity:

$$\begin{aligned}\widehat{f}(m_x, m_y) &= \mathrm{DFT}\{f(n_x, n_y)\}\\ &= \sum_{n_x=-N_x/2}^{N_x/2-1}\sum_{n_y=-N_y/2}^{N_y/2-1} f(n_x, n_y)\exp\left\{-j2\pi\left(\frac{m_xn_x}{N_x} + \frac{m_yn_y}{N_y}\right)\right\}\end{aligned} \tag{22}$$

where DFT{–} denotes the DFT operator, and n_x and m_x are integers: $-N_x/2 \to N_x/2 - 1$, while n_y and m_y each lie in the range: $-N_y/2 \to N_y/2 - 1$. The inverse DFT is defined by:

$$\begin{aligned}f(n_x, n_y) &= \mathrm{DFT}^{-1}\{\widehat{f}(m_x, m_y)\}\\ &= \frac{1}{N_xN_y}\sum_{n_x=-N_x/2}^{N_x/2-1}\sum_{n_y=-N_y/2}^{N_y/2-1} \widehat{f}(m_x, m_y)\exp\left\{j2\pi\left(\frac{n_xm_x}{N_x} + \frac{n_ym_y}{N_y}\right)\right\}\end{aligned} \tag{23}$$

where $DFT^{-1}\{-\}$ denotes the inverse DFT operator. The DFT can be computed efficiently using the fast Fourier transform (FFT) algorithm in $O(N \log N)$ time [21].

2.4.2. Avoiding aliasing in the spatial frequency domain

The convolution theorem of the FT states that multiplying two signals in the space domain is equivalent to convolving their FTs in the frequency domain; see Table 1 or Page 115 in [24]. Furthermore, the resulting spatial frequency distribution will have a bandwidth equal to the sum of the bandwidths of the two product signals, i.e. $B_{1x} + B_{2x}$ and $B_{1y} + B_{2y}$, where B_{1x}, B_{2x} and B_{1y}, B_{2y} represent the bandwidths of the two product signals in k_x and k_y. In order to avoid aliasing in the frequency domain when multiplying two discrete signals, we must therefore sample both signals at a rate greater than or equal to the Nyquist rate, i.e., $1/T_x \geq B_{1x} + B_{2x}$ and $1/T_y \geq B_{1y} + B_{2y}$, prior to multiplication.

2.4.3. Avoiding Aliasing in the Space Domain

When we multiply two discrete signals in the frequency domain, the resulting signal will have a spatial width equal to the sum of the spatial distributions of the individual signals, i.e. $W_{1x} + W_{2x}$ and $W_{1y} + W_{2y}$, where W_{1x}, W_{2x} and W_{1y}, W_{2y} represent the widths of the two signals in x and y [24]. Therefore, in order to avoid aliasing in the space domain we need to ensure that $T_xN_x \geq W_{1x} + W_{2x}$ and $T_yN_y \geq W_{1y} + W_{2y}$, prior to multiplication.

2.4.4. Width and Bandwidth: the OFT

In this section we discuss the width and bandwidth of a signal in terms of simulating the OFT using the DFT. As described in Section 2.4.1, when the DFT is used to simulate the FT, the following relationships exist between the input sampling intervals (T_x and T_y) and the output sampling intervals (δ_x and δ_y) [26]:

$$\delta_x = \frac{1}{N_xT_x};\ \delta_y = \frac{1}{N_yT_y} \tag{24}$$

We recall that N_x and N_y are the numbers of samples in the x and y directions, respectively. Following our relationship between the FT and OFT in Section 2.3, the FT and the OFT can be related by the scaling (similarity) theorem of the FT, see Table 1. Given a wavelength and a lens of focal length q, this results in the following changes in the output coordinates:

$$k_x \to \frac{k_x}{\lambda q};\ k_y \to \frac{k_y}{\lambda q} \tag{25}$$

Accordingly, the DFT may be used to calculate the OFT with output sampling intervals of:

$$\delta_x = \frac{\lambda q}{N_xT_x}\ ;\ \delta_y = \frac{\lambda q}{N_yT_y} \tag{26}$$

Based on the discussion above, in Table 2 the widths and the bandwidths of a 2D signal calculated using DFT necessary to simulate both the OFT and the FT are presented.

2.5 Fractional Fourier Transform (FRT)

In this section, a general case of the Fourier transform, i.e. the fractional Fourier transform (FRT), is presented and followed by a discussion of its optical implementations and numerical algorithms.

Table 2. The width and bandwidth of a 2D signal using DFT to simulate the OFT and the FT.

	DFT simulating OFT	**DFT simulating FT**
Width (m)	$W_x = N_xT_x$	$W_x = N_xT_x$
	$W_y = N_yT_y$	$W_y = N_yT_y$
Bandwidth (1/m)	$B_x = N_x\delta_x = \lambda q/T_x$	$B_x = N_x\delta_x = 1/T_x$
	$B_y = N_y\delta_y = \lambda q/T_y$	$B_y = N_y\delta_y = 1/T_y$

2.5.1. Derivation and Definition of the FRT

The FRT is a generalization of the conventional Fourier transform (FT) with a fractional order parameter, a, indicating the domain into which the signal is transformed [27]. It was previously noted that the Fourier transform (FT) is a linear transformation allowing a signal originally captured in the space domain to be transformed into the orthogonal spatial frequency domain. The FRT, in an analogous way, can be seen as transforming a signal into a mixed frequency-space domain [28]. Using the concept of Wigner distribution function the FT acts as a rotation of 90° in phase space, while the FRT corresponds to a rotation of a different angle φ, which is proportional to order a of the FRT, i.e. $\varphi = a\pi/2$. The FT is a first-order fractional Fourier transform, i.e. of order $a = 1$. Mathematically, the a^{th} order fractional Fourier transform is the a^{th} power of the Fourier transform operator, and interpolates between a function $f(x)$ and its Fourier transform $\widehat{f}(k_x)$. The 0^{th} order transform is simply the function itself.

The existence of the FRT indicates that the frequency domain is simply a special case of a continuum of fractional Fourier domains, each of which contains alternate signal representations, all of which are related through the concept of phase-space distributions [29]. Every property and possible application of the FT is a special case of those of the FRT. In every field in which FT and spatial frequency domain concepts are used, there exists the potential for generalization and possible improvement using the FRT. Therefore, introduction of the FRT to optics resulted in an explosion of interest within the optical signal processing (OSP) community [30]. For instance, as noted using the FT, the output distribution obtained at the back focal plane of a lens for some input distribution at the front focal plane of the lens can be determined. Using the FRT one can predict the output distribution at some arbitrary distance from the lens for an input at any equal distance from the lens. Additionally, the theory of optimal Wiener filtering in the Fourier domain can be generalized to optimal filtering in fractional Fourier domains, to produce reduced mean square errors (MSE) at practically no additional cost [31]. The FRT has lead to a deeper appreciation in the optics community of the importance of phase space representations, which permits the manipulation of a signal over all of the space frequency plane rather than just in the space or the spatial frequency domain.

One optical system which has been generalized using the FRT is the Double Random Phase Encoding (DRPE) method. This technique was proposed in 1995 by Refregier and Javidi, and it is an optical encryption method based on the use of the FT. Returning to examine the system illustrated in Fig 1, it was proposed to insert two random phase screens, one after the input plane, i.e. after $f(x, y)$, and one after the Fourier plane (instead of filter plane in the figure). This results in an output which has the characteristics of the white noise. Insertion of this signal to a similar system in which the two phase screens are replaced by their complex conjugates allows the original signal to be decrypted. The original system employs optical implementations of the FT. It was quickly realized that the FRT provided a means for generalizing this original system advantageously [20, 32]. The extra degrees of freedom it provides, i.e. the independent fractional orders in both x and y directions, improves the systems performance (security and robustness) to blind decryption attacks [33-35].

Returning to our discussions of the FRT, we note that the ath order 2D FRT, $\widehat{f}_a(x_a, y_a)$, of an input image $f(x, y)$ is defined as:

$$\widehat{f}_a(x_a, y_a) = \mathrm{FRT}\{f(x, y)\}(x_a, y_a)$$

$$= A\varphi \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y) \times \exp\left\{ j\pi \left(\frac{x^2 + x_a^2}{\tan\varphi} - \frac{2(xx_a + yy_a)}{\sin\varphi} + \frac{y^2 + y_a^2}{\tan\varphi} \right) \right\} dxdy, \tag{27}$$

where $\varphi = a\pi/2$ and $0 < |a| < 2$. x_a and y_a represent the a^{th} fractional domain coordinates and FRT{(−)} denotes the FRT operation. A_φ is given by:

$$A_\varphi = \frac{\exp[-j\pi\,\mathrm{sgn}(\sin\varphi)/4 + j\varphi/2]}{|\sin\varphi|^{1/2}} \tag{28}$$

A_φ is a constant output phase factor that is dependent on the fractional order. It is possible, in the 2D FRT case, to have two independent fractional orders, a_x in x direction and a_y in y direction. These can both be employed as separate keys in two-dimensional optical encryption systems.

2.5.2. Properties of the FRT

We now review the properties of the FRT [29]. The index additivity property is simply satisfied, i.e. the $a^{th}{}_2$ FRT of the $a^{th}{}_1$ FRT is equivalent taking the $(a_1 + a_2)^{th}$ FRT. The -1^{th} FRT is the inverse Fourier transform and the $-a^{th}$ transform represents the inverse of the a^{th} transform. A summary of properties of the FRT are presented as below:

(i) *Linearity property:*

$$\mathrm{FRT}\left\{\sum_i \alpha_i f_i(x)\right\}(x_a) = \sum_i \alpha_i\, \mathrm{FRT}\{f_i(x)\}(x_a) = \sum_i \alpha_i \widehat{f_{ai}}(x_{ai}) \tag{29}$$

(ii) *Scaling property:*

$$\mathrm{FRT}\left\{\frac{1}{\sqrt{M}} f\left(\frac{x}{M}\right)\right\}(x_a) = \frac{1}{\sqrt{M'}} \exp\{j\pi x_{a'}^2\, q\}\mathrm{FRT}\{f(x)\}\left(\frac{x_a}{\sqrt{M'}}\right)$$

$$a' = \frac{2\varphi}{\pi} = \frac{2}{\pi}\tan^{-1}\left(\frac{\tan\varphi}{M^2}\right)$$

$$q = \frac{\sin^2\varphi' - \sin^2\varphi}{\sin\varphi\cos\varphi}$$

$$M' = \frac{\sin\varphi}{M\sin\varphi'} \tag{30}$$

(iii) *Conjugation property:*

$$\mathrm{FRT}\{f^*(x)\}(x_a) = \widehat{f^*}_{-a}(x_a) \tag{31}$$

(iv) *Additive index property:*

$$\mathrm{FRT}_b\{\mathrm{FRT}_a\{f(x)\}(x_a)\}(x_b) = \mathrm{FRT}\{f(x)\}(x_{a+b}) = \widehat{f}_{a+b}(x_a) \tag{32}$$

(v) *Parseval's theorem:*

$$\int_{-\infty}^{+\infty} |f(x)|^2\, dx = \int_{-\infty}^{+\infty} |\widehat{f}_a(x_a)|^2\, dx_a \tag{33}$$

(vi) *Shift property:*

$$\mathrm{FRT}\{f(x - x_0)\}(x_a) = \frac{1}{M}\exp\left\{j\pi x_0^2\left(\frac{a\pi}{2}\right)\cos\left(\frac{a\pi}{2}\right)\right\} \times \exp\left\{-j\pi x_a x_0 \sin\left(\frac{a\pi}{2}\right)\right\} \times \mathrm{FRT}\{f(x)\}\left\{x_a - x_0\cos\left(\frac{a\pi}{2}\right)\right\}$$

$$\mathrm{FRT}\{\exp\{j2\pi xk\}f(x)\}(x_a) = \frac{1}{M}\exp\left\{-j\pi k^2\sin\left(\frac{a\pi}{2}\right)\cos\left(\frac{a\pi}{2}\right)\right\} \times \exp\left\{j\pi x_a k\cos\left(\frac{a\pi}{2}\right)\right\} \times \mathrm{FRT}\{f(x)\}\left(x_a - k\sin\left(\frac{a\pi}{2}\right)\right\} \tag{34}$$

There is clearly much similarity to the properties of the FT.

2.5.3. The Optical and Numerical Implementations of the FRT

The 2D optical FRT has the integral representation:

$$\widehat{f}_a(x_a, y_a) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y) \times \exp\left\{j\frac{\pi}{\lambda q}\left(\frac{x^2 + x_a^2}{\tan\varphi} - \frac{2(xx_a + yy_a)}{\sin\varphi} + \frac{y^2 + y_a^2}{\tan\varphi}\right)\right\}dxdy, \tag{35}$$

where q is called the standard focal length which is dependent on the physical parameters of the optical FRT system, and λ denotes the wavelength. The product λ_q acts as a scaling factor in the optical FRT implementation. Generally, the FRT can be implemented using some arbitrary length of Gradient-index (GRIN) optics medium, where the length defines the fractional order [36]. Lohmann proposed two types of bulk optical FRT implementations, i.e. Type-I and Type-II as shown in Fig 3. Both are implemented using free space propagation and the action of a thin lens [28]. Type-I involves a single lens of focal length f_L with free space gap of length z on either side, where the following conditions are met:

$$f_L = \frac{\pi}{\sin\varphi},\ z = q\tan\frac{\varphi}{2}. \tag{36}$$

Type-II, Fig 3(b), involves two lenses of focal length f_L separated by free space of length z, with the following conditions required:

$$f_L = \frac{q}{\tan\frac{\varphi}{2}},\ z = q\sin\varphi, \tag{37}$$

where once again f_L denotes the focal length of the lens. A 2D FRT with different fractional orders in x and y directions can be implemented using two orthogonally oriented cylindrical thin lenses (aligned along x and y) having different focal lengths [37].

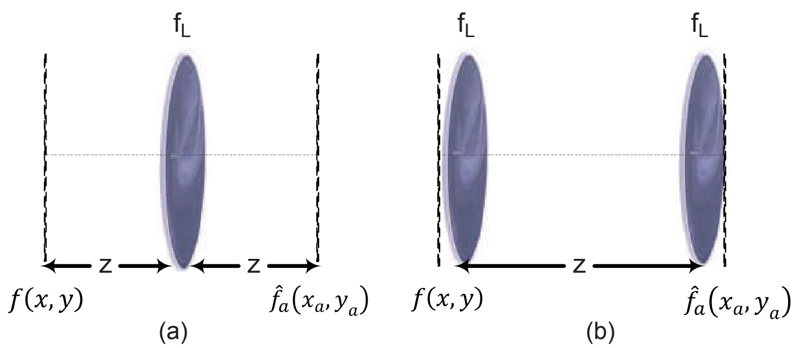

Fig 3. Lohmann's (a) Type-I and (b) Type-II optical implementations of the FRT.

Several numerical implementations of the FRT have been proposed in [38], where the Shannon interpolation formula and a series of mathematical manipulations are used to arrive at a convolution summation that can be implemented using the FFT algorithm. However, these algorithms are not very accurate for small fractional orders due to the high sampling rate needed because of rapid oscillations in the fields. In order to perform an accurate discrete simulation of the optical FRT, it is critically necessary to make sure the discrete FRT employed is unitary, i.e. conserves power and is completely reversible. Significantly without the ability to perform an exact inverse calculation, the ideal decryption process of the optical encryption system cannot be simulated. In [39] an exactly unitary, index additive, and discrete FRT was proposed based on the discrete counterparts of the Hermite-Gaussian functions. This algorithm requires $O(N^2)$ calculations. A method to generate and perform optimized numerical implementations of the FRT has been presented in [20]. In [35], we describe and apply such an unitary FRT algorithm to implement our proposed optical encryption schemes.

2.6. The Fresnel transform (FST)

The Fresnel transform (FST) has the simplest optical implementation as it corresponds to free space propagation [1]. Every FST can be related to an FRT of some order followed by appropriate magnification and additional quadratic phase multiplication [40]. The FST is also a special type of the LCT [20]. In this section we briefly review the FST and its optical and numerical implementations. The FST of a 2D image, $f(x, y)$, at a distance z can be expressed as:

$$\widehat{f}_z(x_z, y_z) = \mathrm{FST}_z\{f(x, y)\}$$
$$= \frac{\exp\{j2\pi z/\lambda\}}{j\lambda z} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y) \exp\left\{\frac{j\pi}{\lambda z}\{(x_z - x)^2 + (y_z - y)^2\}\right\} dxdy \quad (38)$$

where x_z and y_z are coordinates in the Fresnel domain and λ denotes the wavelength.

$\mathrm{FST}_z\{-\}$ represents the Fresnel transform operation involving a propagation distance z. The properties of the FST are discussed in [41, 42], and include additivity and the conjugation:

$$\mathrm{FST}_{z_1}\{\mathrm{FST}_{z_2}\{f(x, y)\}\} = \mathrm{FST}_{z_1+z_2}\{f(x, y)\},$$
$$\mathrm{FST}_z\{f^*(x, y)\} = \mathrm{FST}^*_{-z}\{f(x, y)\}. \quad (39)$$

where '$*$' denotes the complex conjugate operation.

In order to numerically simulate the FST, a unitary numerical implementation is again necessary when simulating optical security system since ideal decryption is necessary to obtain the original authentic information. The "direct method (DM)" of implementing the FST is described in Eq (40) below [43]. It is unitary with appropriate sampling and can be inverted perfectly [44]. Since the integral transform corresponding to the Fresnel approximation is equivalent to the convolution of $f(x, y)$ with the free space operator [see the third expression in Eq. (40)], the FST integral can be interpreted as involving a quadratic phase factor multiplied by the FT of the product of the input image and another quadratic phase factor [see the second expressions in Eq.(40)].

$$\widehat{f}_z(x_z, y_z) = \mathrm{FST}_z\{f(x, y)\}$$
$$= \exp\left\{\frac{j\pi}{\lambda z}(x_z^2 + y_z^2)\right\} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y)\exp\left\{\frac{j\pi}{\lambda z}(x^2 + y^2)\right\}$$
$$\times \exp\left\{\frac{-j2\pi}{\lambda z}(xx_z + yy_z)\right\} dxdy \quad (40)$$
$$= \exp\left\{\frac{j\pi}{\lambda z}(x_z^2 + y_z^2)\right\} \times \mathrm{DFT}\left\{f(x, y)\exp\left(\frac{j\pi}{\lambda z}(x^2 + y^2)\right)\right\}$$
$$= f(x, y) * \exp\left\{\frac{j\pi}{\lambda z}(x_z^2 + y_z^2)\right\}$$

In this equation the operator $*$ here denotes the convolution operator. We note that a phase factor, $\exp\{j2\pi z/\lambda\}/j\lambda z$, which depends on z, has been dropped for the sake of brevity. This "direct method (DM)" provided a numerical way to calculate the FST using the DFT/FFT.

A different numerical algorithm can be used to carry out simulations of the FST, namely the "spectral method (SM)". This is also sampling unitary and can be inverted [45]. This algorithm uses the angular spectrum approach to evaluate the Fresnel integral. With large propagation distances DM can cause aliasing in the output distribution, which indicates that it require oversampling (using ze-ropadding) [46]. With suitable sampling, the numerical results are identical to those obtained using "DM".

2.7. The Linear Canonical Transform (LCT)

The FT, the FRT, and the FST are all special cases of the linear canonical transform (LCT), and can all be optically implemented using quadratic phase systems (QPS) made up of lenses and free space [47]. In this section we briefly review the LCT and its optical and numerical implementations.

The 2D LCT is a three-parameter class of linear integral transform which is defined as follows:

$$\widehat{f}_{\alpha,\beta,\gamma}(x_l, y_l) = \mathrm{LCT}_{\alpha,\beta,\gamma}\{f(x, y)\}(x_l, y_l)$$
$$= \exp\{-j\pi/4\}\sqrt{\beta}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y) \times$$
$$\exp\{\alpha(x^2 + y^2) - 2\beta(xx_l + yy_l) + \gamma(x_l^2 + y_l^2)\}dxdy \tag{41}$$

where $\mathrm{LCT}_{\alpha,\beta,\gamma}\{-\}$ denotes the LCT operator with (in the lossless case) three real transform parameters, α, β and γ, which are independent of signals in the input (x, y) and output (x_l, y_l) domains. We note that α, β, and γ can be complex valued in the case of systems with Gaussian apertures [48]. The definition of the LCT can be further generalized to a five-parameter transform, known as the *Special Affne Fourier Transform (SAFT)* or shifted LCT, in which the additional two parameters are shifts in both the space and spatial frequency domain [19, 49]. The LCT includes as special cases of the FT, FRT, and FST, and the operations of scaling (magnification) and chirp multiplication (effect of a thin lens) [21]. The LCT has the properties of: linearity, scaling, shifting, conjugation, additivity, and obey Parseval's theorem [15, 50]. These are now listed:

(i) Linearity property:

$$\mathrm{LCT}_{\alpha,\beta,\gamma}\left\{\sum_i M_i f_i(x)\right\}(x_l) = \sum_i M_i \mathrm{LCT}_{\alpha,\beta,\gamma}\{f_i(x)\}(x_l)$$
$$= \sum_i M_i \widehat{f}(x_l) \tag{42}$$

(ii) Scaling property:

$$\mathrm{LCT}_{\alpha,\beta,\gamma}\left\{\frac{1}{\sqrt{M}} f\left(\frac{x}{M}\right)\right\}(x_l) = \frac{1}{\sqrt{M}}\widehat{f}_{\alpha,M\beta,M^2\gamma}(x_l) \tag{43}$$

(iii) Conjugation property:

$$\mathrm{LCT}_{\alpha,\beta,\gamma}\{f^*(x)\}(x_l) = \widehat{f^*}_{-\alpha,-\beta,-\gamma}(x_l) \tag{44}$$

(iv) Shift property:

$$\mathrm{LCT}_{\alpha,\beta,\gamma}\{f(x - x_0)\}(x_l) = \exp\left\{-j\pi x_0^2\left(\alpha\frac{\gamma^2}{\beta^2} - \gamma\right)\right\} \times \exp\left\{j2\pi x_l x_0\left(\frac{\alpha\gamma}{\beta} - \beta\right)\right\}$$
$$\times \mathrm{LCT}_{\alpha,\beta\,\gamma}\{f(x)\}\left(x_l - \frac{x_0\gamma}{\beta}\right),$$

$$\mathrm{LCT}_{\alpha,\beta,\gamma}\{\exp\{j2\pi k_x\}f(x - x_0)\}(x_l) = \exp\left\{-j\pi k^2\left(\frac{\gamma}{\beta}\right)\right\} \times \exp\left\{j\pi x_l k\left(\frac{\alpha}{\gamma}\right)\right\}$$
$$\times \mathrm{LCT}_{\alpha,\beta\,\gamma}\{f(x)\}\left(x_l - \frac{k}{\beta}\right). \tag{45}$$

(v) Additive index property:

$$\mathrm{LCT}_{\alpha_2,\beta_2,\gamma_2}\{\mathrm{LCT}_{\alpha_1,\beta_1,\gamma_1}\{f(x)\}(x_l)\}(x_l) = \widehat{f}_{\alpha_3,\beta_3,\gamma_3}(x_l), \tag{46}$$

where

$$\alpha_3 = \alpha_2 - \frac{\beta_2^2}{\alpha_1 + \gamma_2}$$
$$\beta_3 = \frac{\beta_1\beta_2}{\alpha_1 + \gamma_2} \tag{47}$$
$$\gamma_3 = \gamma_1 - \frac{\beta_1^2}{\alpha_1 + \gamma_2}$$

(vi) Parseval's theorem:

$$\int_{-\infty}^{+\infty} |f(x)|^2\, dx = \int_{-\infty}^{+\infty} |\widehat{f}_{\alpha,\beta,\gamma}(x_l)|^2\, dx_l \tag{48}$$

We note that in the case of the FST, in Eq. (39), the three LCT parameters are $\alpha = \beta = \gamma = 1/\lambda z$. The LCT can be used to model any paraxial systems composed of lenses, any sequence of free space, graded-index (GRIN) media, and other quadratic phase systems (QPS) [27, 49]. The LCT can also be described in terms of the FRT using the "extended FRT" with scaling factors relating to the three real transform parameters [33]. In Fig 4, an illustration of one such QPS, using a single lens, is shown. In this case, the parameters α, β, and γ associated with the QPS transformation can be related to the distances d_1, d_2 and the lens focal length f using the following relations:

$$\alpha = \frac{d_1 - f}{\lambda\,[f(d_1 + d_2) - d_1 d_2]};$$

$$\beta = \frac{f}{\lambda[f(d_1 + d_2) - d_1 d_2]};$$

$$\gamma = \frac{d_2 - f}{\lambda[f(d_1 + d_2) - d_1 d_2]}. \tag{49}$$

In previous sections, the numerical algorithms used to implement the FT, FRT and FST in order to perform optical encryption have been reviewed. We note that all these numerical algorithms involve use of the FFT. The kernel of the LCT in Eq (41) can be shown to be equivalent to chirp multiplication (by α) followed by performance of a scaled FT (by β) and then followed by another chirp multiplication (by γ) [29]. Therefore numerical algorithms for the LCT can also be developed using the FFT. It has been shown that the resulting algorithm is unitary and retains the additive properties as in the case for the continuous LCT [51]. Hennelly *et al* [21] proposed a direct $N\log N$ fast numerical algorithms for the LCT, namely the "fast linear canonical transform (FLCT)". The derivation uses the periodicity and shifting properties of the discrete LCT (DLCT). The DLCT has been derived for the LCT following the same derivation procedure employed to derive the DFT for the FT. Likewise a FLCT algorithm was then derived using an approach analogous to that used to derive the FFT for the DFT. This approach also produces fast algorithms for the FFT, FRT, and FST. Healy *et al* [52] reviewed the numerical approximation of the LCT including discretization, sampling, fast algorithms, and identical key results, and proposed a frequency-division fast LCT algorithm, which is directly analogous to the Cooley-Tukey FFT algorithm. Most recently, by deriving a sucient condition on the sampling rates chosen in the discretization, Zhao *et al* [53] presented a generalized unitary discrete LCT. The results are completely consistent with all previous results in the literature.

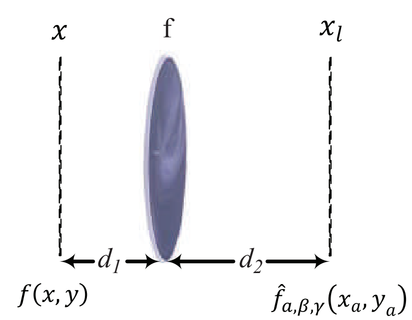

Fig 4. An illustration of a simple single lens quadratic phase system (QPS).

3 Optical Encryption

3.1. General Introduction

Information security is important throughout modern societies. Securing information has implications for virtually all aspects of our lives, including protecting the privacy of our financial transactions and medical records, facilitating the operation of government, maintaining the integrity of national borders, securing critical facilities, ensuring the safety of food and commercial products [54]. In recent years, optical methods, to provide image encryption, have been widely examined.

They offer the possibility of high-speed parallel processing of 2D image data, as well as the ability to hide information using multiple degrees of freedom, i.e. the amplitude, phase, wavelength, polarization, fractional orders, and propagation distances available when using linear lossless paraxial optical systems [55]. The natural 2D imaging capabilities of optics has made optical encryption, (as an application of OSP), receive much attention. Since optical waves can pass through one another, they can additionally be combined in novel passive multiplexing schemes. Such optical security system requires the modulation and capture of the full complex encrypted field information, i.e. both the intensity and the phase, using for example digital holographic (DH) and interferometric techniques [56, 57]. Let us briefly review the fundamental optical encryption methods explored.

One important optical encryption scheme, previously mentioned is the "Double Random Phase Encoding (DRPE)" [58]. It involves multiplication of the field by random phase diffusers (masks) both in the input (space) and the Fourier (spatial frequency) domains. As noted the encrypted image can be shown to be a stationary white noise if the two random phases are statistically independent white noises. Digital holography [56, 57] provides a convenient form of digitally recording the output complex encrypted images after passage through the optical DRPE systems. The random phase diffusers serve as the keys in this encryption system. The DRPE decryption process is the exact reverse of the encryption process with the encrypted field being input and the two random diffusers used being complex inverted conjugates of the original diffusers. This results arises due to the duality property of the OFT [59], with a single lens OFT optical system being used to implement both forward and inverse OFTs. In Fig 5, we illustrate such a DRPE encryption/decryption optical implementation using a spatial light modulator (SLM) to generate the input field to the system. The SLM can be used to display both amplitude and phase information [60].

For encryption, SLM1 displays the input image multiplied by the first diffuser, while SLM2 displays the second diffuser. For decryption, SLM1 displays the encrypted complex image and SLM2 displays the conjugate of the second diffuser. The use of an interfering reference beam allows the output complex data to be captured. We note that in the final stage of the decryption process the reference beam is no longer required at the CCD since when encrypting real input images only the intensity of the decrypted image is of concern.

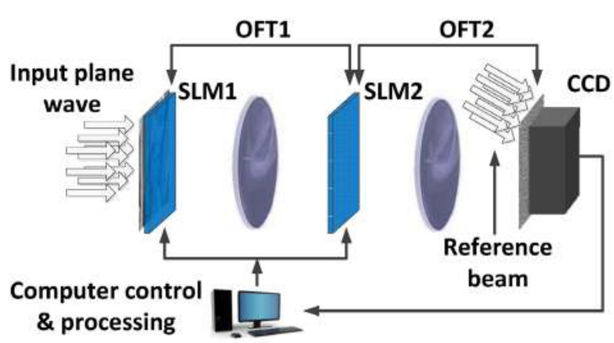

Fig 5. A possible optical DRPE encryption/decryption setup.

The availability of high quality spatial light modulators (SLM) makes it technologically feasible to perform the classic DRPE system in a relatively flexible manner [61]. The SLM can also be used to modulate the input digital data onto a coherent wave-field by modulating the polarization of the field [61]. Therefore, SLMs have been employed for both the 2D and 3D image data in the following ways: (i) encoding the input image; (ii) as part of an optical reconstruction technique; and (iii) providing the encryption key during either encryption or decryption (as the random phase diffusers). However, it is worth noting that serious practical limitations have been encountered when implementing such systems [60], as they can include noise into the system. Furthermore, alignment is critical in these optical systems. Even slight misalignments can lead to serious difficulties.

3.2. Existing Optical Image Encryption Methods

The proposal of the DRPE method generated a new area of activity in optical image and signal processing research and has led to many studies. Much effort has been expended by the community to develop new approaches and modeling techniques. Some of the optically inspired encryption methods proposed in the literature include digital optical stream cipher [62], optical XOR image encryption [63], phase-shifting interferometry [64], information security verification techniques involving the joint transform correlators [65-67], and polarization encoding [68, 69]. Both theoretical and experimental results have been reported, and the security and robustness of the optical encryption systems have been examined. Important techniques to improve the performance of optical encryption systems have also been presented.

Many variations of the original DRPE approach have been developed involving the use of more keys and of extra degrees of freedom to secure information. Extensions involving the use of the fractional Fourier transform (FRT) [20, 32-35, 70-75] and the Fresnel transform (FST) [42, 76-78] have been utilized in encryption algorithms and systems. In such systems the fractional orders and the propagation distances used serve as additional keys. It has been argued that these extra keys improve the security of the optical encryption system since they provide additional difficulties for attackers of the system. As noted the FT, the FRT, and the FST are all special cases of the LCT. Therefore, the three parameter LCT has also been proposed for use in the optical encryption techniques implemented using QPSs [79]. In this case the three independent QPS transformation parameters provide further extra keys for the encryption system. The gyrator transform (GT), which also belongs to the class of the linear canonical transforms, can also be used for optical encryption systems, where the rotation angle parameter provides the extra key of the encryption system [80-90]. The Hartley transform (HT), which is effectively a real Fourier transform without any phase information, has also been incorporated in optical DRPE systems [91-95]. In addition, the DRPE has been investigated for use in multiple-image encryption systems [96-98] and as part of color image encryption systems [99-106].

Scrambling techniques, which can be viewed as computer based numerical preprocessing procedure, have also been applied in conjunction with the DRPE systems, e.g. the so called Jigsaw transform (JT) [107-109] and the non-linear Arnold transform (ART) [110-114]. One possible advantage of employing such inherently digital techniques is that the resulting scrambled encrypted image can be easily stored and transmitted using conventional digital communication channels.

Recently, the computational ghost imaging technique has been proposed for use in optically encrypting and transmitting information, where the encrypted image appears as an intensity vector instead of the complex-valued matrix used in the typical DRPE system [115, 116]. Another optical imaging method, i.e.diffractive imaging, has also been developed in relation to optical encryption. It involves using an iterative phase retrieval algorithm to decrypt the original image from the recorded diffraction patterns [117-120]. As noted, 2D optical encryption has been extended into 3D space-based encryption processing, where each pixel of the image is axially considered as one particle and phase-shifting digital holography technique is applied to the diffraction of all the pixels in space (particles)[121-124].

In the last 60 years it has been realized that security systems that fundamentally rely on the secrecy of the encryption mechanism suffer from serious limitations [125]. Attacking or cracking these optical encryption techniques has thus become important. In the context of cryptography and cryptoanalysis, both chosen plaintext [110] and known-plaintext attack [126-128] on DRPE systems have been examined, as have several other attacking methods [129-133] and the key space of DRPE technique itself has also been analyzed [134-137]. A more detailed and technique specific review of the existing optical image encryption techniques, inspired by the architecture of the classic optical DRPE system, is presented in [138]. Both the optical implementations and numerical algorithms for each technique are investigated, together with an quantitative comparison of their robustness, which allows the performance of the extra degree of freedom (keys) provided by different methods to be appreciated. We classify the reviewed optically encryption approaches into two categories: (I) all optical techniques and (II) image scrambling techniques.

4. Digital Watermarking

4.1. General introduction and properties

Another important optical signal processing applications involves the use of digital watermarking techniques. The rapid growth of the modern communication techniques, especially the internet usage, has increasingly stimulated the development of information tracking and intellectual property protection [139]. To provide copyright protection for digital, audio and video data, two complementary techniques are being developed: *encryption* and *watermarking* [140]. In Section 3, we noted that optical system can offer distinct advantages, e.g. high-speed parallel processing and hiding information in many different dimensions. Therefore, optical signal processing opens up some promising areas of investigation in relation to digital watermarking. In this section, we briefly introduce the basic concepts of digital watermarking techniques.

Watermarking is defined by the practice of imperceptibly altering a host (carrier signal) to embed a message about that host. It can be seen as an effective way to provide copyright protection and data tracking security for many types of information [141]. Watermarking originated from the steganography. This is the science of writing hidden messages in such a way that the presence of the messages cannot be detected [142]. In particular, steganographic systems hide information that may have no relation to the host, with the aim of preventing detection by any party other than the sender and intended recipient, thus allowing both parties to communicate secretly. However, the information hidden by the watermarking systems may be associated with the owner or the digital object to be protected, and watermarking does not necessarily hide the fact of secret transmission of information from a third party [143].

Watermarking can be either “visible” or “invisible” [144, 145]. Invisible digital watermarking involves the embedding of an imperceptible signal into a multimedia data object, such as image, audio or video data. The watermark can then be detected to trace the origins of the object and thus be used to make an assertion about its ownership [140, 146]. Although copyright protection has been the major driving force behind watermarking research, there are a number of proposed or actual applications for which watermarking has been used or suggested. These include broadcast monitoring, owner identification, proof of ownership, transaction tracking, authentication, copy control, device control, and legacy enhancements [141]. In order to be effective, a watermark and its associated detection processes should have several important properties [147]:

(i) *Robustness*: This feature refers to the ability to detect the watermark after some signal processing techniques, common geometric distortions, or subterfuge attacks has taken place. Examples of common operations on images include lossy compression, spatial filtering, printing and scanning. Geometric distortions include rotations, translations, and scaling. We note that no watermarking system is completely impervious to all signal distortions and attacks. However, not all watermarking applications require robustness to every signal processing operation. Therefore, a watermark to be useful needs only survive common signal processing operations which are likely to occur between the time of embedding and detection. Similarly attack resilience must be optimized according to application. With this in mind, watermarking can be categorized under three headings: (i) *fragile*, i.e. watermarks which are destroyed by small distortions and which maybe used for authentication and integrity verification applications; (ii) *semi-fragile*, i.e. those which behave as fragile watermarks against intentional modifications and as robust watermarks against casual manipulations, e.g. the presence of noise. These can be used for image authentication and tamper control; and (iii) *robust*, i.e. watermarks which designed to resist heterogeneous manipulations. These are often used in copy control and monitoring.

(ii) *Fidelity* (*Invisibility*): This requirement preserves the similarity between the watermarked image and the original image based on human perception, i.e. they are perceptually invisible and their presence should not reduce the quality of the host image being protected.

(iii) *Capacity* (*Data Payload*): The number of bits that can be inserted through watermarking varies with different application. In the case of images, a watermark will involve the presence of a static set of bits. In the case of videos, capacity is determined by the quantity of embedded bits per frame, and similarly in the case of audio signals it is the number of embedded bits per second.

(iv) *Unambiguousness* (*Detection Types*): This property requires the watermarking system to be unique and retrievable (detectable). Generally, the type of watermarking detection is dependent on the requirements of the application. In some applications, the watermarking detection process must be applied without access to the original host image, which is also referred to as the *unwatermarked* image. For example, in the case of mass copy control, the detector must be distributed in every consumer recording device. It is thus not practical to distribute the original unwatermarked data to every detector. Such a detection process, that does not require the original host image, is referred to as "Blind Detection". In the watermarking literature, watermarking systems that use blind detection are generally called "*public watermarking systems*". In some applications of digital watermarking, the original host data, is available during the detection procedure. For example, in a transaction tracking application, the owner of the host data usually control the detector for the purpose of discovering illegal distribution of copies. The owner should have access to the original unwatermarked data and is able to provide it to the detector along with the illegal copy. This type of watermarking detection is referred to as "Informed Detection", and watermarking systems that use informed detection are called "*private watermarking systems*".

4.2. Existing digital watermarking techniques

Digital image watermarking is an emerging technology in the areas of signal processing and communications. Generally, the model of a watermarking system contains four stages: Generation, Embedding, Distribution (Transmission), and Detection, and we illustrate this basic model in Fig 6. The embedding methods used in the watermarking system influences both the robustness of the system and the form of the detection algorithm. Generally a watermark can be embedded either in the spatial domain or in the transform (spatial frequency) domain of the host image, referred to as "*spatial watermarking methods*" and "*spread spectrum* (SS) watermarking *methods*", respectively [148]. In the first case, the embedded watermark acts like a noise signal added to the host media and thus changes the characteristics of the watermarked signal. The least significant bit (LSB) [149] and the singular value decomposition (SVD) [150] methods are two common methods of this type. In the second case, the watermark is hidden in the host image's spectrum, commonly using either discrete cosine transform (DCT) [151, 152], discrete wavelet transform (DWT) [153], or lifting wavelet transform (LWT) [154]. Cox *et al* [147] proposed a classic secure spread-spectrum (SS) watermarking technique for multimedia, commonly known as *additive spread spectrum* (*additive SS*), involving the insertion of a watermark (pseudorandom sequence) into the perceptually most significant components of the image spectrum using the DCT. This fundamental spread spectrum method will be further investigated in terms of its robustness in comparison with the recently proposed "*DPRE SS-SS*" algorithm [155]. This traditional *additive spread-spectrum* was later extended by Malvar *et al* [156], who proposed a new SS technique referred to as *improved spread-spectrum* (*ISS*), producing an exceptional improvement in the quality of the watermarking processing by removing the host signal as a source of interference noise.

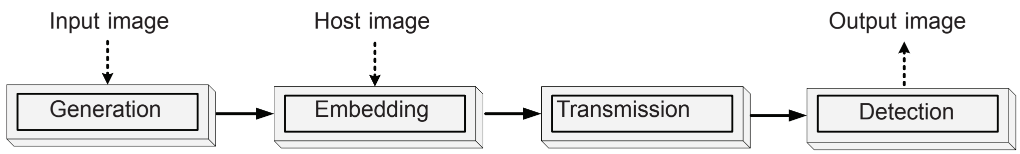

Fig 6. An illustration of basic model of watermarking system.

Recently, digital watermarking systems based on optically inspired techniques have began to attract significant interests, since as noted optical systems offer the possibility of high-speed parallel processing of 2D image data and many degrees of freedom [55]. We have already met the classic optical encryption scheme, Double Random Phase Encoding (DRPE) [58], which involves multiplication of the image by random phase diffusers (masks) both in the input (space) and Fourier (spatial frequency) domains. The encrypted image can be shown to be a stationary white noise if the two random phases are statistically independent white noises. A novel digital image watermarking technique, which involves use of the DRPE, is proposed in [155] and referred to as "*DRPE SS-SS*". This watermarking technique utilizes the capability of the optical DRPE method to reversibly spread the energy of the input information in both the space and spatial frequency domains using the two random diffusers [157]. The statistical properties of various noise terms arising due to the decryption procedure when using this method are investigated in [158]. We show that these noise terms share the same first-order statistical properties as optical speckle. The robustness of *DRPE SS-SS* technique is explored in [159], and it is demonstrated that this algorithm is robust to scaling, JPEG compression distortion, cropping, and both low pass and high pass filtering, and it also has very low false positive rates.

5 The Application of OSP Concepts

The application of the above techniques in a large number of areas has led to many interesting results in optics. The analysis of optical systems (both coherent and incoherent) has allowed significant improvements in our understanding of such systems (and their limitations) and has resulted in many practical improvements [10, 11, 19, 160-169]. Recently a great deal of work using these techniques has concentrated on the area of optical encryption [58, 71, 75, 79, 84, 94, 102, 104, 107-109, 137, 138, 158, 170-189]. Much work has been done studying various mean to attack such system [128, 134, 136, 190-192]. Holographic data storage is also an area of particular interest [187, 193-203]. Application of spread spectrum approaches, which employ optical encryption techniques, permit uniform use of material sensitivity and are of great value [155, 157, 196-203]. Optical speckle is also of continuing interest as it is either a noise term (which one wishes to minimise or remove) or alternatively critical to the operation of the metrology system [163, 165, 166, 204-210]. Finally, digital optical components (SLMs and CCDs) are more widely used than ever. The digital processing of optical information and the requirement to numerically analyze both digital and analogue optical systems (i.e. systems governed by the LCT) is of ever increasing importance [21,46,52, 53], in particular we note the growing importance of computational imaging techniques. Much work remains to be done for those attempting to follow in the footsteps of pioneers like Prof Kehar Singh.

6 Conclusion

In this paper, a brief overview of the optical signal processing (OSP) and its basic concepts and theories have been presented. Both optical implementations and numerical algorithms have been briefly reviewed. The Fourier transform (FT) and its generalizations, including the fractional Fourier transform (FRT), the Fresnel transform (FST), and the most general case linear canonical transform (LCT) are discussed. The DFT is described involving moving from the continuous OFT to its discrete numerical implementation. The Nyquist sampling theorem is considered carefully and it is noted how to avoid aliasing in both the space and the spatial frequency domains. Then two main topics of the OSP applications are introduced: optical image encryption systems and digital image watermarking techniques. It is discussed how, when simulating the OFT using the DFT, the width and bandwidth of a 2D signal evolve. It is noted that when multiplying two signals in the space domain, the resulting spatial frequency distribution will have a bandwidth equal to the sum of the bandwidths of the two product signals. A similar effect occurs when multiplying two signals in the frequency domain. Two widely used applications of OSP, i.e. optical encryption and digital watermarking, have been reviewed in terms of fundamental concepts and existing methods.

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