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# Watermarking of phase-only Fresnel hologram using symmetrical 3D modified Gerchberg-Saxton algorithm

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Dedicated to Padma Shree Prof R S Sirohi, FNAE

A watermarking technique for phase-only Fresnel hologram using symmetrical three-dimensional (3D) modified Gerchberg-Saxton (GS) algorithm has been proposed. The phase-only Fresnel hologram (POFH) of multi-plane object is synthesized using symmetrical 3D modified GS algorithm. Reconstruction from the POFH and the watermarked POFH has been demonstrated through simulation results. © Anita Publications. All rights reserved.

Keywords: Digital holography; Fresnel diffraction; Phase retrieval; Watermarking

## **1** Introduction

Three-dimensional (3D) display has been widely used in a variety of technological applications, such as, 3D movies, multimedia display, and virtual reality [1-3]. Among the many candidates known for 3D display technique, holographic 3D display is the most efficient. It provides true visual depth clues, no requirement of special devices for 3D viewing, fatigue-free and no visual confusion, etc. [4-6]. The 3D visualization is realized by the optical reconstruction of digital hologram (DH) by holographic display system. The full wave field information from the DH of the recorded object is recovered numerically and is reconstructed in a physical plane using a spatial light modulator [5,6]. The intensity distribution of the optical wavefield can be controlled by using a computer generated hologram [7]. The hologram lacks diffraction efficiency, since it is a combination of amplitude and phase and the amplitude part of the hologram modulates the reconstruction beam. The phase-only hologram manipulates only the phase of the optical field at each pixel without modulating amplitude part of the reconstruction beam [8-10]. The phase information of a DH can be extracted from the iterative Fourier transform algorithm, which is computationally efficient and easy to implement using fast Fourier transform, compared to the other methods [10].

In this paper, phase-only Fresnel hologram (POFH) for multi-plane object has been synthesized using iterative phase retrieval algorithm [11-13]. The symmetrical 3D modified Gerchberg-Saxton (GS) algorithm has been used to extract phase in the Fresnel domain [14,15]. A random phase is taken as the input which is Fresnel propagated to the first plane. The desired object is substituted in the Fresnel diffracted random phase, which is then inverse Fresnel transformed. Again, the process is repeated for the second plane. An image is embedded into the phase hologram to obtain the watermarked phase-only hologram [16]. The original and the watermarked POFHs are reconstructed to obtain the desired images in both the planes.

### 2 Symmetrical 3D Modified Gerchberg-Saxton Algorithm

The original GS algorithm is implemented by the fast Fourier transform technique, but in the modified algorithm [11,13,14], the Fourier transform is substituted by the Fresnel transform. In the GS algorithm, during each iteration the amplitude distribution in the signal window is replaced by the desired amplitude in the image plane. The block diagram of modified GS algorithm is shown in Fig 1. The symmetrical 3D modified GS algorithm differs from the GS algorithm only in a way that the amplitude is modified in the signal window, which is given by Eq (1).

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$$I| = \begin{cases} 2\eta |I_0| - |J|, u \in S \\ |J|, u \notin S \end{cases}, \quad \eta = \frac{\sum |J(u \in S)|}{\sum |I_0(u \in S)|} \end{cases}$$
(1)

Here,  $I_0$  denotes the desired output in the signal window, J is the calculated amplitude in the image plane, u is the coordinate in the image plane, S is the signal window, and I is the modified amplitude in the image plane.



Fig 1. Schematic of modified GS algorithm. RP: Random phase, FT: Fresnel transform, FT<sup>-1</sup>: Inverse Fresnel transform, and f(x,y): desired object.

#### **3** Phase-only hologram watermarking

In hologram watermarking technique [16], an image is embedded into the hologram, thereby ensuring the security of the hologram. The phase-only hologram watermarking process can be defined by equation

$$\phi_{\mathsf{w}}(\xi,\eta) = \phi(\xi,\eta) + a \exp\left(i \times I(\xi,\eta)\right) \tag{2}$$

where  $\phi_w$  is the watermarked phase-only hologram,  $\phi$  is the phase-only hologram,  $I(\xi,\eta)$  is the watermark image, and *a* is an arbitrary constant. Through this watermarked phase-only hologram, 3D information can be secured.

The reconstruction of the objects from the original as well as the watermarked hologram is possible in different planes [16]. Further, to claim the authenticity of the phase-only hologram, the watermark image is retrieved from the watermarked hologram. Mathematically, the retrieved watermark can be obtained using Eq (3).

$$I(\xi,\eta) = \left| \log_e \left( \frac{\phi_w(\xi,\eta) - \phi(\xi,\eta)}{a} \right) \right|$$
(3)

### **4 Simulation Results**

Simulation has been carried out on MATLAB 7.0 platform to verify the proposed method of POFH watermarking. The hologram is designed to reconstruct an image in the axial planes at distances of 30 cm and 50 cm, respectively. Figures 2(a) and 2(b) show the images in first and second planes, respectively. The

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size of the desired images is  $128 \times 128$  pixels with a sampling interval of 2.2 µm. The desired image is zeropadded into an image matrix  $256 \times 256$  pixels. Zero padding is a process in which an image matrix is put into a larger matrix, having all its elements as zero. The wavelength used is 632.8 nm. Total 500 iterations were used in the GS algorithm. The POFH is shown in the Fig 2(c). Figures 3(a)-3(c) show the images used for watermark, the watermarked POFH, and the retrieved watermark image, respectively. The value of arbitrary constant (a) used for simulation is 0.15. Reconstruction of images in two different planes from POFH and watermarked POFH has been shown in Fig 4. It can be seen that there is no difference between POFH and watermarked POFH, visually. At the same time, original watermark image is successfully retrieved from the watermarked hologram.



Fig 2. (a) The desired object in first plane, (b) desired object in second plane, and (c) phase-only Fresnel hologram



Fig 3. (a) The desired watermark image, (b) watermarked phase-only Fresnel hologram, and (c) retrieved watermark image.



Fig 4. Phase-only hologram reconstruction of (a) 1st plane and (b) 2nd plane. Watermarked phase-only hologram reconstruction of (c)1st plane and (d) 2nd plane

# **5** Conclusion

A technique of POFH watermarking has been suggested. The POFH of a multi-plane object is obtained by iterative process of the symmetrical 3D modified GS algorithm. The reconstruction of the images in different planes from the original POFH and the watermarked POFH has been shown, which is same for the two holograms. It is seen that the original and the watermarked POFHs look same and the original watermark image is also retrieved. Therefore, this technique of hologram watermarking can be used for authenticating a phase-only hologram. The watermarking technique can also be used for authenticating the POFH in parts by watermarking different planes with different images.

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