Asian Journal of Physics
Vol. 25, No 3 (2016) 387-410
Available on: www.asianjournalofphysics.in

Predictions of Solar Cycle

Sushanta C Tripathy
National Solar Observatory, 950 N Cherry Avenue, Tucson, AZ 85719, USA

The Sun's activity measured through many of its proxies varies in a periodic manner with an average duration of about 11 years. The empirical relations based on the periodicity are considered as the first generation methods to predict the maximum amplitude of the next solar cycle. These methods which are statistical in nature fall into two different categories: precursor methods and extrapolation methods and has been widely used in the later part of the 20th century. Recent advances include model based predictions which combines dynamo models with observational data to predict not only the maximum amplitude of the solar cycle but also the timing of the activity maximum. In this review, we focus on different prediction methods and compare their outcome for previous cycles with an emphasis on cycle 24. We further compare various predictions for solar cycle 25.

Keywords: Solar Cycle; Predictions; Solar Activity

1 Introduction

The level of magnetic activity of the Sun varies over time showing a periodic behavior. The prominent indicator among them is the number of sunspots observed at the solar surface that has been counted for about 400 years. An intriguing aspect of the sunspots is the 11-year periodic variation known as the sunspot cycle. Although the number of sunspots has been carefully counted since the early 1700s, it was Heinrich Schwabe who pointed out that sunspots increase and decrease with a period of about 10 years. Later, it was found that not only the sunspots but other solar activities fluctuate with periods closer to 11 years. The term solar activity cycle is now used to describe any parameter or phenomena on the Sun that vary with a period of about 11 years.

As the study of solar activity progressed, it was apparent that the energetic phenomena e.g solar flares, coronal mass ejections etc. also vary with the solar cycle. These explosive events influences the Earth's geomagnetic field by shooting energetic photons and other charged particles toward the Earth. For example, in 10 March 1989 an X-class flare gave rise to a geomagnetic storm which blocked out the power system in Canada and caused voltage and power fluctuations across North America. In space, some satellites actually tumbled out of control for several hours. NASA's TDRS-1 communication satellite recorded over 250 anomalies as high-energy particles invaded the satellite's sensitive electronics. Even the Space Shuttle Discovery was having its own mysterious problems[1]. It is estimated that the storm cost about $30 million.

During an active solar period, such violent eruptions occur more often and potentially could disrupt satellites, Global Positioning System (GPS) signal transmission and reception, power grids and radio communications and even threaten astronauts in space or airline passengers flying through the polar routes. Another effect that increases during these active periods is the satellite drag and requires boosting of the

[1]*http://www.nasa.gov/topics/earth/features/sun_darkness.html*

Corresponding author :
e-mail: stripathy@nso.edu (Sushanta C Tripathy)

satellites to maintain their orbit. It is estimated that during the solar maximum period, a satellite needs a boosting every 2-3 weeks. All these time varying environmental conditions within the space surrounding the Earth has been defined as the space weather (see the article by Jain and Komm [1] in this volume). The likelihood of the occurrence of disruptive events follow closely the intensity of the activity cycle, hence accurate forecasting of solar activity on time scales of few years is of considerable importance to National Oceanic & Atmospheric Administration (NOAA) and National Aeronautics and Space Administration (NASA) to plan and operate space missions. Thus the need to forecast solar activity cycle became a critical component of space weather.

Predicting the activity levels of the Sun is a stimulating topic since it reflects our current understanding of the generation of the magnetic field and the progress in the solar dynamo modeling. Due to the close relationship between the relative sunspot number and solar activity demonstrated over the last 60 years and the existence of long historical records of the sunspot observations, most of the efforts to forecast solar cycle have been concentrated to predict the peak amplitude of the smoothed sunspot number together with the time of the solar maximum. Recently, effort has begun to use the forecasted sunspot number as predictors of other solar indices through the relationship between them. The sunspot number will remain the basic quantity for solar cycle forecasting till we gain a better understanding of the solar activity cycle. We therefore begin the review with a brief description of the sunspot number. In section 2, we will describe different prediction methods. In section 3 we will assess how the past predictions agreed with the observed amplitudes and timing. Section 4 will be devoted to the prediction from dynamo models while in Section 5, we will summarize the prediction for cycle 25. A short conclusion is presented in Section 6.

1.1 The sunspot number

Although a large number of solar activity indices are now measured from different observatories, the sunspot number is still the most commonly used index as it has been counted over the last 4 centuries. Currently the activity index is available in two main forms: the relative sunspot number initiated by R. Wolf in 1849 and the group sunspot number constructed by Hoyt and Schatten [2]. The definition of the relative sunspot number, R, as defined by Wolf [3] can be expressed as

$$R = k\,(10 \times G + N) \tag{1}$$

where G is the number of sunspot groups, N is the total number of spots visible on the solar disk, and k is a correction factor which accounts for the differences in the number of recorded sunspot number by different observers. Wolf set this constant to be 1 since he was the primary observer, however, successive observers counted smallest spots and as a result the counts resulted in a larger value. In order to reconcile the pre-Wolf counting to the new ones, so that a homogeneous data set is produced, the correction factor was reduced to $k = 0.6$ for subsequent determination of R [4, 5]. In addition to the relative sunspot number, Wolf also used historical observations to reconstruct the monthly mean values dating back to 1749. In this way, a single series depicting the 11 year sunspot cycle was constructed retaining the sunspot numbering cycle. In most analyses, the sunspot series is largely assumed to be a homogeneous series.

Since the sunspot number prior to the early 19^{th} century were uncertain and discrepancies were noticed specially during the transitions of observers, Hoyt and Schatten [2] introduced the group sunspot number (GSN) as an alternate proxy of solar activity to serve as a long term index. This discards the number of spots in each group and counts only the sunspot groups which are generally reliable and defines it as a weighted average of all observables available for a given day. The GSN series has been constructed for the whole period starting from 1611. It is generally agreed that these are more robust during the period of 1611-1818 compared to the relative sunspot number.

Unfortunately, the two series do not match and several efforts were recently made to recalibrate and reconstruct two composite series which appears to be consistent compared to the old series (for a review, see Clette *et al* [6]) and data can be downloaded from http://sidc.oma.be.silso/home. Figure 1 depicts the recalibrated, new sunspot time series. This still shows large uncertainties prior to 1800 due to poorly observed periods. The revised sunspot series clearly shows the progressive decline of solar activity before the onset of the Maunder Minimum (during which the sunspots were absent) and a slow rising trend after the Maunder Minimum to the recent levels. However please note that most of the predictions that we will discuss here are based on the old sunspot time series known as the International (Wolf/Zurich) sunspot number which is now calculated at the Solar Influences Data Center (SIDC, formerly Sunspot Index Data Center) in Brussels, Belgium starting from 1981 and serves as the authoritative source of archive sunspot data[2]. A comparison plot of International sunspot number and group sunspot time series can be found in [7].

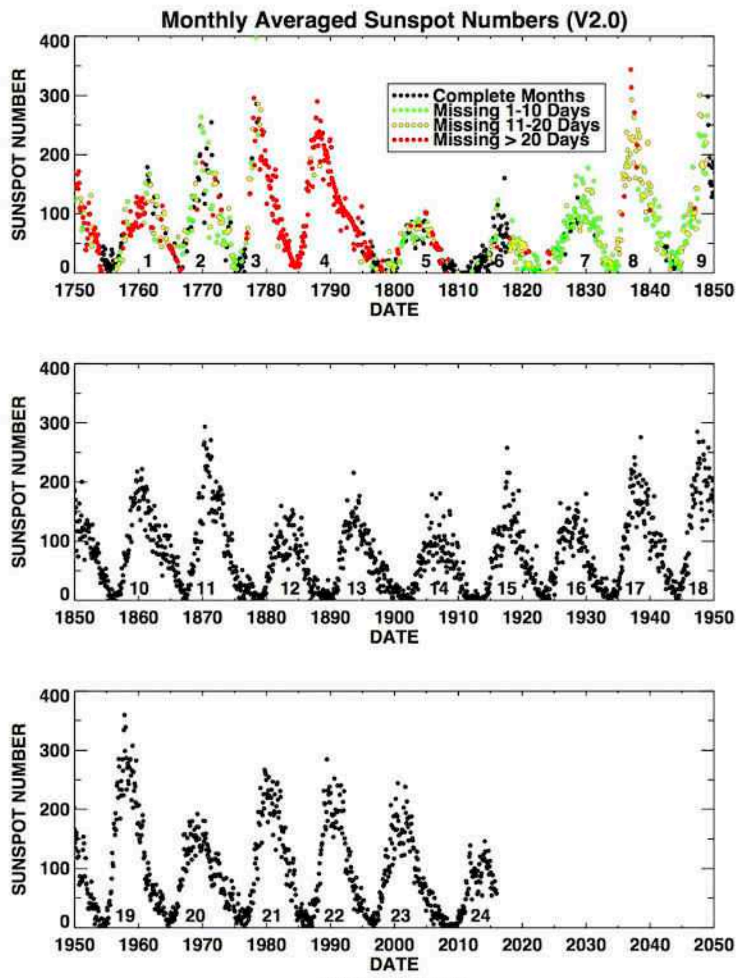

Fig 1. The variation of the monthly smoothed revised sunspot number during the period 1749 – 2016. The numbers in each panel represent the solar cycle number. V2.0 refers to the revised sunspot series, for details, see Clette *et al* [6]. Credit *http://solarscience.msfc.nasa.gov*

In addition to the sunspot number, sunspot area (available from 1874) and geomagnetic indices (the prominent of which is the *aa* index available since 1868) have been also used in the cycle prediction. Other

[2]*http://sidc.oma.be*

recent indices of solar activity available in the second half of 20th century includes magnetic and plage indices and more popular spectral irradiance flux at radio wavelength of 10.7 cm ($F_{10.7}$) which are available from 1947 onwards. However, due to the availability over a time scale which is considered to be of insufficient length from a prediction point of view, uses of these activity indicators are limited.

A search on Astrophysics Data System (ADS)[3] dealing with solar cycle predictions returned more than 2300 abstracts. Obviously, it is nearly impossible to review all the existing literature with a wide variety of prediction methods and observables. Thus the review presented here reflects my personal bias.

2 Prediction techniques

A sunspot cycle as shown in Fig 1 is defined as occurring from one sunspot minimum to the next sunspot minimum and the interval 1755-1765 is designated as cycle 1. Although sunspot cycles display a wide range of amplitudes as well as variations in length and shape, the time series analysis have demonstrated some consistent behavior that can be utilized for prediction of sunspot maxima in successive cycles. The NOAA's involvement to examine different predictions (and making a consensus predictions) for the last two solar cycles have not only accelerated the development of better forecasting methods but has also added emphasis to understand the physical formulation behind these predictions. In addition, the extended minimum between the cycle 23 and 24 and the weak cycle 24 has also provided impetus to improve the prediction methods.

Since Yule [8] first proposed a technique called autoregressive method, a great variety of techniques have been formulated to predict the solar cycle. In the absence of realistic dynamo models, statistical methods have been developed to predict both mid- and long-term values of the cycle amplitude and in some cases, timing of the sunspot maximum. Broadly these can be divided into four different categories: (i) Regression techniques also known as empirical methods or time series analysis (more recently categorized as climatology by solar cycle 23 panel) (ii) Precursor techniques (iii) Non-linear techniques such as pattern recognition and (iv) Predictions based on the dynamo models. For recent reviews on this topic, see [9, 10].

2.1 Regression Techniques

The technique assumes that the future is predictable from the statistical properties of the past. It has the advantage of predicting many cycles into the future since it is assumed that all the important periodicities are contained in the past data. The simplest example of this method is the average of all observed maxima which for solar cycle 24 would be 114.15 ± 40 where the uncertainty is one standard deviation from the mean. In the reminder of this article, the amplitude of the annually averaged sunspot number for solar cycle n will be labelled as Rn.

For medium-term prediction, the statistical method of McNish and Lincoln [11] remains the reference which was used to predict the annual sunspot number of the current cycle one year into the future. This technique is primarily based on the average shape of the sunspot cycle which could be modelled mathematically as a time series and has been widely used for operational purposes in many of the forecasting centers. In this method, as a first approximation, the future value in a cycle is considered as the mean of all past values for that part of the cycle. This estimate is then improved by adding a correction factor proportional to the departure of the earlier values to the cycle from the mean cycle. Mathematically, this can be expressed as

$$R'_{n+1} = \bar{R}_{n+1} + C_n \, \Delta \, R_n \tag{2}$$

where $\bar{R}_{n+1}$ is the smoothed sunspot number for year n+1 averaged over the previous cycles and Cn is the regression coefficients for year n and $\Delta \, R_n$ is the deviation from the averaged value. The regression coefficients

[3] *http://adsabs.harvard.edu/abstract_service.html*

are determined by minimizing the sum of the RMS differences between the predicted and observed variations from the mean cycle at year *n* [12]. This method has been improved to consider monthly sunspot values to predict monthly mean values [13]. Hildner and Greer [14] examined the skill of this method and confirmed that the method is well adapted for predicting solar cycle a few months in advance. However, the method suffers from an inherent problem that it depends on the average shape of the cycle as the base prediction and do not account for the systematic changes in the shape of the cycle with time. For example, Wolf [15] noted that small cycles tend to be longer than the big cycles while Waldmeier [16] reported that small cycles tends to take longer to reach maximum than do big cycles i.e. strong cycles rise faster. This is known as the "Waldmeier effect".

Waldmeier [17] has proposed a different method to predict solar cycle profiles from the steepness of the ascending phase of the cycle and is currently used as a standard method at SIDC. Hathaway, Wilson and Reichmann [18] have proposed another prediction method based on the same empirical ground but using cycle profiles simulated with an analytical function using two parameters: the start time of the solar cycle and its amplitude which was an improvement over the five parameter fit proposed by Elling and Schwentek [19]. The prediction is carried out by fitting the past sunspot number observations to that of the current cycle. Several new predictions also use amplitude –time relations at different latitudes [20, 21, 22]. As an alternate approach, Hiremath [23] modeled the solar cycle as a forced and damped harmonic oscillator and then used the autoregressive model to predict future cycles.

Although regression techniques have been widely used for prediction, a common disadvantage is the length of time required to find a good estimate for the behavior of the current cycle. Thus these methods give better predictions when the solar cycle has progressed about 3 to 4 years after the minimum and has not been very successful for cycle-to-cycle predictions.

2.1.1 Secular Cycles:

This method relies on the long term secular trends seen in the sunspot time series and a number of periodicities have been found in the cycle amplitude. A simple example is the existence of the Gleissberg cycle with a period of about 80-90 years. This relation has been used as a predictive tool after removing the secular trend [24]. A two-cycle periodicity has also been noted where the odd numbered cycle has a larger amplitude than the preceding even-numbered cycle (odd-even rule, [25]). Further a three-cycle saw tooth shaped periodicity has been noted in the *Ap* index [26].

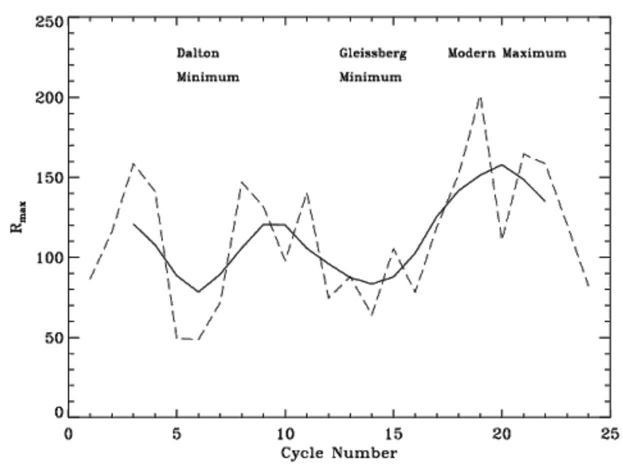

Fig 2. Amplitudes of the sunspot cycles (dashed line) and their Gleissberg filtered values (solid line) plotted as a function of the solar cycle number.

The amplitudes of the sunspot cycles and the Gleissberg filtered sunspot number is plotted in Fig 2 and shows three distinct characteristics. The first is the elevated sunspot activity level starting from mid-20th century and has been termed as “Modern Maximum”. The second is the unusually weak cycles 5, 6, and 7 denoting the “Dalton Minimum”. The final is the moderately weak cycles 12-16 referred to as “Gleissberg Minimum”. However note that these are local minima while the Maunder minimum (Fig 3), the period where sunspots were scarce (1640 – 1705), represents a low activity state of the Sun. Such extended periods of low and high activity are usually referred to as grand minima and grand maxima.

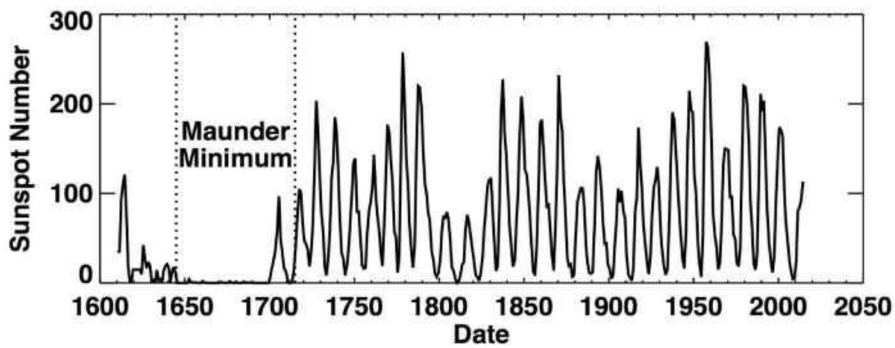

Fig 3. Temporal evolution of yearly smoothed sunspot number showing the period of Maunder minimum where solar activity was very low and very few sunspots were observed. Credit: *http://solarscience.msfc.nasa.gov*.

By their nature, secular methods can provide predictions for few cycles in advance. In general, it is suggested that a minimum in the current high-activity cycle may be expected around the end of 20th century with low amplitude cycles 22 and 23 and even a Maunder Minimum around the year 2200.

2.1.2 Spectral Methods

The spectral method is a class of techniques involving the use of some form of the harmonic analysis with orthogonal basis functions. The classical example is the Fourier analysis besides the Lomb-Scargle periodogram (commonly known as wavelet analysis). All these methods have been applied to the sunspot time series from the beginning of the 20th century. The initial predictions [27-30] underestimated the amplitude of the next cycle. The most recent entrants to this category are the maximum entropy method (MEM) and Singular Spectrum Analysis (SSA).

In MEM method, one attempts to fit sharp spectral features with essentially a polynomial. Using a combination of MEM and Multiple Regression Analysis, Kane [31] estimated the maximum sunspot number to be 140 ± 9 for cycle 23 and 105 ± 9 for cycle 24 to occur in 2000 and 2010-2011, respectively. Both of these predictions do not agree with the observed value.

The SSA method involves the orthogonal decomposition of a time series which isolates significant signal components from the background noise. The method was first applied to the sunspot record by Rangarajan [32] who used this method to pre-filter the data before applying MEM. The predicted sunspot maximum for cycle 23 was 130 during late 2000 and early 2001 and appears to be in good agreement with observed value. Loskutov et al. [33] also used the method and predicted a peak amplitude of 117 for cycle 24 which was later revised to 106 [34].

It should be noted that the solar cycle panel to summarize cycle 23 forecast categorized these class of predictions as climatology (and recent climatology if the prediction used data after solar cycle 17) for their analysis. The same terminology has also been followed by Pesnell [35].

2. 2 Precursor Methods:

Precursor methods are the most common and successful technique in the forecasting field and are considered as half-way between the extrapolation method and dynamo models. These are based on the correlations between certain measures of solar activity at a specified time (usually activity minimum) to predict the maximum strength of the next solar cycle. The precursor may be any proxy of solar activity or other indicators e.g. interplanetary magnetic field or a combination of indices. Due to their significant prediction skill in the past, the two most dominant precursors are the polar flux and the geomagnetic indices besides the sunspot number. Some other possible precursors not discussed here are (i) sunspot area [36] (ii) average latitude of the magnetic field [37] (iii)) solar cycle lengths [38] (iv) Ca II K bright points, (v) coronal emission line intensity, (vi) north-south asymmetry [21] (vii) asymmetry of the ascending and descending phases [39] (viii) the shape and structure of the corona at minimum, (ix) polar coronal holes etc. More details can be found in Wilson [40] and Tlatov [41].

2.2.1 Sunspot number as precursors

A number of patterns e.g. cycle maximum, minimum, length and period have been detected in the observed sunspot record and have been used as precursors. The simplest among these is the average of all the previous cycle which yields a value of $R_{24} = 114.15 \pm 40$ for cycle 24, where the uncertainty represents one standard deviation from the mean.

Another class of prediction technique is based on trends and periodicities in the cycle amplitudes. A simple approach is the correlation between the amplitudes of different cycles. But this turns out to be marginal (35%) yielding a poor forecast. In the amplitude-period method [42], the amplitude of the next cycle maximum is inversely proportional to the period of the previous cycle. Brown [43] showed that the amplitude of the following cycle is correlated with the smoothed sunspot number at the preceding minimum (Fig 4). If we neglect cycle 19, which appears to be an outlier [44], we get a best fit [45]

$$R_{max} = 67.4\ (\pm\ 10.6) + 6.\ 9\ (\pm\ 1.5)\ R_{min} \qquad (3)$$

with a correlation coefficient of 0.72. This relation yields a value of 80 ± 25 for cycle 24 if we use the observed value of 1.7 for cycle 23 minimum. Cameron and Schüssler [46] pointed out that the activity level three years before the minimum is a better predictor of the next maximum which was later found to be 2.5 years [9]. In a slightly different form, Kane [47] used the ratio of the maximum sunspot to the smoothed

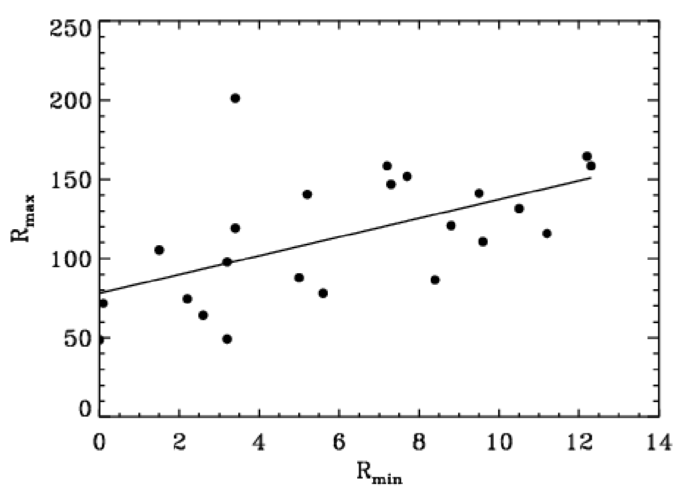

Fig 4. The peak smoothed monthly sunspot number as a function of the same quantity in the preceding minimum. In this plot, cycle 19 is considered an outlier (top left most point)

monthly mean sunspot number over 36 months after the cycle minimum. This predicted that the cycle 24 will be below average in size with a sunspot number of 77 ± 13.4.

2.2.2 Geomagnetic indices

Geomagnetic indices which are based on the changes in the Earth's magnetic field have also been used to predict the amplitude of future cycles. The most common among them is the *aa* index which represents the three hourly averaged global geomagnetic activity index measured at two antipodal stations [48] and is available since 1868. The upper panel of Fig 5 shows the time evolution of the yearly *aa* index and the smoothed sunspot number. The lower panel shows the correlation between the minimum values of *aa* index (aa_{min}) with the maximum of the sunspot number. It is evident that the aa_{min} is directly related to R_{max} for the following cycle which was first found by Ohl [49]. This method is similar to the Maximum-Minimum method but uses the *aa* index at the minimum instead of the sunspot number.

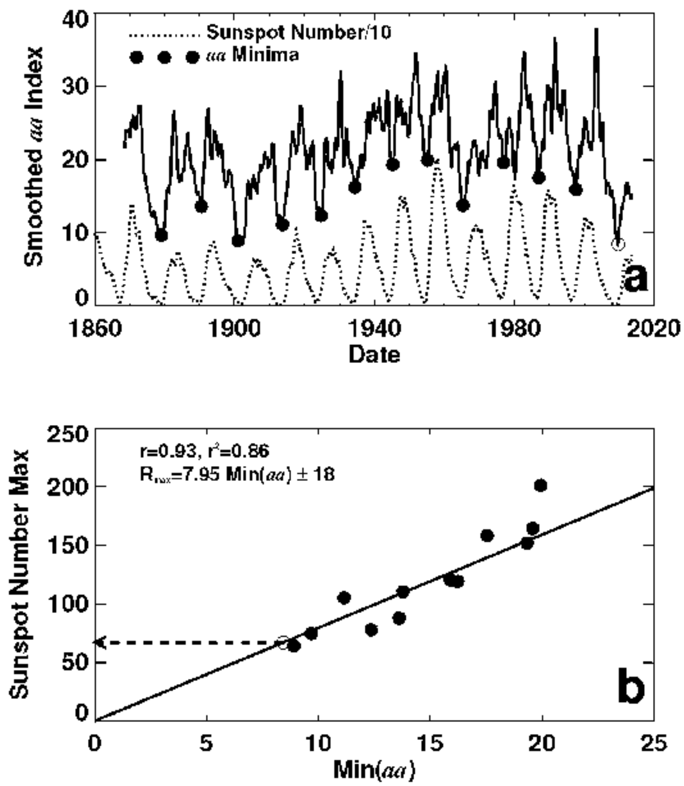

Fig 5. Ohl's method for predicting cycle amplitudes using the minima in the smoothed *aa* index (panel a) as precursors for the maximum sunspot numbers of the following cycle (panel b). Figure Credit: David H. Hathaway, "The Solar Cycle", Living Reviews in Solar Physics, 12(2015)4.

There are several alternative prediction methods which uses *aa* index. Feynman [50] observed that the index could be separated into two components one of which is in phase with the current sunspot number while the other component associated with the interplanetary disturbances is out of phase with the activity cycle. Using this method, Hathaway and Wilson [51] found that the cycle 24 would be large cycle amplitude comparable with the prediction of Dikpati et al. [52] and Hathaway & Wilson [53]. Another method is due to Thompson [54] who used the number of geomagnetically disturbed days (defined as $Ap > 25$ where Ap is the average of the geomagnetic disturbances measured at many stations distributed over the globe) that occurred during a sunspot cycle is proportional to the sum of the amplitudes of the sunspot number of that cycle and the following cycle. Lantos and Richard [55] examined several geomagnetic precursor methods using both single, and multivariate regressions and predicted that the maximum amplitude should be about 168 ± 15 peaking some time in 1999-2000 which as we now know is not a very good prediction. Recently Pesnell [56] has used the *Ap* index and $F_{10.7}$ as a precursor pair where $F_{10.7}$ was used to remove the direct component of the solar activity from *Ap*. The study reports a maximum smoothed sunspot amplitude of cycle 24 to be 65 ± 20 around 2014.5 ± 0.5.

Although the use of geomagnetic activity has gained some degree of success in predicting the future activity cycle, it is really intriguing that this should be used as a precursor since the source of the geomagnetic disturbances happen to be the Sun. Thus, one would believe that the prediction should have been reversed; the solar activity should be predicting the geomagnetic activity indices. However it appears that the behavior of the *aa* index around the solar minima is similar to the polar flux around the same time and appears to be the reason for its success [57].

Similar to the sunspot number, it has also been demonstrated that the *aa* index has not been calibrated uniformly throughout the observation period and this should be borne in mind when long time series of *aa* index are used in the forecasting method. To mitigate, Svalgaard, Cliver and Le Sager [58] have reconstructed a new daily index, the Inter-hour Variability index (IHV) which successfully reconstructs yearly averages of *aa* and *Ap* indices from 1959 through 2000 but fails when extended to 1900.

2.2.3 Polar Precursor:

Shatten *et al* [59] was the first to propose that the polar field could be used as a precursor since the field at the minimum has a good correlation with the strength of the next solar cycle. This was based on the Babcock-Leighton scenario of the origin of the solar cycle where the toroidal field is carried towards the pole to form the poloidal field and hence often this method is considered as the first prediction from a dynamo model. However, accurate measurement of the Sun's polar field is a difficult task due to (i) the field is weak and (ii) the field is radially directed and hence transverse to our line of sight. Nevertheless, Wilcox Solar Observatory (WSO) has been measuring the magnetic field in the polar regions of the Sun[4] since 1976 [60, 61]. Figure 6 shows the polar field strength as a function of time since 1976 and the smoothed sunspot number. It is evident that the polar field and sunspot number have an inverse relation; the polar field reaches its minimum amplitude when the strength of the cycle is near the maximum. It is also clear that the polar fields vary in strength from cycle to cycle.

Figure 7 shows the correlation between the observed maximum polar fields of cycle n with maximum strength of the sunspot number of cycle n+1 and confirms the basis of this prediction method. Although the plot has only 4 data points and does not provide a convincing correlation statistically, it is remarkable that the method has consistently predicted the strength of the cycles 21, 22 and 23 in the correct range. For cycle 24, R_{24} was predicted to be about 75 ± 8 [62, 63] four years before the minimum which is not always feasible. Early polar field predictions of cycle 22 and 23 had to be corrected at a later time and only forecasts made shortly before the actual minimum were close to the observed value.

Thus the predictive capability of the polar field as a precursor appears to be limited to few years before the maximum activity level since the minimum in sunspot data, by virtue of the definition of 13-month mean, cannot be known earlier than 6 months after the minimum has passed. Schatten and Pesnell [64] suggested a solution to this problem by introducing a new activity index, the Solar Dynamo Amplitude (SODA) which combines the polar field strength with the 10.7 cm radio flux. This index is believed to be sensitive to the magnetic flux trapped within the solar convection zone but not to the phase of the solar cycle.

---

[4] *http://wso.stanford.edu/gifs/Polar.gif*

The prediction yielded a value of 170 ± 25 as the cycle 23 amplitude and 1999.7 ± 1 year as the time of occurrence.

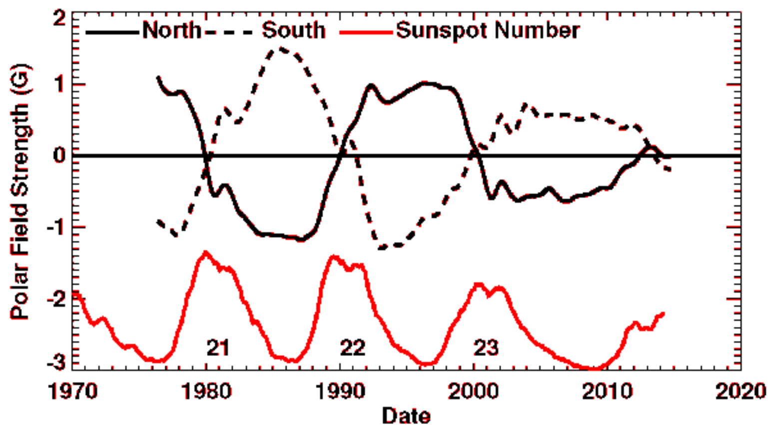

Fig 6. Observed polar field strength as a function of time as observed by Wilcox Solar Observatory. The line styles have the following meaning: Solid line: Northern hemisphere; Dashed: Southern hemisphere; thick solid line at the bottom is the sunspot number. Courtesy of David H. Hathaway, "The Solar Cycle", Living Reviews in *Solar Physics*, 12, (2015), 4.

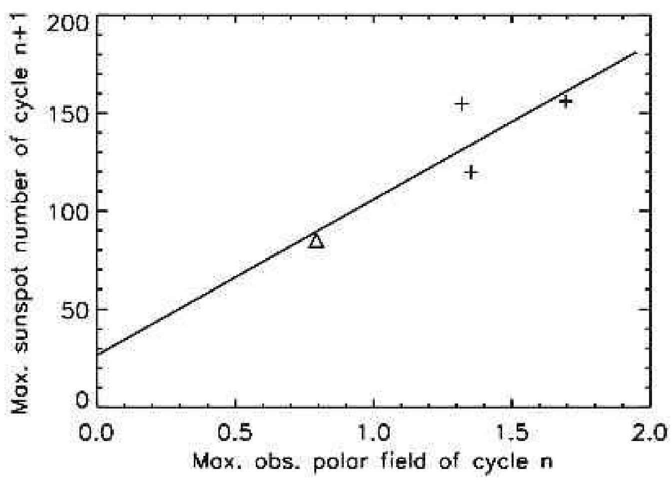

Fig 7. Correlation between the maximum polar field near the end of solar cycle n and the maximum strength of cycle n+1. Adapted from Jie Jiang [57].

In order to extend the time series further back in time, several other proxies have been used. For instance, the number of polar faculae available from Mt. Wilson was recognized as an indicator of polar fields [65]. Using these data for each year between 1906 and 1976, Layden *et al* [66] found that the polar faculae have little predictive power. Makarov and Makarova [67] have also used polar faculae counts observed at Kislovodsk to predict the sunspot cycle 23 with a time lag of 5-6 years. The success is attributed to the fact that Makarov *et al* considered all faculae poleward of 50° latitude although a similar analysis using the Mitaka data base found that the amplitude of the next solar cycle could only be predicted 4 years in advance [68]. Later, recalibration of the Mt. Wilson data also revealed that the polar faculae counts could be reliably used in the prediction of cycle amplitudes [69]. We reproduce their Fig 3 here (Fig 8) which shows the relation between the polar flux at solar minimum and amplitude of the next solar cycle over the last century for different hemisphere separately, where the polar faculae have been used as a proxy for the polar flux. The Pearson's correlation is found to be 0.6 with a 99% confidence level. Obridko and Shellting [70] have also used Hα synoptic maps to reconstruct the polar field strength at the surface back to 1915. In a different approach, Tlatov [41] linked the amplitude of the new cycle with the amplitude of the preceding cycle and the moments of polar reversal and maximum of sunspot activity and found a relation:

$$R_{max}^{(n+1)} = C_1 + C_2 R_{max}^{(n)} \left(T_{rev}^{(n)} - T_{max}^{(n)}\right) abs\left(T_{rev}^{(n)} - T_{max}^{(n)}\right) \tag{4}$$

where $C_1 = 83 \pm 11$ and $C_2 = 0.09 \pm 0.02$ and is the epoch of the polarity reversal in cycle n (typically about a year after). This relation gives a good correlation coefficient ($r = 0.86$ based on 12 cycles) and the time lag of ~ 10 years. Based on ≈ 2011.9 ± 0.7, the predicted $R_{24} = 92 \pm 21$.

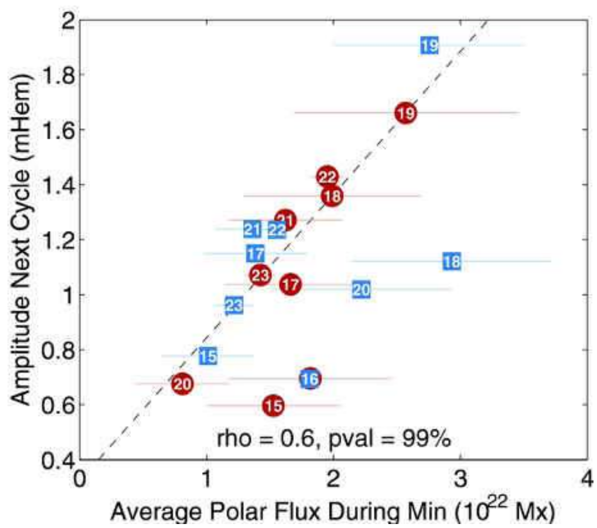

Fig 8. Correlation between polar flux at solar minimum and the amplitude of the next cycle. Squares (circles) denote the northern (southern) hemispheres. The numbers represent the solar cycle. The dashed line is the linear fit using the least absolute residuals method. Here rho is the Pearson's linear correlation coefficient with a confidence level, pval, of 99%. Credit: Muñoz-Jaramillo [69].

2.3 Non-linear Method

The long term behavior of the solar activity exhibiting phenomena like grand minima and grand maxima is suggestive of a system which is chaotic and hence non-linear in nature. Here we only consider one particular type of non-linear method e.g. neural network algorithms that determine and model complex relationships between inputs and outputs to find patterns in the data that can be extrapolated to future. But before the method can be applied to forecast, the network needs to be trained. However, the prediction for cycle 23 was far away from the observed value. In spite of the claims of the technique, the prediction for cycle 23 was much higher than the observed value. Similarly, the six predictions for cycle 24 showed large deviations.

2.4 Dynamo Models:

Since the activity cycle is believed to be a natural outcome of the dynamo mechanisms, one may postulate that the dynamo models would be the best predictor of the solar cycle. But in reality, predictions based on the dynamo models started with the solar cycle 24. The two models that were submitted to the solar cycle 24 panel presented two different predictions, one stronger [52] and the other weaker [71] than the average cycle indicating the challenges associated with predictions based on such dynamo models. Before we try to understand why the two models produced vastly different results, let me briefly discuss the solar dynamo theory. For details, the reader may consult many extensive reviews written on the subject [72, 73, 74].

The central idea of solar dynamo theory is that the toroidal and poloidal components of the magnetic field sustain each other through a feedback loop. Thus, there are three basic processes:

(i) The strong toroidal field is produced from the stretching of the poloidal field through the differential rotation in the tachocline

(ii) The toroidal field then rises through the convection zone due to magnetic buoyancy to produce sunspots which decay to produce the poloidal field

(iii) The meridional circulation then advects the poloidal field to the high latitudes and subsequently to the tachocline

The two main groups of dynamo models, interface dynamos and flux transport dynamos, differ mainly in their assumptions of the generation of the poloidal field from the toroidal field. In interface dynamos, the helical turbulence twists the toroidal field near the base of the convection zone to produce the poloidal field and the mechanism is often termed as α-effect [75]. The alternative idea due to Babcock [76] and Leighton [77] is based on the active region tilt. After the decay of the bipolar sunspots, their magnetic fields diffuse around to give rise to a poloidal field. Thus this is a surface effect. The resultant poloidal field are then advected to the poles and there down to the tachocline by the meridional circulation. Dynamo models in which the meridional circulation plays an important role are called flux-transport dynamo. A simple dynamo based on this principle was considered by Wang *et al* [78] in early nineties. First two-dimensional models were constructed by Choudhuri *et al* [79] and Durney [80]. These are the only models which have successfully reproduced the observed butterfly diagram [81].

Table 1. Observed monthly smoothed minimum and maximum International sunspot values for solar cycle 21-24. These are obtained from *http://www.ngdc.noaa.gov/stp/space-weather/solar-data/solar-indices/sunspot-numbers/cycle-data/table_cycle-dates_maximum-minimum.txt*

Parameter	Cycle 21		Cycle 22		Cycle 23		Cycle 24	
R (min)	12.2	03/1976	12.3	09/1986	8.8	05/1996	1.7	10/2008
R (max)	164.5	12/1979	158.5	07/1989	120.8	04/2000	82	04/2014

3 Assessing the solar cycle predictions

It has now become clear that the sunspot cycle is crucial for many terrestrial effects such as operation of satellites, electric power transmission grids, high-frequency radio communications, GPS as well flight paths of airlines to name a few. Since all these require planning many years ahead in time, the solar cycle forecasts were used to aid their planning. Thus, forecasting became serious business rather than fun. This initiated studies to evaluate how well the forecasts matched with the actual observations and which method prevailed over others. In early seventies and eighties several working groups were organized to scrutinize the results and several workshops devoted to solar-terrestrial predictions were held. Starting with cycle 23, as discussed earlier, NOAA created solar cycle panels to examine the predictions. There are several publications which exclusively deal with this subject. We summarize the results here starting with cycle 21. For cycle 20 predictions, the reader is referred to King-Hele [82, 83]. The minima and maxima of sunspot cycles 21-24 taken from NOAA is given in Table 1. The smoothed monthly mean sunspot number quoted for each cycle is defined as the arithmetic average of two sequential 12-month running means of monthly mean numbers.

3.1 Cycle 21 (*1975-1986*)

Sergent [84] discussed the early rise of cycle 21 in the context of 15 predictions. Kane [85] has tabulated some of the predictions for cycle 21 based on three analysis methods; single variable, bivariate analysis, and other. Most of the predictions were closed to the observed smoothed sunspot value of 164.5 by one standard deviation except those of [86] based on the second minimum of the geomagnetic *aa* index which predicted a value of 125 which was an underestimation. Schatten *et al* [59] introduced the use of polar flux as precursor and estimated the strength of the Sun's poloidal field near sunspot minimum in four different ways (i) the coronal flattening, (ii) the bending of high latitude plumes from the shape of the corona, (iii) the flattening of the wrapped current sheet in interplanetary space, and (iv) by counting the numbers of faculae at the solar poles. His average predictions were 140 ± 20 which turned out to be a lower value. Brown [87] divided the 38 predictions summarized by McIntosh [88] into two groups, the precursor and statistical methods. We reproduce his Fig 1 here (Fig 9). Considering the twelve predictions based on precursor methods, indicated by hatched markings in the figure, the mean predicted value is 168 very close to the observed value. As the chairman of the working group, he summarized these findings as: "This agreement may be fortuitous, but coupled with the fact that there is a wealth of data which supports the dynamo theory it does provide certain degree of confidence". In summary the predictions based on the precursor method were closer to the observed value centered at December 1979 by 20% while those made by the statistical methods underestimated the observed value.

3.2 Cycle 22 (*1986-1996*)

Solar cycle 22 started in 1986.8 and reached a maximum monthly mean sunspot number of 158.5 in July 1989 indicating that this cycle is slightly weaker compared to cycle 21. An initial report, before cycle 22 reached the minimum phase, is presented in [87]. Similar to cycle 21, 31 predictions were divided into four different categories. The median predicted sunspot number peak was found to be: 89 ± 60 for statistical methods (11), 106 ± 30 for use of secular cycles (11) and 115 ± 20 for precursor methods (6),where the number in the brackets represent the total number of predictions in that category. Methods based on solar geometry were not considered since there only 3 predictions. Brown [89] revisited the predictions in retrospect. Once again, he emphasized that the precursor methods are better predictors of activity cycle. However, the prediction of Schatten and Sofia [90] once again based on the polar flux predicted a value of 170 ± 25 which is higher than the observed value.

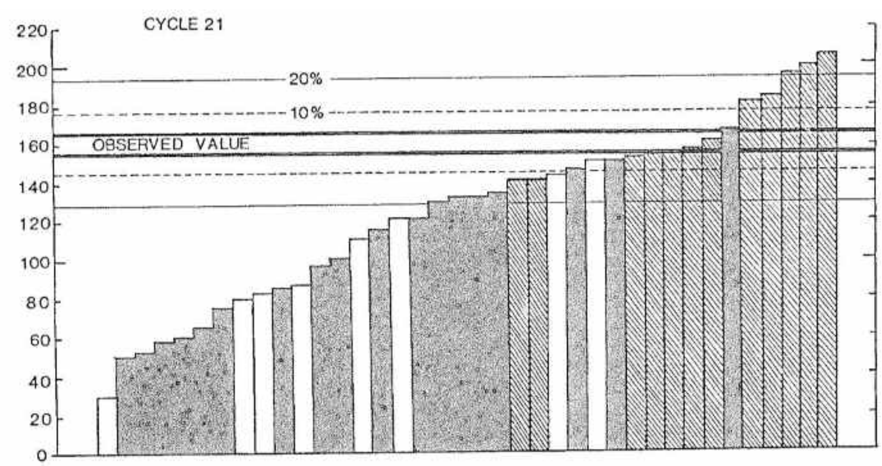

Fig 9. 38 published predictions for the peak amplitude of smoothed sunspot number for cycle 21 with the observed value. Cross-hatched predictions use methods based on precursors near sunspot minimum or during the preceding cycle; solid shadings are predictions based on statistical treatment of the past sunspot number series. Adapted from Brown [87].

Kane [85] and Li *et al* [91] have also summarized the results of solar cycle 22. Kane followed the same classification scheme described for cycle 21. Based on 20 predictions, he concluded that both for cycles 21 and 22, the choice between single variable and bivariate analysis is somewhat ambiguous due to the complex nature of the *aa* index having two minima while the sunspot had a single minima. For cycles 12-20, however, the bivariate analysis is superior to a single variable formulation.

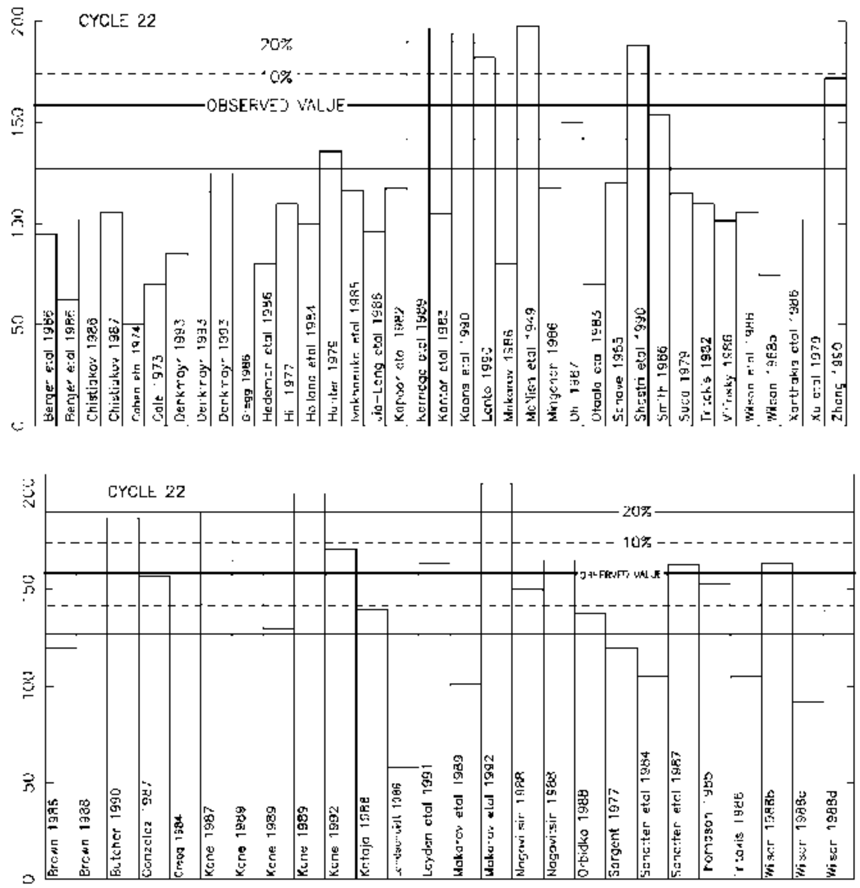

Fig 10. Prediction of the maximum sunspot number of solar cycle 22. The bold horizontal line represents the observed value of 157.6 and the two dashed lines are the upper and lower bounds of ± 10% of the observed value, respectively. The two thin lines refer to the bound of ±20%. Top panel is the prediction based on the statistical methods while the bottom panel is based on the precursor methods. Figure reproduced from [91].

Li *et al* [91] have collected about 63 predictions out of which 37 values were categorized as based on statistical methods, 24 were based on precursor methods and the remaining 2 used data inherent to solar system geometry such as the configuration of planets with respect to the Sun. We reproduce their Fig 1 and 2 here (Fig 10). In the first category, only 3 out of 37 values fall within ± 10% of the observed annual mean value of 157.7 and only 6 values within ±20% confirming that the statistical methods, in general, are not good predictors of the cycle. In the second category, 10 values fall within ±10% while 16 (67%**)** are within ± 20%. On the other hand 10% of the precursor based methods lie within 10% and 16 values within 20% of the observed value signifying the merit of the method. However, the average of the 24 values (neglecting the 2 methods based on the geometry) gave R_{22} = 149.9 with a large variance of 30.9.

3.3 Cycle 23[*1996-2008*]

The onset of cycle 23 is generally regarded as May 1996 with a peak around April 2000 with a monthly smoothed value of 120.8 (many other indices recorded a higher secondary maximum in late 2001). Li *et al* [91] have once again collected 48 predictions among which 17 were classified as based on precursor methods and the other 31 as statistical. On a close examination, the predictions based on the statistical method showed two distinct groups; 16 cases with lower values were put into one group and the remaining 15 cases with larger value into the other. The average amplitude of the first part is 107.9 ± 27.9 while the average of the second part is 202.1 ± 14.9. On the other hand, the precursor methods yielded a mean value of 162.3 ± 11.2. At the time of the writing, Li *et al* did not know the observed maximum value and hence could not judge the validity of the predictions. With retrospect, none of the average predictions are closer to the observed value. Hathaway, Wilson, and Reichman [18] also evaluated the predictions based on geomagnetic precursor methods and found the averaged amplitude exceeds the observed value by nearly two standard deviations.

Kane [92] have also examined the prediction of the cycle 23. The 20 predictions predicted peak amplitudes between 80 and 210. The study concluded that among the various methods of prediction, Ohl's precursor method seems to yield consistently more accurate predictions.

One of the critical operations of Space Environment Center (SEC) of NOAA is to issue space weather alerts and notify storms occurring on the Sun that could impact Earth. In order to issue accurate alerts SEC was interested to evaluate different forecasting methods so that a consensus prediction of future solar activity could be outlined. This in view, SEC constituted a panel of experts on solar cycle predictions and invited the scientific community to submit their predictions to the panel. In addition to the submitted entries, the panel also considered published results in the scientific journals. To place all forecasts on the same footing, 10.7 cm flux predicted values were converted to an equivalent sunspot number. Based on the forecasting methods, 28 predictions were divided into six classes:

(i) Empirical results based on the observation that odd-numbered cycles are larger than their preceding cycle (even/odd behavior),

(ii) Precursor methods using either solar polar magnetic field or geomagnetic activity near minimum,

(iii) Spectral forecast which included wavelet based and autoregressive methods,

(iv) Non-linear methods including neural network,

(v) Climatology which is based on the average behavior of the cycle and includes most of the statistical analyses of sunspot number, and

(vi) Recent climatology where the forecast was related to the recent past (solar cycle 17 onwards)

Table 2. Forecasts of maximum smoothed sunspot number for six different classes of prediction and the consensus forecast for cycle 23.

Technique	Maximum Amplitude
Even/Odd cycle behavior	165 - 235
Precursor	140 - 160
Spectral	135 - 155
Recent Climatology	125 - 185
Neural Network	110 - 170
Climatology (all)	75 - 155
Consensus Monthly R_{23}	130 - 190

The panel combined representative predictions to obtain a consensus prediction for the sunspot number (Table 2). The panel of 12 experts agreed that a large amplitude solar cycle with a smoothed maximum sunspot number of approximately 160 will be observed around 2000-01 with a rise time of about 3.4 years (between the extremes of 2.8 and 4.3 years). The panel forecasted that the 10.7 cm radio flux will have a peak value of 205 solar flux units, within a range of 175 to 235 [93].

3.4 Cycle 24[*2008-ongoing*]

Continuing the tradition, SEC in October 2006 constituted the solar cycle 24 prediction panel to investigate the predictions of cycle 24. The panel invited scientists to submit their predictions of the maximum amplitude and timing of solar cycle 24 and received 15 entries. However, this time the goal to obtain a consensus prediction was a challenge as the submitted predictions clearly showed two different distributions in the amplitudes of the smoothed sunspot number: a moderately strong cycle of 140 ± 20 peaking in October 2011 and 90 ± 10 peaking in August 2012. This lead the panel to make the following statement ".. the prediction panel has been unable to resolve a sufficient number of questions to reach a single, consensus prediction for the amplitude of the cycle". Most surprisingly, as already discussed, the two predictions based on flux transport dynamo models predicted very different answers, one predicting a high value of 155-180 [52] while the other a maximum amplitude of 80 [71].

Table 3. Summary of Predictions for Solar Cycle 24. Adapted from [94].

Category	Number of predictions	Average Peak Value	Range	Skill Score
Climatology	32	103 ± 31	40 -185	– 0.19
Recent Climatology	4	143 ± 17	120 - 160	–2.38
Spectral	24	96 ± 29	42 - 180	0.15
Neural Network	6	114 ± 30	65 - 145	–0.55
Precursor	34	111 ± 31	53 - 180	–0.53
Dynamo Model	5	131 ± 45	80 - 168	–0..96
All	105	106 ± 31	40 - 185	

A detailed description of different predictions including those submitted to the panel and published in scientific journal has been compiled by Pesnell [35]. The 54 predictions compiled showed the peak amplitude of sunspot number to vary from 40 to 185 with an average value of 117 ± 33. Recently Pesnell [94] extended his previous investigation to consider predictions not included in his earlier study [10, 35]. A total of 105 predictions were investigated and separated into the same six different categories mentioned earlier. The recent set of predictions retained the earlier range but reduced the average peak amplitude value to 106 ± 31. Other significant conclusions drawn from this study are (i) no category predicted a higher value than the earlier studies implying that predictions which rely on values closer to the minimum are better predictors, (ii) predictions based on precursor and neural network showed a decrease in the minimum range, (iii) the prediction based on the dynamo model predicted a peak value of 84 around mid-2014 [95].

Pesnell [94] further discuss the merits of each prediction category by calculating a skill score which depends on the square of the difference from the observed value which is assumed to be 80. A perfect forecast would have a skill score of 1 and a forecast which is less skillful than the reference value would have negative values. Table 3 summarizes the skill scores for each category. It is evident that the best method appears to be the spectral method with a positive skill score while the worst method happens to be those categorized under recent climatology. This is possible that the skill score is biased as this category had the fewest number of predictions. It is also to be noted that these methods are restrictive in nature as they use data later than solar cycle 17 and thus the scope of the prediction is restricted to the amount of prior information. The dynamo modes as expected had a low skill score of –0.96.

Figure 11 adapted from Fig 3 of Pesnell [94] shows the categorized predictions with the standard deviation and the error bar represents the prediction range. A dashed line is drawn at R_{24} = 82 which is conjectured to be the smoothed monthly maximum value during April 2014 based on the sunspot record maintained at Space Weather Prediction Center (SWPC). The number of predictions and estimated skill score for each category is also displayed. Only spectral method has a positive skill score indicating that they are an improvement over the climatological average.

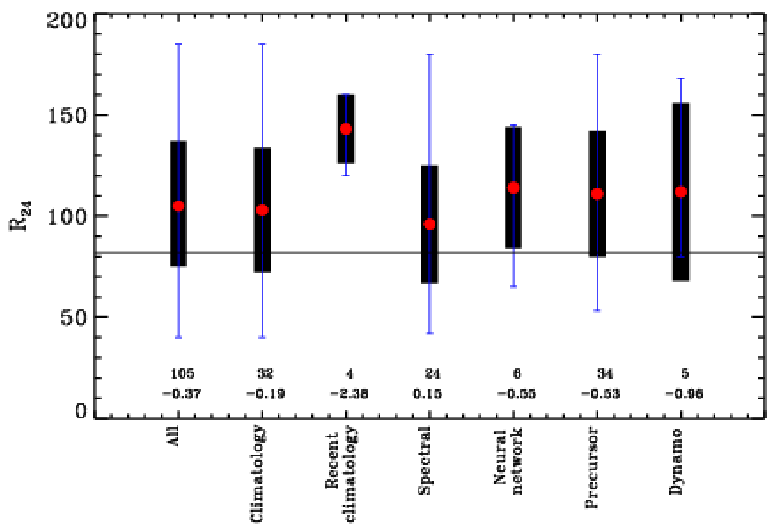

Fig 11. The average predictions for cycle 24 for different categories are shown as dots. The filled boxes represent 1 σ error limits and the error bars show the range of the prediction. The solid horizontal line is drawn at 82, probable value of the maximum based on the sunspot number archived at SWPC. Figure adapted from [94].

The SEC prediction panel updated their forecast in May 8, 2009 and predicted consensus peak amplitude of 90 occurring in May 2013. While awaiting final confirmation (at the time of writing), it is very likely that the observed monthly smoothed sunspot record will agree with this result but occurring significantly later than predicted.

4 Predictions based on dynamo models

As discussed earlier, two predictions based on flux transport dynamo (FTD) models with assimilated data were submitted to the solar cycle 24 panel. Dikpati *et al* [52] predicted amplitude of 150-180 using FTD that included a rotation profile and near surface meridional flow based on helioseismic observations after 1996; the meridional flows was constant prior to 1996. The model was driven with a surface source of poloidal field that depends on the sunspot area observed since 1874. The prediction was based on the strength of the toroidal field produced in the tachocline and gave an excellent agreement between the field strength and the amplitude of the last eight cycles. In fact, the correlation was significantly better than that found with the geomagnetic precursor.

Choudhuri *et al* [71] also predicted the amplitude of the cycle 24 as 80 using a similar FTD but with surface poloidal field at minimum as the assimilated data. The use of polar field is based on the earlier work of Schatten et al. (1978) that the polar field at a minimum gives an indication of the strength of the next solar cycle. This was further verified by plotting a proxy of the dipole moment (DM) of the Sun computed by Svalgaard *et al* [63] as a function of the next cycle strength (Fig 12 left panel). Since the observation of polar flux at WSO started in 1976, there are only three points but they lie very close to a straight line (solid line in the figure). Assuming that this is a real correlation and not a statistical coincidence, the predicted strength of cycle 24 corresponds to the point where the dashed line (the observed value of DM for cycle 23) cuts the solid line. This gives the maximum of about 78 [Choudhuri [96] obtained a value of 75) for cycle 24.

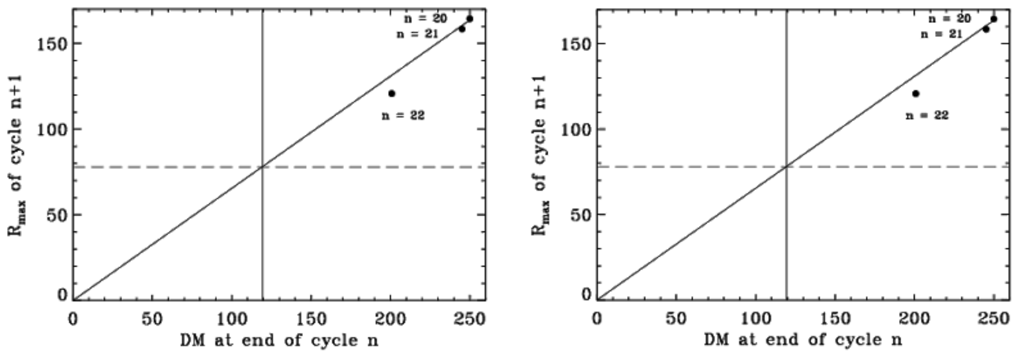

Fig 12: (Left) Strengths of solar cycles plotted against the dipole moment values of polar fields at the preceding minima, (Right) dipole moment values of polar fields at the minima plotted against the strengths of the previous solar cycles. Adapted from [96]

However, an interesting thing happens when DM at the end of cycle n is plotted against the strength of the cycle as shown in the right panel of Fig 12. It is evident that there is no good correlation implying that the strength of the cycle does not completely determine the polar field at the end of the cycle. In other words, a strong cycle does not necessarily produce a strong polar field at the end. Based on these observations, Choudhuri argued that the rising phase of the cycle is reasonably deterministic allowing the prediction of solar cycle a few years ahead of time by using the polar field data. On the other hand, the declining phase of the cycle is not predictable since it is dominated by the Babcock-Leighton mechanism which involves randomness. Thus predicting the maximum strength of the cycle is not feasible more than 7-8 years ahead of time even when better dynamo models are developed.

The other major differences with the Dikpati model are (i) the diffusivity remained high throughout the convection zone and (ii) the meridional flow extended below the convection zone. Another weakness of the model was that the output of the model at each minimum was changed instantaneously to make it match with the dipole moment obtained from the Wilcox Solar Observatory observations. They found an excellent fit to the last three cycles and found that the peak amplitude would be 78.

The high discrepancy between the two models raised doubts about the use of FTD models in solar cycle predictions. Dikpati *et al* [52] criticized the use of poloidal field strengths to predict the cycle amplitude since the fields could not be carried down to the low latitude tachocline in a short time of four years. Using a simplified 1D FTD model and parameters similar to those used by Dikpati *et al*, Cameron and Schüssler [46] noted that the magnetic flux diffusing across the equator was an excellent predictor for the amplitude of the next cycle but the predictive skill was lost when the observed active region emergence latitudes were used. They argued that this result is a consequence of the fact that the precursor amplitude is determined by the sunspot activity a few years before the solar minimum.

Yeates *et al* [97] have argued that the flux transport dynamos have the prediction capability by virtue of their inherent memory. In the advection dominated model of Charbonneau and Dikpati [98], the memory was about 17 years so the polar field at the end of cycle n correlates strongly with the toroidal field of cycle n+2 rather than that of cycle n+1. However, in the diffusion-dominated models like that of Choudhury *et al* [71], the memory of past cycles is governed by downward diffusion of poloidal field into the tachocline which primarily results in a one-cycle memory. Unfortunately, the length of memory depends on the meridional flow profile and other chosen convective properties of the convection zone which are not known. In addition, the return meridional flow that is supposed to carry the polar flux deeper to the base of the tachocline has not been detected through helioseismology in spite of the best efforts. Another problem is the strength of the diffusivity in the convection zone which strongly affects the dynamo models. Based on this comparative analysis, Yeates *et al* [97] asserted that the observed solar cycle amplitude-period dependence arises more naturally in the diffusion-dominated regime. In a subsequent analysis, Dikpati *et al* [99] used polar fields and cross equatorial flux as predictors of cycle amplitudes and concluded that their tachocline toroidal flux was best indicator. Furthermore they argued that the polar fields followed the current cycle so that the weak polar fields at the minimum are due to the weak meridional flow. However, recent studies favor diffusion-dominated solar convection zone [100] and this type of dynamo has been successful in many other aspects of the solar cycle e.g. grand minima [101] and Maunder minimum [102] . Recently, Nandi and Karak [103] have shown that the forecasting ability of the solar cycle is affected by the turbulent pumping of magnetic flux. With significant turbulent pumping the memory of the dynamo is severely degraded and hence a long term prediction of the solar cycle is questionable. Going further, Tobias *et al* [104] noted that even weak stochastic perturbations to the parameters in the flux transport models produce substantial changes to the activity cycle. Thus, they concluded that the solar dynamo is deterministically chaotic and thus inherently unpredictable.

5. Prediction for cycle 25

We are now in the descending phase of the solar cycle 24 and it has been a weak cycle compared to the cycle 23. If we consider the methods that predicted the cycle to be a small amplitude cycle, the minimum will occur beyond 2019 (Fig 13). Several predictions of the amplitude of solar cycle 25 have already appeared in the literature. A search in ADS gave about 12 abstracts (not a complete list by any means) but some of them are presentations in different conferences and omitted if a number was not found in the abstract. The remaining predictions are presented in Table 4 following the same classification scheme of Pesnell [35].

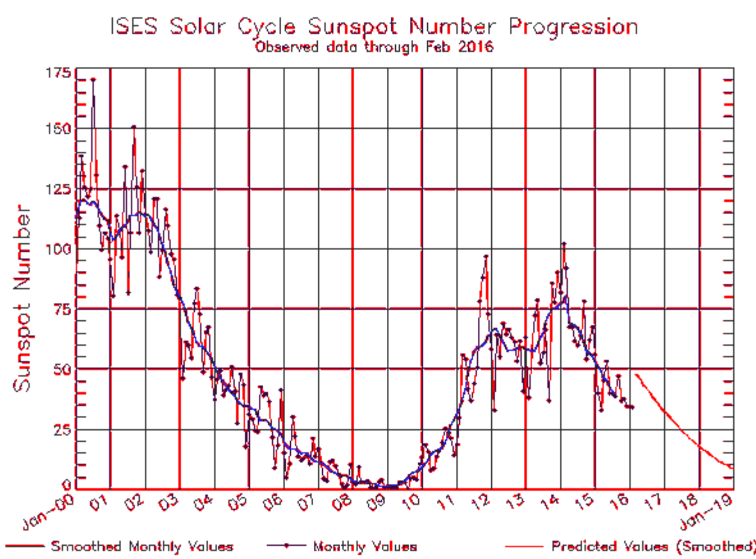

Fig 13. Sunspot number progression. The line joining the dots are the monthly values, solid blue line is the smoothed monthly values. The small red solid line at the end is the smoothed predicted values with cycle 25 minimum occurring beyond 2019. "Figure credit http://www.swpc.noaa.gov/products/solar-cycle-progression"

Table 4. Sunspot number prediction for cycle 25; C stands for climatology, P for Precursor methods, N for non-linear techniques and S for spectral methods.

Author/Date	R_{25}	Error σ	Date	Category	Summary
Attia *et al* 2013 [105]	90.7	8	2022	N	Neuron fuzzy modeling
Bishoi *et al* 2015 [106]	56.0	12	-	P	Correlation between Heliosphere magnetic field and high latitude surface fields
Hathaway and Wilson 2004 [36]	70	30	2023	P	Properties of sunspot number
Helal and Galal 2013 [107]	118.2	-	2023	P	Spotless days
Javaraiah 2015 [108]	50.0	10		S	MEM and Wavelet analysis
Li, Peng, and Li 2015 [109]	109.1		Oct 2023	C	Cycle profiles
Pishkalo 2008 [110]	112.3	33.4	Apr 2023	C	Regression between parameters
Rigozo *et al* 2011 [111]	132.1		Apr 2023	C	Extrapolation of sunspot number
Yoshida A 2012 [112]	112.0	15.1		C	Odd-even rule

Similar to the previous solar cycle predictions, we find a large distribution of the peak amplitude ranging from 50 to 132.1, the higher values mostly arising from climatology models. The solar cycle 25 prediction of Helal and Galal [107] used the new revised calibration of the sunspot number and found that the number of spotless days is a better predictor than with the old calibration and estimated R_{25}, to be 118.2 peaking 4 years after the upcoming solar minimum. If we average the last 24 cycles, we find that R_{25} would be 112.8 ± 40 where the uncertainty is one standard deviation from the mean. Both these estimations appear to be on the higher side, if the trend of a weak cycle continues for cycle 25. A schematic view of cycle 24 and 25 prediction adapted from David Hathaway's prediction is shown in Fig 14.

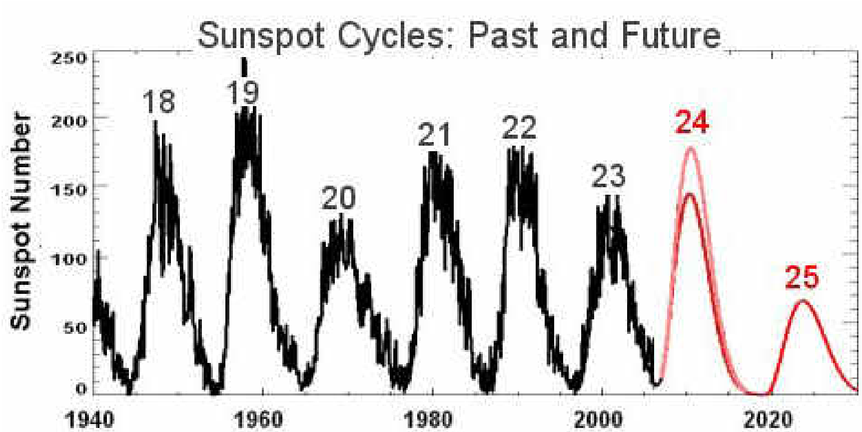

Fig 14. David Hathaway's prediction of cycle 24 and 25. The pink solid line with higher amplitude corresponds to Dikpati *et al*'s [52] prediction for cycle 24. Figure credit http://science.nasa.gov/science-news/science-at-nasa/2006/10may_longrange/

Helioseismic measurement of the migrating zonal flow pattern known as torsional oscillation, first detected by Howard and Labonte [113] in surface Doppler measurements, has also been used to indicate the onset of the next solar cycle. The poleward branch of these oscillations generally becomes visible 10 to 12 years before the appearance of the magnetic activity associated with the next solar cycle. However, the poleward flow for cycle 25 which was expected to appear in 2008-2010 was not observed at first. Subsequent analysis showed that it was very weak implying that the cycle 25 will be a weak cycle [114].

The weak cycle 24 and the prediction that cycle 25 would also be of low amplitude suggests that we are at the end of the Modern Maximum which has lasted about 60 years. Some scientists even think that we may be going back to another period of grand minima like the Maunder minimum where the solar activity was low. In this context, Penn and Livingston [115] and Livingston and Penn [116] reported that their measurements of the magnetic field strength and emergent intensity at the darkest point in sunspot umbrae indicated a trend that would lead to a total loss of sunspots by 2015. But the data had long gaps and was limited in time; hence the linear trend is questionable. Schad and Penn [117] examined this effect using the full disc daily line-of-sight magnetograms from the Kitt Peak Vacuum telescope between 1993 and 2003 but did not find any secular trend. But we may be in for a surprise. Independent of whichever course the Sun takes in the coming years, the scientific community would make its best use to unravel the secrets of the magnetic Sun.

6 Concluding Remarks

A synopsis of the prediction methods clearly indicate that information about the next cycle is available at the time when the solar minimum is clearly identified; not always an easy task as the prolonged minimum of cycle 24 just demonstrated. However, if the solar cycle is a genuinely chaotic system in which small perturbations in the initial conditions can produce widely divergent trajectories, then our attempts to make long-term forecast of future cycles are bound to be a failure. As the flux-transport dynamo models indicated, physical models are not at a stage where reliable predictions can be made. Although, models with high-diffusivity are preferred, it appears that they have short memory of few years and hence the cycle amplitude can only be predicted few years in advance complementing the polar precursor methods which do not require a physical relation between subsequent cycles. But space missions require at least 10-15 years of advanced planning. It appears that we are not at a state where reliable solar cycle predictions can be made 10-15 years in advance. In spite of these shortcomings, solar cycle forecasting has made extensive progress during the last few years particularly due to the controversial predictions from the dynamo models. Once Arnab Rai Choudhuri commented that "we shall have a verdict from the Sun-god himself within the next 4-5 years whether the prediction of Dikpati *et al* [52] or our prediction [71] comes closer to the truth" [96]. The verdict is now out but as many modelers believe it is probably a coincidence rather than a validation of the model used for prediction. The science of prediction has a long way to perfection.

References

1. Jain K, Komm R, *Asian J Phys*, 25(2016)363-386.
2. Hoyt D V, Schatten K H, *Sol Phys*, 181(1998)491-512.
3. Wolf R, *Naturf Gesell Bern Mitt*, 229/230(1852)41-48.
4. Waldmeier M, Schulthess u. Co. / Swiss Federal Observatory, Zurich, 1961.
5. Hathaway D H, *Living Rev Sol Phys*, 12(2015)4.
6. Clette F, Svalgaard L, Vaquero J M, Cliver E W, *Space Sci Rev*, 186(2014)35-103.
7. Cliver E W, Ling A G, *Sol Phys*, (2016) in Press
8. Yule G U, *Phil Trans Roy Soc London A*, 226(1927)267-298.
9. Petrovay K, *Liv Rev Sol Phys*, 7(2010)6.
10. Pesnell W D, Sol Phys, 281(2012)507-532.
11. McNish A G, Lincoln J V, *Trans Am Geophys Union*, 30(1949)673.
12. Hathaway D H, Wilson R M, Reichmann E J, *J Geophys Res*, 104(1999)22375-22388.
13. Stewart J Q, Panofsky H A A, *Astrophys J*, 88(1938)385-407.

14. Hildner E, Greer M S, in Proceedings of Solar-Terrestrial Prediction Workshop III, (eds) R I Thompson, D Cole, P J Wilkinson, M A Shea, D Smart and G Heckman (1990)689.
15. Wolf R, *Mon Not Roy Astron Soc*, 21(1861)77-78.
16. Waldmeier M, *Astron Mitt Zurich*, 14(1935)105-130.
17. Waldmeier M, *Astron Mitt Zurich*, 285(1968)1.
18. Hathaway D H, Wilson R M, Reichmann E J, *Sol Phys*, 151(1994)177-190
19. Elling W, Schwentek H, *Sol Phys*, 137(1992)155-167.
20. Kane R P, *Sol Phys*, 246(2007)471-485.
21. Javaraiah J, *Mon Not Royal Astron Soc*, 377(2007)L34.
22. Miletskii E V, Ivanov V G, *Geomagnetism and Aeronomy*, 55(2015)872-876.
23. Hiremath K M, *Astrophys Space Sci*, 314(2008)45-49.
24. Gleissberg W, Sol Phys, 2(1967)231-233
25. Gnevyshev M N, Ohl A I, *Astron Zh*, 25(1948)18-20.
26. Ahluwalia H S, *J Geophys Res*, 103(1998)12103-12109.
27. Kimura H, *Mon Not Royal Astron Soc*, 73(1913)543-548.
28. Turner H H, *Mon Not Royal Astron Soc*, 73(1913a)549-552.
29. Turner H H, *Mon Not Royal Astron Soc,* 73(1913b)714-732.
30. Michelson A A, *Astrophys J*, 38(1913)268-274.
31. Kane R P, *Sol Phys*, 189(1999)217-224.
32. Rangarajan G K, *Earth Planets Space*, 50(1998)91-100.
33. Loskutov A Y, Istomin I A, Kotlyarov O L, Kuzanyan K M, *Astron Lett*, 27(2001)745-753.
34. Kuzanyan K, Obridko V N, Kotlyarov O L, Loskutov AY, Istomin I A, in 12th European Solar Physics Meeting, (ed) P Hardy, 12(2008)2-69.
35. Pesnell W D, *Sol Phys*, 252(2008)209-220.
36. Hathaway D H, Wilson R M, *Sol Phys*, 224(2004)5-19.
37. Kane R P, *Sol Phys*, 246(2007)471-485.
38. Kane R P, *Sol Phys*, 248(2008)203-209.
39. Du Z L, *Astron Astrophys*, 457(2006)309-312.
40. Wilson R M, Solar and Stellar Activity Cycle*s*, (Cambridge University Press), 1994.
41. Tlatov A G, *Sol Phys*, 260(2009)465-477.
42. Wilson R M, Hathaway D H, Reichmann E J, *J Geophys Res*, 103(1998)6595-6603.
43. Brown G M, *Mon Not Roy Astron Soc*, 174(1976)185-189.
44. Wilson R M, *Sol Phys*, 125(1990)133-141.
45. Brajša R, Wohl H, Hanslmeier A, Verbanac G, Ruzdjak D, Cliver E, Svalgaard L, Roth M, *Astron Astrophys*, 496(2009)855-861.
46. Cameron M, Schüssler M, *Astrophys J*, 659(2007)801-811.
47. Kane R P, *Sol Phys*, 282(2013)87-90.
48. Mayaud P N, *Geophys Monogr Ser*, 22(1980)82-84.
49. Ohl A I, *Solice Danie*, 9(1966)84.
50. Feynman J, *J Geophys Res*, 87(A8)(1982)6153-6162.
51. Hathaway D H, Wilson R M, *Geophys Res Lett*, 33(2006)L18101.
52. Dikpati M, de Toma G Gilman P A, *Geophys Res Lett*, 33(2006)L05102.
53. Hathaway D H, Wilson R M, *Sol Phys*, 224(2004)5-19.
54. Thompson R J, *Sol Phys*, 148(1993)383-388.
55. Lantos P, Richard O, *Sol Phys*, 182(1998)231-246.

56. Pesnell W D, *Sol Phys*, 289(2014)2317-2331
57. Jiang J, in Solar and Astrophysical Dynamos and Magnetic Activity, (eds) A G Kosovichev, E M de Gouveia Dal Pino, Y Yan, *Proc IAU Symp*, 294(2012)49-60.
58. Svalgaard L, Cliver E W, Le Sager P, *Adv space Res*, 34(2004)436-439.
59. Schatten K H, Scherrer P H, Svalgaard L, Wilcox J M, *Geophys Res Lett*, 5(1978)411-414.
60. Svalgaard L, Duvall T L (Jr), Scherrer P H, *Sol Phys*, 58(1978)225-239.
61. Hoeksema J T, *Space Sci Rev*, 72(1995)137-148.
62. Schatten K, *Geophys Res Lett*, 32(2005)L21106.
63. Svalgaard L, Cliver E W, Kamide Y, *Geophys Res Lett*, 32(2005)L01104.
64. Schatten K H, Pesnell W D, *Geophys Res Lett*, 20(1993)2275-2278.
65. Sheeley N R (Jr), *Astrophys J*, 144(1966)728-732.
66. Layden A C, Fox P A, Howard J M, Sarajendini A, Schatten K, Sofia S, *Sol Phys*, 132(1991)1-40.
67. Makarov V I, Makarova V V, *Sol Phys*, 163(1996)267-289
68. Li K J, Irie M, Wang J, Xiong S, Yun H, Liang H, Zhan L, Zhao H, *Publ Astron Soc Japan*, 54(2002)787-792.
69. Muñoz-Zaramillo A, Dasi-Espuig M, Balamaceda L A, DeLuca, E E, *Astrophys J Lett*, 767(2013)L25.
70. Obridko V N, Shelting B D, *Sol Phys*, 248(2008)191-202.
71. Choudhuri A R, Chatterjee, P, Jiang J, *Phys Rev Lett*, 98(2007)131103.
72. Petrovay K, in The Solar Cycle and Terrestrial Climate, Sol and Space Weather, (ed) A Wilson, ESA-SP-463(2000) 3-14.
73. Charbonneau P, *Liv Rev Sol Phys*, 7(2010)3.
74. Choudhuri A R in Solar and Astrophysical Dynamos and Magnetic Activity, (eds) A G Kosovichev, E M de Gouveia Dal Pino, Y Yan, *IAU Symp*, 294(2012)37-47
75. Parker E N, *Astrophys J*, 122(1955)293-314
76. Babcock H W, *Astrophys J*, 133(1961)572-587
77. Leighton R B, *Astrophys J*, 140(1964)1547-1562
78. Wang Y-M, Sheeley N R (Jr), Nash A G, *Astrophys J*, 383(1991)431-442.
79. Choudhuri A R, Schüssler M, Dikpati M, *Astron Astrophys*, 303(1995)L29-L32.
80. Durney B R, *Sol Phys*, 160(1995)213-235.
81. D'silva S, Chudhuri A R, *Astron Astrophys*, 272(1993)621-633.
82. King-Hele D G, *Nature*, 199(1963)226-227.
83. King-Hele D G, *Nature*, 209(1966)285-286.
84. Sargent H H III, in 28th IEEE Vehicular Technology Conference, 28(1978)490.
85. Kane R P, *Sol Phys*, 140(1992)171-180.
86. Kane R P, Sol Phys 122(1989)175-185.
87. Brown G M, Simon P A, in Solar-Terrestrial Predictions, (eds) P A Simon, G Heckman, M A Shea, Proc of a workshop at Meudon, (1986)1-7.
88. McIntosh P S, in *Solar-Terrestrial Predictions Proc*. (ed) R F Donnelly, 2(1979)246-257.
89. Brown G M, *Ann Geophys*, 10(1992)453-461.
90. Schatten K H, Sofia S, *Geophys Res Lett*, 14(1987)632-635.
91. Li K J, Yun H S, Gu X M, *Astron Astrophys*, 368 (2001)285-291.
92. Kane R P, Sol Phys, 202(2001)395-406
93. Joselyn J A, Anderson J B, Coffey H, Harvey K, Hathaway D, Heckman G, Hildner E, Mende W, Schatten K, Thompson R, Thomson W P, White O R, *Eos Trans*, 78(1997)211-212
94. Pesnell W D, *Space Weather*, 14(2016)1-12
95. Passos D, *Astrophys J*, 744(2012)172

96. Choudhuri A R, *J Astrophys Astr*, 29(2008)41-47
97. Yeates A R, Nandy D, Mackay D H, *Astrophys J*, 673(2008)544-556.
98. Charbonneau P, Dikpati M, *Astrophys J*, 543(2000)1027-1043.
99. Dikpati M, de Toma G, Gilman P A, *Astrophys J*, 675(2008),920-930.
100. Miesch M S, Featherstone N A, Rempel M, Trampedach R, *Astrophys J*, 757(2012)128.
101. Choudhuri A R, Karak B B, *Phys Rev Lett*, 109(2012)171103.
102. Karak B B, Nandi D, *Astrophys J Lett*, 761(2012)L13.
103. Nandi D, Karak B B, in Solar and Astrophysical Dynamos and Magnetic Activity, (eds) A G Kosovichev, E M de Gouveia Dal Pino, Y Yan, *Proc IAU Symp*, 29 (2012)439-444.
104. Tobias S, Hughes D, Weiss N, *Nature*, 442(2006)26.
105. Attia A-F, Ismail H A, Basurah H M, *Astrophys Space Sci*, 344(2013)5-11.
106. Bishoi S K,Janardhan P, Ananthakrishnan S, Tokumaru M, Fujiki K, in *IAU General Assembly*, 29(2015) 2256637.
107. Helal H R, Galal A A, *J Adv Res*, 4(2013)275-278.
108. Javaraiah J, *New Astron*, 34(2015)54-64.
109. Li K J, Peng W, Li F Y, *J Atmos Sol-Terres Phys*, 135(2015)72-76
110. Pishkalo M I, *Kinematics and Physics of Celestial Bodies*, 24(2008)242-247.
111. Rigozo N R, Souza Echer M P, Evangelista H, Nordemann D J R, Echer E, *J Atmos Sol-Terres Phys*, 73(2011) 1294-1299.
112. Yoshida A, Sayre R J, in "AGU Fall Meeting", Abstract #SH41D-2133 (2012).
113. Howard R, Labonte, B J, *Astrophys J Lett*, 239(1980)L33-L36.
114. Howe R, Christensen-Dalsgaard J, Hill F, Komm R W, Larson, T P, Rempel M, Schou J, Thompson M J, *Astrophys J Lett*, 767(2013)L20.
115. Penn M, Livingston W L, *Astrophys J Lett*, 649(2006)L45-48.
116. Livingston W L, Matt P, *EOS Trans*, 90(2009)257-258.
117. Schad T, Penn M, *Sol Phys*, 262(2010)19-33.

[*Received*:18.3.2016; *accepted*: 10.4.2016]