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Note on the use of interference terms of Wigner distribution function for specklegram analysis

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Interference term produced by Wigner distribution function is proved to be useful for specklegram analysis. Displacement information of specklegrams can be numerically extracted by low pass filtering the unwanted interference terms of the Wigner distribution output. This study is verified through computer simulations.

1 Introduction

Fourier transform is a classical method used for representing frequency spectrum of stationary signals [1]. For non-stationary signals, time or space variation of frequency information is obscured, because its frequency domain information is defined over an infinite period of time or space. There are alternative non-stationary signal representations such as short time Fourier transform (STFT), wavelet transform (WT) and Wigner distribution function (WDF) which are computed by applying a particular window function to the original signals and taking the conventional Fourier transform of the resultant windowed signals which have finite length [2-4]. By using this technique, it is assumed that the signal is stationary within the window length. As a result, an approximation to the frequency spectrum localized at a midpoint of the window interval can be achieved. Consequently, the time or space-frequency resolution of the signal representation technique is inversely related to the window length. Increasing the window length increases the frequency resolution while, at the same time, it reduces accuracy of the frequency resolution of the representation.

Unlike the STFT or the WT which use particular window functions, the WDF employs a window function which is the analyzed signal itself. As a result, its resolution is dependent upon the length of the original signal. For this reason, the WDF has advantages over the other signal representations in that firstly, both of the time/space and frequency resolutions are inherently optimized. Secondly, varying the width of the analyzing window functions is not needed. Thirdly, its computation is non-iterative. However, although having these attractive properties, it is widely known that the WDF suffers from an inherent interference between either multicomponents of the analyzed signal or instantaneous positive and negative frequency components [5,6].The latter problem can be avoided, provided the analyzed signals are analytic [7].

In the present paper, we show that the interference term of the WDF can be used for measuring space-varying displacement from specklegrams [8,9]. Since the WDF of 2D signal pattern gives 4D quantity, the present study considers the 2D specklegram as consisting of 1D signal patterns. This consideration is implemented by converting digitally pairs of 2D speckles existing along a certain row of the specklegram into pairs of 1D speckles. The advantage of this measurement method is that firstly, since specklegrams can be directly recorded by a charge-coupled device image sensor, the 2D to 1D optical conversion of speckles is not needed [10]. Secondly, its displacement information can be numerically extracted by using the WDF. Thirdly, the measurement accuracy can be improved. The present study is done via computer simulations.

2 Theory

2.1 Specklegrams

Specklegram contains subjective speckle patterns of an object before and after displacements recorded on single photographic film. If a correlation of the two speckle patterns is maintained between the

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two exposures, then information of the displacement can be extracted by using an optical Fourier transform. If $I(x,y)$ represents the intensity distribution of the speckle pattern recorded by an image sensor, the total exposure of the specklegram can be written as [11]

$$I_t(x,y) = I(x,y) + I[x - d_x(x,y), y - d_y(x,y)], \tag{1}$$

where $d_x(x,y)$ and $d_y(x,y)$ correspond to the displacement components in the x and y directions, respectively. By assuming that the average intensities of the first and second exposures are uniform in the entire area of patterns, the recorded intensity can be expressed as a convolution

$$I_t(x,y) = I_0 \otimes \sum_n [\delta(x-x_n, y-y_n) + \delta(x-x_n-d_x(x,y), y-y_n-d_y(x,y)], \tag{2}$$

with I_0 and $\delta(x-x_n, y-y_n)$ represent approximately the uniform intensity over the entire patterns and the random distribution of speckles, respectively.

2.2 The WDF

The WDF is a mathematical technique which has been introduced in signal analysis to overcome the inability of Fourier analysis to provide local frequency spectra. The WVD of a signal pattern $g(x)$ is defined as [4]

$$W_g(x, f_x) = \int_{-\infty}^{\infty} g(x+\xi/2)g^*(x-\xi/2)\exp(-i2\pi f_x\xi)d\xi, \tag{3}$$

where * stands for the complex conjugate: Equation (2) can be considered as a Fourier transform of the instantaneous autocorrelation. It gives simultaneously space and frequency representations of the signal $g(x)$. In order to get better insight into the WDF, the WDF of a summation of two delta functions located at $x = x_0$ and $x = x_1$

$$g(x) = \delta(x - x_0) + \delta(x - x_1) \tag{4}$$

is considered. The WDF of Eq (4) is equal to

$$W_g(x, f_x) = \delta(x - x_0, f_x) + \delta(x - x_1, f_x) + 2\delta\{x - (x_0 + x_1)/2, f_x\} \cos\{2\pi f_x(x_1 - x_0)\}. \tag{5}$$

The first two terms are the 2D delta functions appearing at the original spatial position along the spatial frequency axis, because the delta function contains infinite frequency components. The last term is the interference term of the two delta functions which is equal to the product of delta and cosine functions. The position of the interference term in the WDF plane is in the middle of the two delta functions. The frequency of the cosine function is proportional to the separation information. The 3D plot of this $W_g(x,f)$ output is shown in Fig 1. If the two delta functions of Eq (1) represent two correlated 1D speckles obtained before and after displacements, measurement of the frequency of the cosine function gives the displacement information.

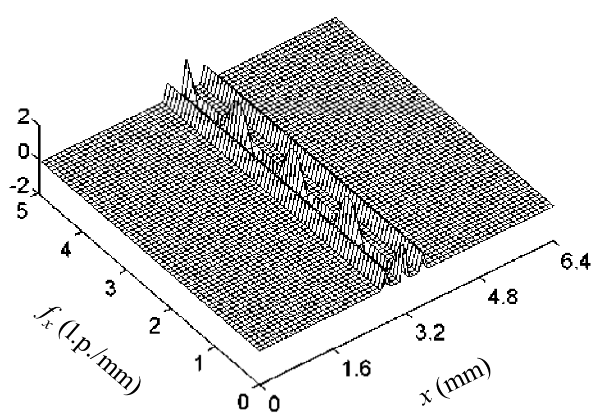

Fig 1. WDF of two delta functions given by Eq. (4)

3 Computer simulations

In order to study feasibility of the use of the interference term, 1D specklegram consisting of three pairs of speckles was simulated by using RAND command of Matlab. Figure 2 shows the simulated specklegram whose displacement increases linearly with respect to the position. Figure 3(a) shows the WDF of this 1D specklegram.The figure shows that besides six uniform spatial frequencies, the interference terms appear almost everywhere, in particular the region between the first and the last pairs of the correlated speckles. These unwanted interference terms are produced by uncorrelated speckles. Consequently, it is hard to measure the displacement. Since the displacement produced by the pair of the correlated speckles is usually smaller than that of the uncorrelated speckles, the corresponding frequency is also lower.

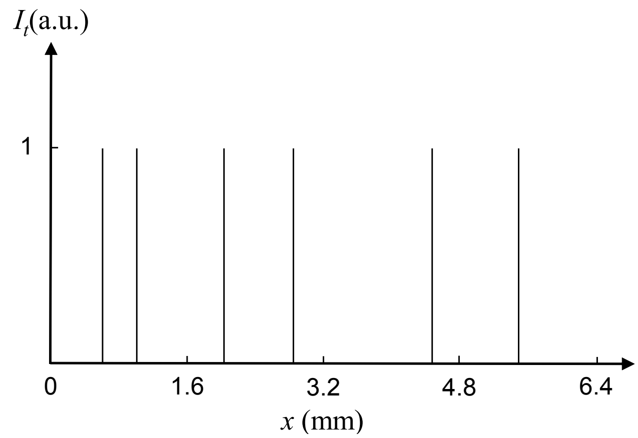

Fig 2. Three pairs of correlated speckles with space-varying displacements.

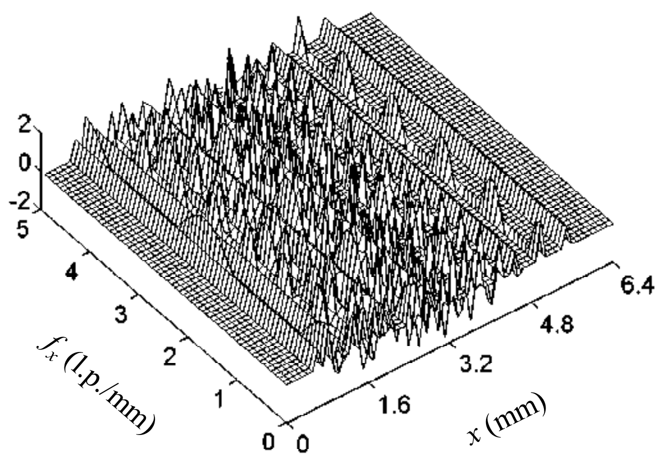

Fig 3. WDF output of the simulated specklegram shown in Fig 2.

In order to solve this problem, a low pass filter can be used to remove the unwanted interference terms appearing along the frequency axis. Figure 4 shows the WDF output obtained by low-pass filtering with the cut-off frequency which is equal to a half of the frequency caused by a maximum displacement. The cosine interference terms of the pair of the correlated speckles appear undistorted parallel to the frequency axis. It is obvious that the frequency of the interference term increases as a function of the position. This is in agreement with the characteristics of the space-variant specklegrams.

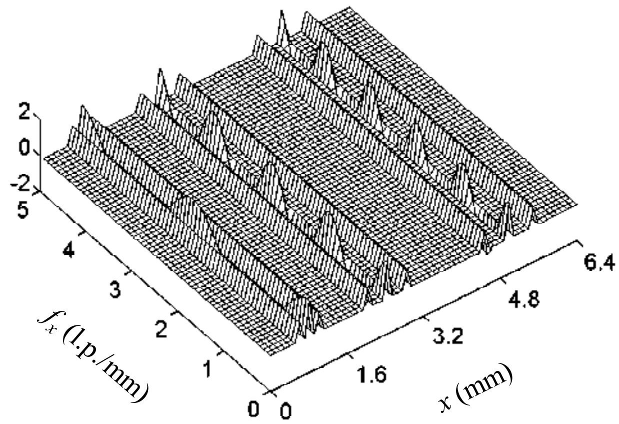

Fig 4. WDF output by low pass filtering.

4 Conclusions

We have studied the application of the interference term of the WDF for measuring local displacement from the specklegrams. In conjunction with the use of the low pass filter, the result shows that the interference term is useful for the displacement measurement.

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