ap

Asian Journal of Physics

Vol. 28, Nos 10-12 (2019) 983-988

Available on: www.asianjournalofphysics.in

Phase reconstruction using compressive parallel phase shift digital holography with Haar wavelet sparsification

Prakash Ramachandran[1] and Anith Nelleri[2]

[1]*Vellore Institute of Technology (VIT), Vellore-632 014, Tamilnadu, India*

[2]*Vellore Institute of Technology (VIT), Chennai, 600 127, Tamilnadu, India*

This article is dedicated to Prof Kehar Singh for his significant contributions to Optics and Photonics

Parallel phase shift digital holography (PPSDH) is a single exposure linear holographic scheme and much suitable for 3D imaging of moving specimens. The linearity of this scheme fits well in to the compressive sensing (CS) frame work. In this paper, we have proposed a method in which the compressive sensing is applied to a two-step parallel phase shift digital holography with Haar wavelet sparsification. A superior phase reconstruction was obtained by this method since the CS approach compensates the noise in the retrieved Fresnel field computed from PPSDH holograms that aroused due to the loss of pixels and approximations involved in parallel phase shift digital holography scheme. The robustness of this CS based method was demonstrated by performing the reconstruction from holograms in which only 50% of the detected Fresnel field sample points were retained. Three methods have been compared such as conventional PPSDH, CS based PPSDH and CS-PPSDH with Haar wavelet sparsified object field. The results show that wavelet sparsified CS-PPSDH is superior to other methods in quantitative phase information reconstruction. The results are presented from numerical experiments to demonstrate the concept.

Keywords: Digital holography, Phase reconstruction, Compressive sensing, Parallel phase shift digital holography, Sparsity, Wavelet sparsification.

1 Introduction

Compressive sensing (CS) is a paradigm shift in sampling theorem that suggests a much lesser sampling rate than Nyquist rate. In compressive Fresnel digital holography [1-10], the original sparse object wave is reconstructed from lesser number of hologram samples. The CS is basically an iterative procedure to reconstruct the original signal from the incomplete linear measurements [11-15] exploiting the sparsity of the signal. The signal is said to be sparse if it has a very less number of non-zero or significant samples. The signal shall be sparse either in the spatial domain or in the transform domain like FFT, wavelet, DCT etc. The CS has been applied in many holographic schemes and of late, the compressive parallel phase shift digital holography (PPSDH) [9-10] is gaining its attention since it is a linear single exposure scheme suitable for 3-D imaging of moving specimens.

The ability of phase reconstruction is the strength of digital holography. This ability of quantitative phase reconstruction and measurement has great implications in 3-D microscopy of moving samples and phase contrast imaging. Even though the phase reconstruction is accurate in classical phase shift digital holography (PSDH) [16], it is not suitable for moving specimens as it involves multiple exposures. The PPSDH is a single exposure scheme used for imaging moving specimens. The PPSDH, which is evolved from the four-step classical PSDH, uses a spatial array device to give the four-phase shifts in parallel to the plane reference wave in an interleaved manner to obtain the single shot in-line hologram. The four individual phase shifted digital holograms are later separated from the single shot PPSDH for the reconstruction. This procedure leads

Corresponding author :

e-mail : anith.nelleri@vit.ac.in (Anith Nelleri)

to loss of information, when the four separate holograms are used for the reconstruction [17]. In two-step parallel phase shift digital holography, two phase shifts are applied in a parallel and an interleaved manner to record the single-shot in-line hologram [18,19]. This is similar to the procedure used in the four-step PPSDH method. The two step parallel phase shift digital holography scheme is evolved from the two-step PSDH. The two-step PSDH, in which, only two holograms are recorded with reference wave phase shifted by two angles is a variation of classical PSDH. Even though the two-step PPSDH is a single exposure scheme with better reconstruction accuracy than the four- step PPSDH, it involves loss of information. To compensate this loss of information, CS was successfully applied to the two-step PPSDH to reconstruct the quantitative phase information [10]. Also, it is shown in the referred work that, the phase reconstruction accuracy using CS based two-step PPSDH is superior to that of CS based single exposure in-line digital holography (SEOL) [8]. It has been shown that the phase reconstruction is superior in CS-PPSDH than CS-SEOL as there involves a nonlinear term in the measurement model of SEOL. The linearity in the measurement model is an important requirement of CS implementation. The PPSDH is a pure linear single exposure model [10] and thus fulfils this requirement. If the object wave is sparse, for example a complex grating, the separate sparsification is not required for the implementation of CS framework to obtain good quality complex image reconstruction. It is required to extend the application of compressive PPSDH for specimens that are not sparse in spatial domain, a separate sparsification is required. In the present work, the CS-PPSDH is demonstrated with the Haar wavelet based sparsification for improving the reconstruction accuracy. The present paper demonstrates that the phase reconstruction accuracy of CS-PPSDH with Haar wavelet sparsification is superior to that of the conventional methods. In the present study, three methods such as conventional PPSDH, CS-PPSDH and CS-PPSDH with Haar wavelet sparsified object field have been compared for reconstruction accuracy. The comparison shows that the accuracy of wavelet sparsified CS-PPSDH method is superior to other methods for quantitative phase information reconstruction. The wavelet based CS-PPSDH algorithm applied to the complex object wave field has better reconstruction accuracy by using just 50% of the Fresnel field pixels detection. This demonstrates the robustness of the wavelet based CS-PPSDH. The necessary mathematical framework of all our implementations is discussed and the results of the numerical simulations are analysed quantitatively and reported in this study. The improvement in phase reconstruction accuracy by the proposed method will be highly impactful when it is extended to three-dimensional object recognition and classification applications [20-23].

2 Mathematical framework for compressive PPSDH with sparsification

The complete mathematical model of two-step PPSDH and the application of CS framework in PPSDH are demonstrated by Ramachandran *et al* [10]. In this section, a brief mathematical discussion is done on the numerical simulations considered for this study. The single exposure two-step PPSDH [18] scheme is evolved from double exposure two-step PSDH scheme. In two-step PSDH scheme [19], two holograms are obtained in two different exposures in which the reference wave is retarded by a phase shift of 0 in the first exposure and of $(-\pi/2)$ in the second exposure. In this scheme the intensity of the reference wave has to be larger than that of object wave. The intensity and phase of the Fresnel field is calculated from these two phase shifted digital holograms using Meng's formula [18,19]. The inverse Fresnel propagation of the Fresnel field reconstructs the original object wave. In two-step PPSDH, single exposure detection is done by passing the reference wave through a spatial array device in which the alternate pixels are designed to provide phase shifts of 0 and $(-\pi/2)$ to the reference carrier in an interleaved manner. During the reconstruction stage, the holograms corresponding to 0 and $(-\pi/2)$ are individually separated from the single exposure recording with periodic loss of pixels. The separated holograms are linearly interpolated and the Fresnel field is obtained by using Meng's formulae [19] using these two holograms. This process of obtaining Fresnel field from the object wave using two-step PPSDH is symbolized as a linear operator $\widetilde{\mathcal{F}}_d$ and the Fresnel field obtained will be an approximation of the original Fresnel field. Thus the object wave reconstructed by inverse Fresnel propagation of the approximated Fresnel field will be manifested by approximation noise. Let us model this with an operator $\tilde{\mathcal{F}}_d$, and $\tilde{U}$ be the noisy and ill conditioned version of the original Fresnel field U as shown in Eq (1).

$$\tilde{U} = \tilde{\mathcal{F}}_d O_0 \tag{1}$$

As the linearity of the operator $\tilde{\mathcal{F}}_d$ is justified [10], CS can be applied on these noisy incomplete measurements of Fresnel field to reconstruct the sparse complex input function O_0 by solving the unconstrained optimisation [15] shown by the Eq (2),

$$\underbrace{\min}_{O(x,y)} \frac{1}{2} \|\tilde{U} - \tilde{\mathcal{F}}_d O(x,y)\|_2^2 + \tau \|O(x,y)\|_1 \tag{2}$$

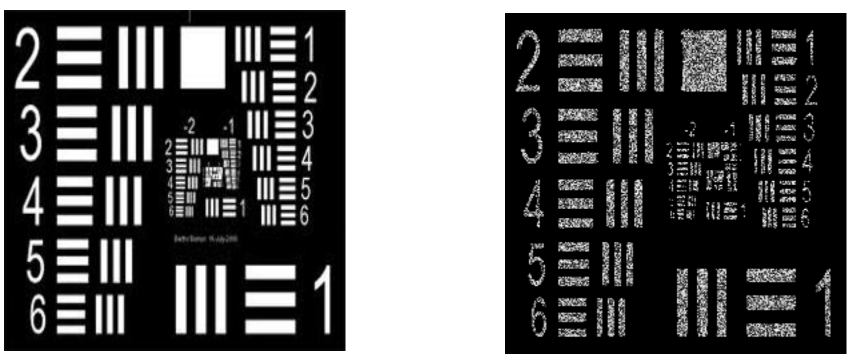
In Eq (2), if $O(x, y)$ is not sparse in spatial domain but sparse in transform domain such that $O(x,y) = \phi s$, ϕ is an $N \times N$ basis matrix and $\boldsymbol{s}$ is sparse in nature, Eq (2) can be expressed as in Eq (3)

$$\underbrace{\min}_{O(x,y)} \frac{1}{2} \|\tilde{U} - \tilde{\mathcal{F}}_d \phi_s\|_2^2 + \tau \|\boldsymbol{s}\|_1 \tag{3}$$

Here $\tilde{\mathcal{F}}_d$ and ϕ are termed as sensing and sparsifying matrices, respectively. The properties of $\tilde{\mathcal{F}}_d$ and ϕ are to be mutually non-coherent for the accurate reconstruction of the signal $O(x, y)$ [11-15]. In the present simulation, Eqs (2) & (3) are solved using Gradient Projection Sparse Representation (GPSR) [15].The Haar wavelet is used for sparsification in place of the operator ϕ.

3 Numerical simulations

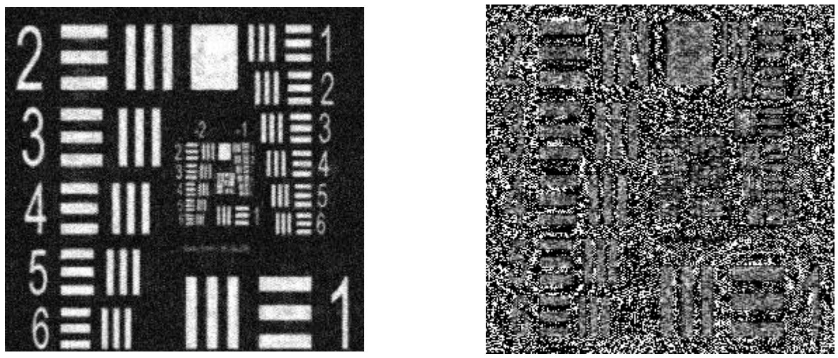
A single shot two-step PPSDH is obtained for a simulated complex object wave of size 256 × 256 pixels. The simulated complex object wave is created using USAF chart as intensity matrix and randomly generated real values ranges in (0-1) as phase information. The amplitude and the phase of the simulated complex object wave is shown in Fig 1.



(a) (b)

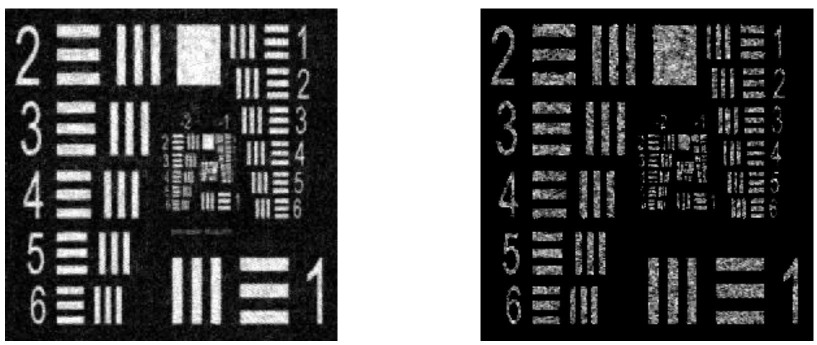
Fig 1. Input complex object wave (a) Intensity and (b) Phase.

It should be noted from Fig 1 that the object field is not sparse as required by the CS. In the numerical experiment, the propagated field matrix, Fresnel field $U(x,y)$ of the object wave at the recording plane is generated by using Fresnel transform of the object function. The parameters used are, propagation distance, d = 10 mm, wavelength λ = 514.4nm, pixel pitch of the detector, $\Delta x = \Delta y$ = 5µm, and the reference wave $R(x,y) = 1$. This Fresnel field $U(x,y)$ is used to simulate the two-step PPSDH. The approximated Fresnel field is computed numerically using Meng's formulae and the interpolated holograms obtained from the PPSDH. The intensity and phase of the reconstructed object wave obtained by direct method (without CS) are shown in the Fig 2. The CS-PPSDH is applied and the results are shown in the Fig 3. This CS-PPSDH method has been implemented by solving Eq (2) without any sparsification. The reconstruction results obtained by extending CS-PPSDH method with Haar wavelet as the sparsifier by solving Eq (3) are shown in Fig 4. The reconstruction was performed using CS-PPSDH with Haar wavelet sparsification by retaining just 50% of the detected Fresnel field pixels. This validates the ability of CS to reconstruct the object wave from incomplete measurements and the results are shown in Fig 5.



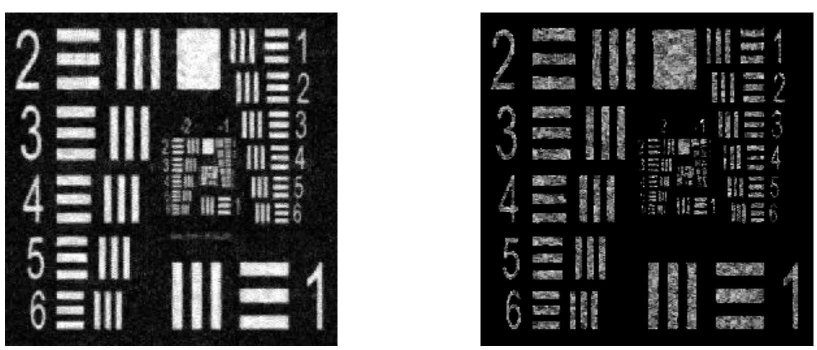
(a) (b)

Fig 2. Reconstructed complex object wave using direct PPSDH (a) Intensity and (b) phase.



(a) (b)

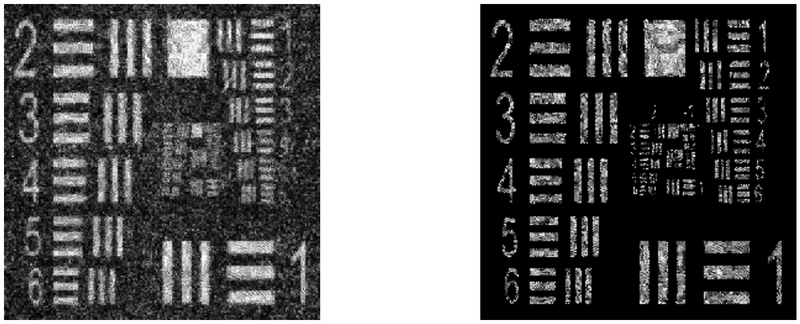
Fig 3. Reconstructed complex object wave using CS-PPSDH without sparsification (a) Intensity and (b) Phase.



(a) (b)

Fig 4. Reconstructed complex object wave using CS-PPSDH with Haarwavelet sparsification (a) Intensity and (b) phase.

The deviation of the intensity and the phase of the reconstructed complex object wave with that of original complex object wave are determined as mean square error (MSE) and are tabulated in Table 1. To appreciate the phase reconstruction capability of the methods the deviation is determined for intensity and phase separately and tabulated in the table.



(a) (b)

Fig 5. The complex object wave reconstructed with just 50% of the detected Fresnel field pixels and using CS-PPSDH with Haarwavelet sparsification (a) Intensity and (b) Phase.

Table 1. Deviation between original and reconstructed complex object waves (MSE)

Methods	Intensity Deviation	Phase Deviation
Direct PPSDH	15.4	39.4
CS PPSDH (without sparsification)	12.3	36.3
CS PPSDH (with Haar wavelet sparsification)	9.1	22.3
CS PPSDH (with Haar sparsification and using only 50% of pixels of Fresnel field)	18.1	37.9

4 Results and discussion

The input complex object wave used in the simulation shown in Fig 1 is not a sparsefield and the phase of the object wave is not uniform but randomly varying at every point. The intensity and the phase of the complex object wave reconstructed using direct method are shown in Fig 2. It is subjectively visible that the reconstructed intensity is blurred and the phase is constant throughout the image, whereas the original phase of the object wave varies in each and every pixel. It can be inferred from the Figs 4 & 3 that, the reconstructed results of CS-PPSDH with and without sparsification are subjectively superior than the direct method in terms of accuracy of both intensity and phase. The variations in the reconstructed phase in the CS method are similar to that of the phase of the original input complex object wave. The quantitative analysis presented in Table 1 shows that CS-PPSDH with Haar wavelet as sparsifier is the most superior method as its reconstruction accuracy of complex object wave is very high giving the least deviation with the original complex object wave. In addition, the CS based algorithm reconstructs the original complex object wave using only the 50% of the Fresnel field pixels detection. The results are subjectively good and shown in Fig 5.

5 Conclusions

In this paper, we have demonstrated a compressive two-step parallel phase shift digital holographic technique (PPSDH) that enables an accurate and quantitative complex object wave reconstruction. Both intensity and phase reconstruction using this method was found to be much superior to that of the conventional direct method. The linear reconstruction approach in PPSDH technique has fulfilled the linearity requirement of CS algorithm and also the CS approach compensates the noise in the retrieved Fresnel field computed from PPSDH holograms that aroused due to the loss of pixels and approximations involved in parallel phase shift digital holography scheme. It has also been shown that the object wave which is not sparse in spatial domain can be reconstructed with greater accuracy using CS-PPSDH with Haar wavelet sparsification. Three methods have been compared such as conventional PPSDH, CS based PPSDH and CS-PPSDH with Haar wavelet sparsified object field. The results show that wavelet sparsified CS-PPSDH found to be superior to other methods in quantitative phase information reconstruction. The single exposure recording involved in the PPSDH is useful for imaging the dynamic events or moving objects. On the whole, we conclude that the CS based approach to PPSDH has promising applications in the field of quantitative phase contrast microscopic imaging of moving objects.

Acknowledgement

This work was supported by the Science and Engineering Research Board (SERB), Department of Science and Technology (DST), Government of India under the grant no. CRG/2018/003906.

References

1. Brady D J, Choi K, Marks D L, Horisaki R, Lim S, Compressive Holography, *Opt Express,*17(2009)13040-13049.
2. Rivenson Y, Stern A, Javidi B, Overview of compressive sensing techniques applied in holography, *Appl Opt*, 52(2013)A423-A432.
3. Rivenson Y, Stern A, Javidi B, Compressive Fresnel Holography, *J Disp Technol*, 6(2010)506-512.
4. Rivenson Y, Stern A, Conditions for practicing compressive Fresnel holography, *Opt Lett*, 36(2011)3365-3367.
5. Ramachandran P, Alex Z C, Nelleri A, Compressive Fresnel digital holography using Fresnelet based sparse representation, *Opt Commun*, 340(2015)110-115.
6. Ke K, Ashok A, Neifeld M A, Block-wise motion detection using compressive imaging system, *Opt Commun*, 284(2011)1170-1180.
7. Wu X, Yu Y, Zhou W, Asundi A, 4f amplified in-line compressive holography, *Opt Express*, 22(2014)19860-19872.
8. Rivenson Y, Stern A, Javidi B, Improved depth resolution by single-exposure in-line compressive holography, *Appl Opt*, 52(2013)A223-A231.
9. Wan Y, Man T, Wu F, Kim M, Wang D, Parallel phase-shifting self-interference digital holography with faithful reconstruction using compressive sensing, *Opt Laser Eng*, 86(2016)38-43.
10. Ramachandran P, Alex Z C, A. Nelleri A, Phase Reconstruction Using Compressive Two Step Parallel Phase Shifting Digital Holography, *Opt Eng*, 57(2018)043105; doi.org/10.1117/1.OE.57.4.043105.
11. Candès E, Compressive sampling, Proceedings of the International Congress of Mathematicians, Madrid, Spain, (2006)1-20
12. Donoho D L, Compressed sensing, *IEEE Trans on Inf Theory*, 52(2006)1289-1306.
13. Candès E, Romberg J, Sparsity and incoherence in compressive sampling, *Inverse Problems*, 23(2007)969-985.
14. Candes E, Wakin M, Gradient projection for sparse reconstruction, *IEEE Signal Process Mag*, 25(2008)21-30.
15. Figueiredo M A T, Nowak R D, Wright S J, Gradient projection for sparse reconstruction, *IEEE J Sel Topics Signal Process*, 1(2007)586-597.
16. Yamaguchi I, Zhang T, Parallel quasi-phase digital holography, *Opt Lett*, 22(1997)1268-1270.
17. Awatsuji Y, Sasada M, Kubota T, Parallel quasi-phase digital holography, *Appl Phys Lett*, 85(2004)1069-1071.
18. Awatsuji Y, Tahara T, Kaneko A, Koyama T, Nishio K, Ura S, Kubota T, Matoba O, Improving image quality of parallel phase-shifting digital holography, *J Phys: Conf Ser*, 139(2008)012009; doi:10.1088/1742-6596/139/1/012009.
19. Meng X F, Cai L Z, Xu X F, Yang X L, Shen X X, Dong G Y, Wang Y R, Two-step phase-shifting interferometry and its application in image encryption, *Opt Lett,* 31(2006)1414-1416.
20. Nelleri A, Gopinathan U, Joseph J, Singh K,Three-dimensional object recognition from digital Fresnel hologram by wavelet matched filtering, *Opt Commun*, 259(2006)499-506.
21. Nelleri A, Joseph J, Singh K, Recognition and classification of three-dimensional phase objects by digital Fresnel holography, *Appl Opt*, 45(2006)4046-4053.
22. Nelleri A, Joseph J, Singh K, Digital holography for three-dimensional information processing: Application to object recognition and information security, Asian J Phys, 15(2006)253-273.
23. Nelleri A, Joseph J, Singh K, "Phase reconstruction in lensless digital in-line holographic microscopy, Opt Lasers Eng, 48(2010)27-31.

[*Received*: 25.11.2019; *revised recd*: 11.12.2019; *accepted*: 19.12,2019]