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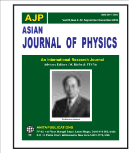

Three-dimensional correlation properties of speckles produced by diffractal-illuminated diffusers

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This Article is dedicated to Prof T Asakura

Three-dimensional correlation properties were studied experimentally for speckled-speckle patterns produced by a rough surface on which the speckle field due to a random fractal object is incident. The speckled speckles observed in some lateral planes with different propagation distances did not exhibit a definite speckle size, having many intensity clusters with various sizes which tend to increase with an increase of the fractal dimension of the fractal object. The fractality across the lateral planes was confirmed by the existence of a power-law behavior in the intensity correlation, and was practically independent of the propagation distance. The longitudinal fractality was also revealed by finding a nearly power-law behavior in the longitudinal intensity correlation. It was shown that the longitudinal fractal dimension was larger than the lateral fractal dimension for each dimension of the fractal object, indicating an anisotropic fractality of the speckle field.

Keywords: Speckled speckle, Fractal speckle, Speckle clustering, Longitudinal correlation, Correlation tail, Power law

1 Introduction

It is recognized nowadays that various fractal geometries are found in many structures that are formed naturally and artificially. Typical examples are found in physical processes such as phase transitions, aggregations, and interface formations, as well as in biological structures such as neurons and vessel networks [1,2]. To apply the optical technology to such structures with fractal properties, it is desired to understand how the fractality of a given object influences the properties of the diffracted, transmitted or scattered field. Since the first explicit attempt made by Berry [3], who coined the word *diffractals* to denote such optical fields, extensive studies have been made from this viewpoint in the past decades, some of them being reviewed in Refs [4,5].

On the other hand, if one considers applications of the concept of fractal to the optical technology, it is attractive to create an optical field having a fractal property which is controllable in a certain manner. The generation of coherent optical field with a random fractal property was discussed theoretically [6], and was subsequently confirmed experimentally [7]. In the experimental study [7], random fractal intensity distributions were formed as speckled speckles by a ground glass plate which was illuminated by the speckle patterns produced by random fractal apertures. The speckle fields were detected across the Fraunhofer diffraction plane, namely the focal plane of the lens placed behind the ground glass. The fractality of the fields was investigated in terms of the power-law behavior of the intensity correlation functions of the speckles. As a result, it was revealed that the examined intensity correlation functions obey approximately a power law with the exponent which depends on the fractal dimension of the first fractal objects. Therefore, such speckles can be called fractal speckles.

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More recently, experimental generation of fractal speckles in the configuration not of speckled speckles but of a single scattering was performed by producing a power-law intensity distribution with a given exponent by means of a spatial light modulator for the illumination of a ground glass plate [8,9]. More extensive study on fractality of fractal speckles was made on the basis of the concept of multifractals, and it was shown that fractal speckles produced by these methods are actually regarded to be multifractals [10]. A method for generating much brighter fractal speckles was proposed on the basis of digital holographic approach [11,12]. Fractal speckles in an image plane was also discussed, and were actually generated experimentally by using an aperture with a power-law transmittance in the pupil plane of the imaging system, an application of the image fractal speckles to the displacement measurements of diffusers being shown experimentally [13].

In the present paper, we extend the observation plane of the previous experiment [7] to the three-dimensional region around the focal plane to see whether the fractality of the field has a three-dimensional extent or not. To this end, we first study the autocorrelation functions of the speckle patterns in some observation planes with different longitudinal distances from the lens in comparison with that observed in the focal plane. Then, the existence of the fractal property in the longitudinal direction is examined by evaluating the longitudinal intensity correlation function of the speckle fields.

The main part of the present study was performed soon after the previous experimental work [7], and some results were presented in conferences and partly reported in their proceedings [14,15]. For some reason, however, the full description and discussions are presented in this paper dedicated to Prof T Asakura.

2 Theoretical background

First, we summarize the theoretical background for the fractal property of the speckle fields discussed in the previous study [6]. Consider an experimental system shown in Fig 1. A random fractal object placed in the object plane P_1 is illuminated by coherent light from a He-Ne laser after passing through a beam expander consisting of lenses L_1 and L_2 with a pinhole between them. The light scattered by the fractal object illuminates a diffuser located in the back focal plane P_2 of a lens L_3 lying focal distance f_1 away from P_1. The field scattered by the diffuser is observed in the focal plane P_3 of a lens L_4 placed at the focal distance f_2 away behind the diffuser.

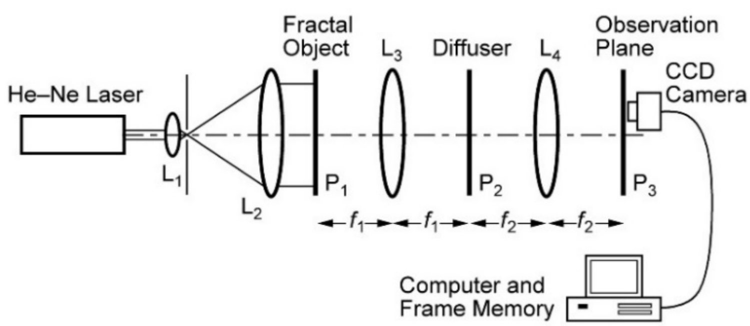

Fig 1. Experimental set-up for producing speckles with fractal property at different longitudinal distances.

Since the field illuminating the diffuser is a speckle pattern due to the random nature of the fractal object, the pattern observed in P_3 is a kind of speckled-speckle pattern. Statistical properties of general speckled speckles were studied by several researchers [16-20]. However, the special case of the current optical configuration was studied theoretically by Uno *et al* [6], with the main result that the correlation function of the intensity distribution is given, after certain approximations and manipulations, by

$$C_I(\boldsymbol{r}) = \frac{\langle I(\boldsymbol{r}')I(\boldsymbol{r}'+\boldsymbol{r})\rangle - \langle I(\boldsymbol{r}')\rangle\langle I(\boldsymbol{r}'+\boldsymbol{r})\rangle}{\langle I(\boldsymbol{r}')\rangle\langle I(\boldsymbol{r}'+\boldsymbol{r})\rangle} \propto \begin{cases} \left(\frac{r}{MR}\right)^{2(D-2)} & \text{for} \quad 1 < D < 2, \\ \left(\log\frac{r}{MR}\right)^2 & \text{for} \quad D = 2, \\ 1 & \text{for} \quad 2 < D < 3, \end{cases} \tag{1}$$

for $r << MR$, where $M = f_2/f_1$, R is a quantity proportional to the maximum scale of the fractal object generating the first speckle pattern, and D is a fractal dimension of the object.

Here it is to be mentioned that there are some confusing notions in our previous paper [6] as explained in the following. The parameter α in Eq (10) should be read as $\alpha = (2\pi/\lambda f)R$ while the same symbol α in Eq (12) and thereafter should be read as $\alpha = MR$, where $M = z/f$ in the configuration of Ref [6]. With these corrections, Eq (15) of Ref [6] leads Eq (1) given above.

This equation indicates that, as long as the fractal dimension of the first object lies between one and two, the generated speckled speckle is also fractal having the power-law intensity correlation with the exponent dependent on the dimension D of the object. This prediction was confirmed experimentally in the earlier paper [7] using three objects generated by the band-limited Weierstrass functions with different fractal dimension $D = 1.2$, 1.5, and 1.8. The generated fractal speckles were found to have different appearance from the normal speckles in the sense that they do not have definite speckle sizes but instead exhibit clusters or clumps of speckles. In the next section, we will extend the discussion to the speckles observed in planes other than the focal plane P_3.

3 Experiment and Discussion

3.1 Fractal speckles at different propagation distances

The setup employed in the present experiment was the same as that assumed in Sec 2. It was also the same as one employed in the previous study [7] except that, in the present case, the observation plane was not restricted to P_3 but located at an arbitrary distance z from L_4. The same scattering objects as with the previous study were employed, the fractal objects being the self-similar trails with different fractal dimensions D = 1.2, 1.5, and 1.8, and the diffuser being the ground glass plate. A positive lens with a focal length f_2 = 10 cm was used as L_4.

In the present experiment, we first examined the speckles produced by the three fractal objects at some distances other than the focal distance f_2. The obtained speckles are shown in Fig 2 for nine combinations of the parameters z and D; D = (a) 1.2, (b) 1.5, and (c) 1.8 for z = 10 cm (focal plane): D = (d) 1.2, (e) 1.5, and (f) 1.8 for z = 50 cm; and D = (g) 1.2, (h) 1.5, and (i) 1.8 for z = 150 cm. Hence, Figs 2(a)–(c) have the same parameters as the previous results [7] and are shown here for comparison with the other figures.

It is noted from Fig 2 that the speckle patterns have peculiar appearance not only at the focal plane P_3 but also at other detection distances, being different from ordinary speckles which are seen, for example, in Fig 5(c) in Ref [7]. To examine the peculiarity of these speckles, an intensity distribution along a certain line was derived from each of Figs 2(a)–(c), and is shown in Figs 3(a)–(c), respectively. A similar plot obtained from an ordinary speckle pattern is shown in Fig 3(d) for comparison. Figure 3(d) shows that the lateral extents of the bright spots are not very different while speckles in Figs 3(a)–(c) have spots with wide range of lateral extents, which give rise to a clustering appearance. While the sizes of the clusters grow as D increases in Figs 3(a)–(c), speckle spots with smaller sizes still remain. This is recognized well by noting that most intensity spots have very sharp peaks for each of Figs 3(a)–(c). The sharpness of the peaks implies the existence of the components of small speckle spots which are not resolved sufficiently by the pixels of the CCD. On the other hand, the speckles in Fig 3(d) are not so sharp as the other figures, indicating that the speckles are well resolved by the detector.

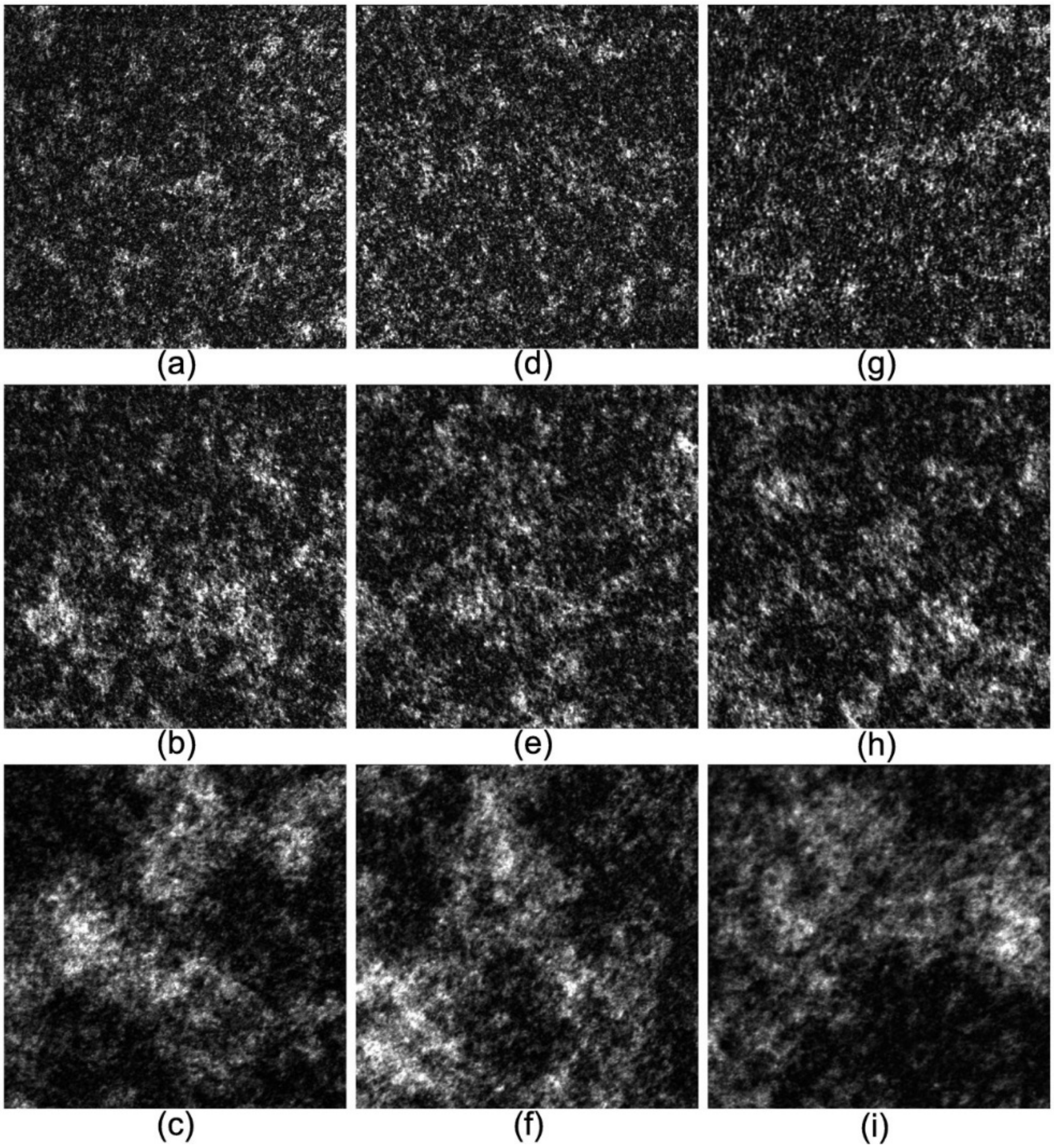

Fig 2. Speckled speckles observed at longitudinal distances of z = (a)–(c) 10, (d)–(f) 50, and (g)–(i) 150 cm, and for different fractal dimensions of D = (a), (d), (g) 1.2; (b), (e), (h) 1.5; and (c), (f), (i) 1.8.

It is also noted from Fig 2 that the three speckle patterns for each value of D look similar in a statistical sense. Namely, the speckles seem to depend only on the fractal dimension D of the first scattering object, and not on the propagation distance z.

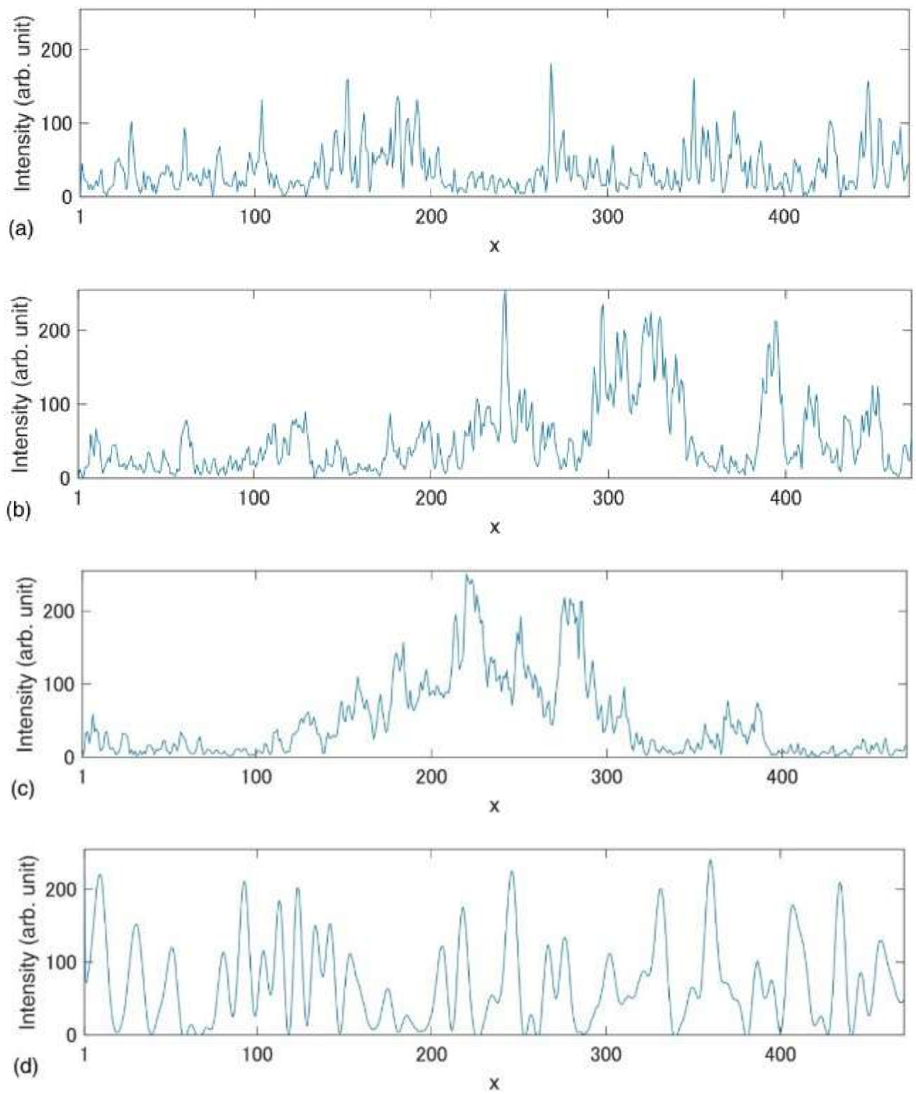

Fig 3. One-dimensional intensity distributions of the three fractal speckles with D = (a) 1.2, (b) 1.5, and (c) 1.8 at z = 10 cm, and (d) of an ordinary speckle pattern. The abscissa is the position in pixel numbers along a line in the observation plane.

3.2 Lateral correlations at different propagation distances

In the previous study [7], we showed that the peculiarity of the speckle clustering found in P_3 is attributed to the power-law behavior of the lateral intensity correlation function. From the above observation that the speckle patterns look statistically similar for different z, it is expected that the intensity correlation function across the lateral observation planes does not change with z. This conjecture can be proved theoretically. In the present experiment, the diffuser is placed at the front focal plane of L_4. Our previous analysis [21] shows that, in this configuration, the speckle field observed behind the lens is weakly stationary not only laterally but also longitudinally. This means that the form of the lateral correlation function does not depend on the propagation distance z and, hence, that a same power function governs the lateral correlations for different z provided D is constant.

To verify this prediction experimentally, the intensity correlation functions $C(r)$ were calculated from the observed speckle patterns of the nine combinations of the parameters shown in Figs 2(a)−(i), and the results are shown in Fig 4. It is seen that the correlation function for the nine speckle patterns are approximately linear in this plot for the range of $2 \lesssim r \lesssim 10$ mm, showing power-law behaviors in the correlations. It is also noted that the correlation functions take substantially the same form for different distances for each value of D, though the correlation values seem to decrease slightly with increasing z.

As noted in the previous paper [7], however, there are deviations from the power laws at the marginal values of r in Fig 4. To make this point evident, local slopes were calculated from every three successive points of the curves in Fig 4, and the results for $z = 150$ cm are shown in Fig 5. Now it is clear that the slopes are almost constant for $2 \lesssim r \lesssim 10$ mm, and decrease for $r \gtrsim 10$ mm. This steep decrease in the local slope for $r \gtrsim 10$ mm is consistent with behavior given in Eq (1). This upper cutoff of the power-law behavior is attributed to the finite extent of the fractal object placed in P_1. It is also noted that, though the power function in Eq (1) diverges at the origin, any actual intensity correlation should saturate at a finite value, which is set to unity in the normalized form of Eq (1).

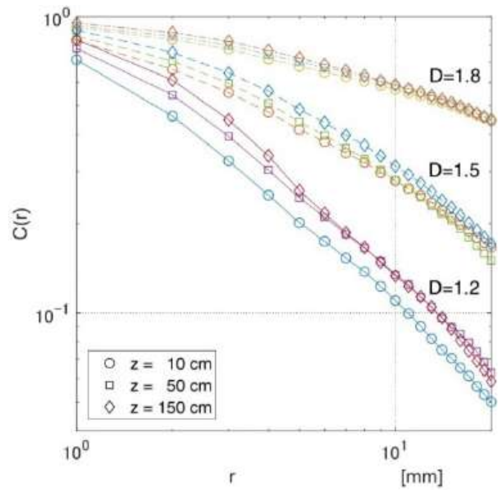

Fig 4. Autocorrelation functions $C(r)$ of the speckle intensities observed at distance $z = 10$, 50, and 150 cm for three objects of $D = 1.2$, 1.5, and 1.8.

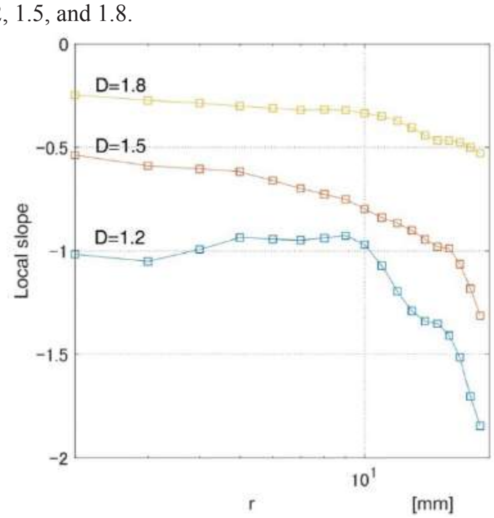

Fig 5. Local slopes of the lateral correlations of the speckles produced at $z = 150$ cm for three fractal objects with $D = 1.2$, 1.5, and 1.8.

The values of the local slopes should be compared with the theoretical ones; $2(D-2) = -1.6, -1$, and -0.4 for the fractal objects with the dimensions of $D = 1.2$, 1.5, and 1.8, respectively. The experimental values are smaller in their moduli than, but approach for larger r, the theoretical ones. These behaviors of the local slopes for $z = 150$ cm are almost the same as the case of $z = 10$ cm which was shown in Fig 9 of Ref [7] and this is in agreement with the three-dimensional stationarity of the speckle field. Here, we note that the fractal dimension D_s of the speckle intensity distributions is given theoretically by $D_s = 2D - 2$, which yields $D_s = 0.4$, 1, and 1.6 for $D = 1.2$, 1.5, and 1.8, respectively.

3.3 Longitudinal correlations

We are now in a position to investigate whether or not these speckle fields have a fractality in the longitudinal direction as well as in the lateral directions. To this end, the speckled speckles were recorded at different longitudinal distances from 11 to 21 cm with a step interval of 2 cm, and also $z =$ at 10, 10.1, and 10.5 cm. As discussed in the previous paper [7], the longitudinal intensity correlation can be expressed by the z-component $R_I(0,0,\Delta z)$ of the three-dimensional intensity correlation function

$$R_I(\Delta x, \Delta y, \Delta z) = \iiint \Delta I(x, y, z)\, \Delta I(x-\Delta x, y-\Delta y, z-\Delta z)\, dxdydz \times \left[\iiint I(x, y, z) dxdydz \iiint I(x-\Delta x, y-\Delta y, z-\Delta z) dxdydz\right]^{-1} \quad (2)$$

which requires a huge amount of data to be recorded and handled to evaluate it. To avoid it, we consider the following function by dropping out the integration with respect to z in Eq (2):

$$R_I'(\Delta x, \Delta y; z_1, z_2) = \iint \Delta I(x, y, z_1)\, \Delta I(x-\Delta x, y-\Delta y, z_2) dxdy \times \left[\iint I(x, y, z_1) dxdy \iint I(x-\Delta x, y-\Delta y, z_2) \mathrm{d}x\mathrm{d}y\right]^{-1} \quad (3)$$

which is actually a two-dimensional cross-correlation function of two intensity distributions across the planes at different longitudinal distances $z_1 = z$ and $z_2 = z - \Delta z$. The desired longitudinal property can be approximated by the central peak $R_I'(0,0; z_1, z_2)$ of Eq (3). It is noted that Δz, which is regarded to be independent of a specific value of z, can be safely replaced by $z_2 - z_1$ in Eq (3) because of the longitudinal stationarity as discussed in Sec 3.2. The calculations of Eq (3) is accomplished by firstly computing the two-dimensional FFT of the data sets for two intensity distributions, secondly multiplying one transformed data set by the complex conjugate of the other, and finally performing the two-dimensional inverse FFT of the product.

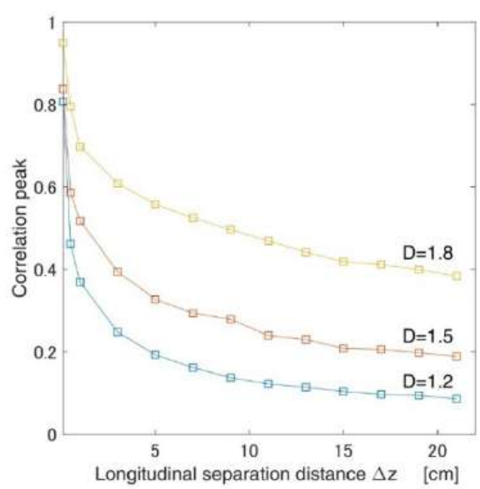

Fig 6. Longitudinal intensity correlations as a function of the separation Δz for three fractal objects with $D = 1.2$, 1.5, and 1.8.

The resultant longitudinal correlations of the speckles are shown in Fig 6. Each data point was calculated from two speckle patterns: one was fixed at z_2 = 10 cm as a reference pattern and the other was chosen from the patterns observed at different longitudinal distances z_1. It is clear from Fig 6 that, for each fractal object, the longitudinal intensity correlation of the speckle field has a steep central peak and a gradually decreasing tail, and that the correlation values increase in the entire region of Δz with an increase in the fractal dimension D of the object. To test for the longitudinal fractality, Fig 6 is replotted in Fig 7 in a log-log scale. We see that the three experimental curves in this plot is approximately linear though the approximation is not so good as the case of the lateral correlations shown in Fig 4. The important point is, however, that the longitudinal correlations of the present speckle fields have much longer tails than expected for ordinary speckles and, hence, we can conclude that the speckled speckles produced by this experiment have a fairly high fractality in the longitudinal direction as well as in the lateral direction.

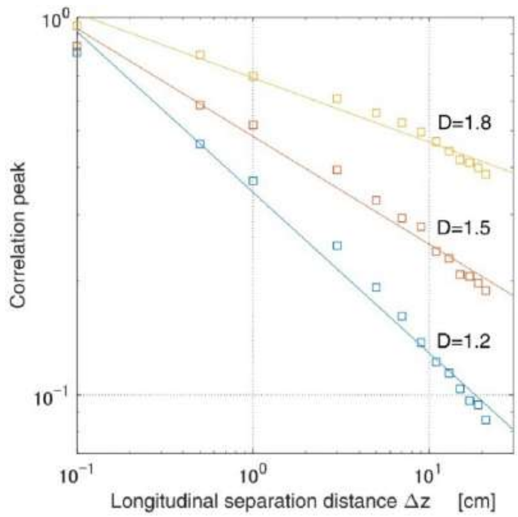

Fig 7. Logarithmical plot of the longitudinal intensity correlations shown in Fig 6.

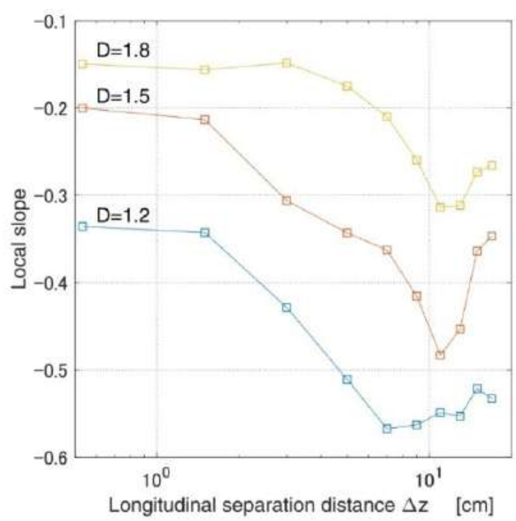

Fig 8. Local slopes of the longitudinal correlation of the speckles produced by three fractal objects with D = 1.2, 1.5, and 1.8.

Finally, the local slopes for the curves in Fig 7 were calculated and shown in Fig 8. It is noted that data for the marginal values of Δz were lost because three successive points were required to obtain the results. This figure shows that the slopes are almost constant for $\Delta z \lesssim$ 2cm and decrease for larger Δz, ending up with small increases. This trend is qualitatively similar to the case of the lateral correlation. However, the moduli of the local slopes, in particular in their constant regions, are clearly smaller than those of the corresponding lateral correlations shown in Fig 4. This means that, for each D, the longitudinal fractal dimension is larger than the lateral one and, hence, that the speckle field has an anisotropic fractality.

4 Conclusions

As a continuation of the previous work [7], we investigated the particular type of speckled-speckle patterns which were produced by a ground glass on which the speckles due to random fractal objects were incident. Properties of the speckle patterns were studied experimentally not only at the Fraunhofer diffraction plane of the ground glass but also as a function of the propagation distance from the lens behind the diffuser.

The observed speckles were different from ordinary speckles in the sense that they do not have definite speckle sizes but show a clustering appearance, and that the sizes of the intensity clusters tend to increase with the dimension of the fractal object. This appearance seemed to be independent of the propagation distance, which is also suggested by the theoretical consideration developed in the previous paper.

The fractality in the lateral intensity distributions of the speckle patterns was confirmed by finding power-law behaviors in the intensity correlations. The lateral correlation function was found to be practically independent of the propagation distance. Local slopes evaluated from the lateral intensity distributions at z = 150 cm were almost constant in the range of $2 \lesssim r \lesssim 10$ mm, being substantially the same as the case of z = 10 cm.

The longitudinal intensity correlations were also evaluated by calculating the peak values of the cross-correlation functions of the intensity distributions across two observation planes. It was found that the longitudinal correlation follows approximately a power law in the limited range of $z \lesssim 2$ cm. The modulus of the exponent of the power function, corresponding to the slope in its log-log representation, decrease with an increase of the fractal dimension of the fractal object. These slopes for the longitudinal correlations were small as compared with the corresponding lateral slopes, implying three-dimensional but anisotropic fractal fields for the speckled speckles examined.

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