



Spectral Interferometry: Some aspects of complex optical fields

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Dedicated to Professor D N Rao for his significant contributions and pioneering works in the fields of spectroscopy, optics, nonlinear optics and photonics

Interference fringes in the space-frequency domain are as important to the understanding of complex light fields and their interaction with material as are the traditional interference fringes observed, measured and used in the space-time domain. A short journey through this fundamentally important area of research is presented here, as one begins to understand and unravel the underlying connection between interference fringes in complementary domains. With the recent knowledge of the significance of optical singularities and the non-separability of polarization – optical mode, connecting the behavior of and the information contained in complex optical fields in the space-frequency domain has only become richer. Some recent results are presented to emphasize the underlying connections. © Anita Publications. All rights reserved.

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1 Introduction

The fundamental aspect of spectral invariance of radiation on propagation in free-space stems from the ‘questionable’ assumption, though largely for mathematical convenience, that light in general is treated as a monochromatic plane wave [1]. The development of statistical treatment of optical fields and optical coherence theory led to the discovery that spatial coherence properties of a source indeed affect the emitted spectrum. The subsequent development of coherence effects in the space-frequency domain [1] provided credence to the fundamentally significant anticipation by Wolf [2,3] that the spectrum of light will differ from the source spectrum, and more importantly, may change on free-space propagation. Experimental verification of this prediction [4,5] quickly made this wave phenomenon an area of research of fundamental importance [1,6].

Correlation-induced changes to the source spectrum apart, it was demonstrated theoretically and experimentally that partial coherence between two same spectral density broadband light beams can also affect the source spectrum, leading to spectral modulations [7,8]. Unlike quasi-monochromatic light beams, the superposition of broadband light beams does not lead to the facile appearance of interference fringes in the spatial domain beyond a narrow region and hence monitoring of changes in the spectral domain provides us with access to the critical missing information. This led to complementarity (in space and frequency domain), involving fringe formation due to two-beam superposition [9,10]. A large variety of mechanisms and methods have since been developed and used to modify the source spectra for different applications [see Refs. 1, 6 and citations there in].

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Of relevance here are the spectral modulation arising from the superposition of light beams in a Mach-Zehnder interferometer with different degrees of correlation [11] and its experimental verification [12]. The spectral modulations due to superposition of broadband light beams in the space-frequency domain were also used for precise determination of the refractive index of normal dispersive material [13], film thickness [14], group velocity dispersion [15] and complex degree of spectral coherence [16]. Subsequent realization that around phase singularity spectral modulations go through anomalous behavior led us to the demonstration of spectral shifts and spectral switches in a Michelson interferometer, which was also found to influence the temporal behavior of broadband light and ultra-short pulses [17,18] and the measurement of nanometer scale displacements using wavelength shifts [19]. This article summarizes some of the important results from University of Hyderabad (India), in the larger context of superposition in space-frequency domain and extends them to understand anisotropic and chiral materials via spectropolarimetric measurements [20] and spectral phase metrology [21] using broadband light beams

2 Results and Discussion

Superposition of complex light fields in the space-time and space-frequency complementary domain and the resulting interference pattern was proposed [10] and demonstrated [8,9,12] to address the fundamental issue of interference in the complementary domain, where one normally thinks that interference is not possible. Following the notations used in Ref [10], the superposition of two complex light fields of amplitudes $\varepsilon_1(t)$ and $\varepsilon_2(t)$ gives the field amplitude at the detector $\varepsilon(t) = \alpha \varepsilon_1(t) + \beta \varepsilon_1(t + \tau)$, which results in the interference equation $I = |\alpha|^2 I_1 + |\beta|^2 I_2 + \{\alpha^* \beta \Gamma_{12}(\tau) + c.c.\}$, where $\Gamma_{12}(\tau)$ is the cross-correlation function between the beams. For quasi-monochromatic light beams which are delayed beyond the correlation time (τ_c) and then superposed, $\Gamma_{12}(\tau) \cong 0$ and so the interference term vanishes, leading to the statement that ‘no interference is possible if the path difference between the beams is larger than the correlation time’. However, taking Fourier transform of the complex fields, the spectrum of the superposed beams given by $S(\omega) = |\alpha|^2 S_1(\omega) + |\beta|^2 S_2(\omega) + \{\alpha^* \beta S_{12}(\omega) e^{i\omega\tau} + c.c.\}$, shows that even if $S_{12}(\omega)$ were white noise (of zero correlation time), the spectrum will always exhibit ‘cosine’ modulation. This unequivocally demonstrates that even if the interference is lost in the time domain, it is restored in the frequency domain, due to complementarity of the time and frequency domains.

The appearance of spectral modulations when the path difference between the superposed beams in a Mach-Zehnder interferometer (MZI) is delayed beyond coherence time (or coherence length) – τ_c (l_c) was experimentally demonstrated using a broadband ($\Delta\lambda = 8$ nm) low-coherence ($\tau_c = 140$ fs, $l_c = 42$ μm) laser [12]. The appearance and the behavior of spectral fringes measured in the interferometer output were understood using simplified interference equation in the spectral domain: $S_o(\lambda) = 1/2 S_i(\lambda)[1 + \cos(2\pi\Delta/\lambda + \theta)]$, where $\Delta = L_1 - L_2$ is the path difference between the two arms of the MZI (Fig 1 (a), (b)). The number of fringes in the spectral domain were found to increase linearly with the path difference as can be expected. We, however, noticed a small linear change in the periodicity of the fringes as one goes from the blue to the red region of the source spectrum. This was exploited subsequently by increasing the bandwidth of the light source by changing to spatially and temporally partially coherent white light source [13,14]. The spectrally broad, incoherent white light source was used in a Michelson interferometer (MI). As the coherence length is much smaller (~ 0.35 μm) as compared to the broadband laser, it was much easier to obtain spectral fringes, as aligning the interferometer to within the coherence length is not any more a critical requirement. More importantly, introducing a normal dispersive medium (like transparent glass plate or polymer film) in one of the interferometer arms and adjusting the interferometer arms to near zero path difference results in dramatic changes to the spectral fringes, with the appearance of zero-order fringe, at a wavelength of choice (Fig 1 (c), (d)). The zero-order fringe appears for the condition $\Delta = n(\lambda_{st})2t - L_0 = 0$, where λ_{st} is the wavelength corresponding to the stationary phase point, and can be adjusted to be anywhere within the source bandwidth

by adjusting L_0 . This feature was used to accurately calculate the refractive index - thickness [13,14] and the group velocity dispersion (GVD) [15] of the transparent sample introduced in one of the interferometer arm.

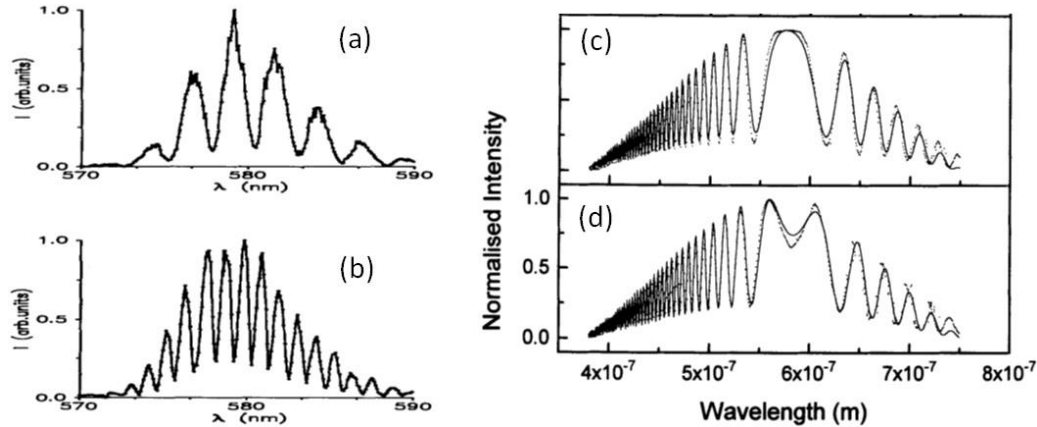


Fig 1. (a), (b) Spectral modulations obtained in a MZI illuminated by a broadband laser [12], (c), (d) spectral modifications due to introducing a normal dispersive medium in one of the arms of the MI [14] for broadband white light source.

The complex degree of spectral coherence is a quantity which determines the source correlation aspects arising due to superposition of optical waves. Written as $\mu_{12}(\lambda) = |\mu_{12}(\lambda)|\exp[i\beta_{12}(\lambda)]$, $|\mu_{12}(\lambda)|$ and $\beta_{12}(\lambda)$ give respectively the modulus and the phase of the complex degree of spectral coherence. The wavelength-dependent visibility of the interference fringes obtained in the amplitude division MI experiment directly gives information about $|\mu_{12}(\lambda)|$. On the other hand, measuring correlation-induced spectral changes at the on-axis and different off-axis positions in the Young's double slit interference experiment gives information about the real and imaginary parts of $\mu_{12}(\lambda)$ from which the modulus and phase of the complex degree of spectral coherence were determined [16]. This demonstrates that spectral modifications including spectral peak shift and spectral modulations arising due to complete and partial correlation of two complex light fields, derived from a broadband white light source, can give the complete spectral coherence information of the source.

Phase singularities are points or lines in 2D or 3D space where the optical intensity becomes null and the phase there becomes indeterminate [22]. The behavior of the optical field around such locations has complicated structure including the appearance of dislocations and scalar and vectorial vortices [23]. Gbur *et al*, showed dramatic spectral changes in the vicinity of phase singular points in the focal region of polychromatic field [24]. These spectral changes around phase singularity were also experimentally demonstrated [25]. The anomalous spectral shift and spectral switch behavior arising due to diffraction of spatially coherent and spectrally broad ultrashort pulse around the phase singularity was found to affect the pulse evolution in the complementary space-time domain [17]. Experimental demonstration of anomalous spectral behavior due to temporal correlation around intensity minima in a Michelson interferometer has been also reported [18]. The spectral switch behavior was subsequently used for accurate nano-displacement measurements [19].

Introducing an anisotropic material in a polarization interferometer operated in the spectral domain offers additional control on the spectral characteristics and a possibility to unravel dispersion characteristics of the material as well. We briefly present our results from three such measurements using a polarization interferometer constructed using a broadband white light source and two Glan-Thompson polarizers, used

as polarizer and analyzer and a fiber-coupled spectrometer. After ensuring that no spectral feature appears in the crossed P-A condition, an anisotropic sample is introduced in between. Rotating the anisotropic sample with respect to the crossed P-A condition to minimum intensity position gives spectral modulations as shown in Fig 2 (a), measured for two different analyzer angles. A small wavelength-dependent phase difference can be seen from the shift in the spectral modulations measured which can be used to accurately calculate dispersion characteristics of the anisotropic medium. Next, we introduce a waveplate in between the polarizer and analyzer. The waveplate used is a combination of two plates with fixed orientation angle between their optical axes and with respect to the input polarization. By rotating the analyzer angle, for a fixed relative orientation of the waveplates, one can realize spectral switch behavior at a particular wavelength of ~ 620 nm as shown in Fig 2 (b). Introducing a chiral crystal in between the orthogonal oriented P-A changes the spectrum in the entire visible range as shown in Fig 2 (c), reminiscing the spectral switch behavior realized using a MI in Ref 18. Selectively introducing absorption in a particular range of the spectrum via interference coating to the anisotropic crystal also modifies the spectral characteristics as a function of analyzer orientation as shown in Fig 2 (d). We thus have demonstrated the versatility of spectral interference fringes manipulated by using polarization as a degree of freedom, by passing a polarized broadband light through different types of anisotropic media. These results can be understood based on Jones matrix calculations and a detailed discussion on the results and analysis will be reported elsewhere.

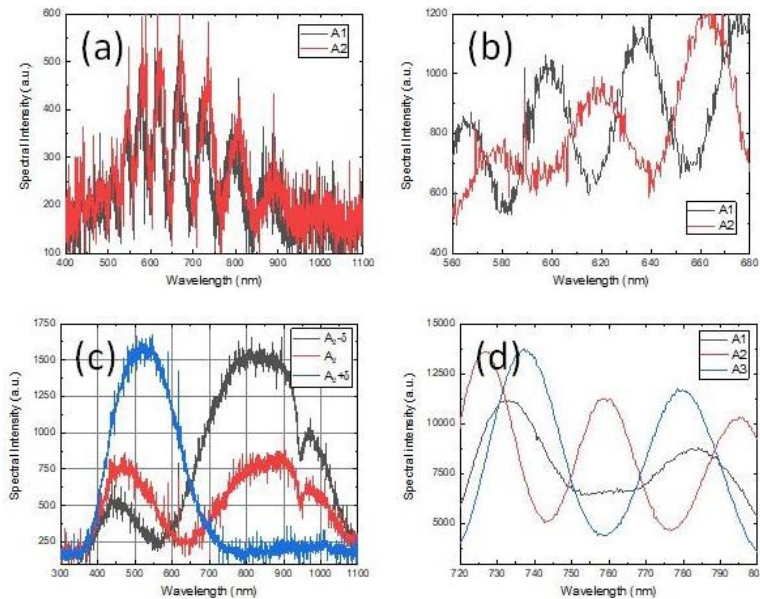


Fig 2. (a) – (d) Interference patterns measured in the spectral domain and their modifications accessed via polarization degree of freedom. See text for details on the graphs.

3 Summary

This article presents some of the results from the research work carried out in the research labs in School of Physics, University of Hyderabad, related to the spectral interferometry. This by no means is a review of the area of research discussed herein which is vast. The mere connections made to interferometry and the information that can be obtained in the complementary time-frequency domains ensures rich and nuanced nature of this area of research which continues to grow.

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References

1. Wolf E, James D F V, Correlation-induced spectral changes, *Rep Prog Phys*, 59(1996)771; doi.org/10.1088/0034-4885/59/6/002.
2. Wolf E, *Phys Rev Lett*, Invariance of the spectrum of light on propagation, 56(1986)1370; doi.org/10.1103/PhysRevLett.56.1370.
3. Wolf E, *Nature*, Non-cosmological redshifts of spectral lines, 326(1987)363–365.
4. Morris G M, Faklis D, Effects of source correlation on the spectrum of light, *Opt Commun*, 62(1987)5–11.
5. Faklis D, Morris G M, Spectral shifts produced by source correlations, *Opt Lett*, 13 (1988)4–6.
6. Mandel L, Wolf E, *Optical Coherence and Quantum Optics*, (Cambridge University Press) 1995.
7. James D F V, Wolf E, Some new aspects of Young's interference experiment, *Phys Lett*, 157A(1991)6–10.
8. Santarsiero M, Gori F, Spectral changes in a Young interference pattern, *Phys Lett*, 167A(1992)123–128.
9. Rauch H, Phase space coupling in interference and EPR experiments, *Phys Lett*, 173A(1993)240–242.
10. Agarwal G S, Interference in complementary spaces, *Found Phys*, 25(1995)219–228.
11. Agarwal G S, James D F V, Spectral changes in the Mach-Zehnder interferometer, *J Mod Opt*, 40(1993)1431; doi.org/10.1080/09500349314551491.
12. Rao D N, Viswanathan N K, Experimental demonstration of spectral modification in a Mach-Zehnder interferometer, *J Mod Opt*, 41 (1994)1757–1763.
13. Viswanathan N K, Rao D Narayana, Using interference in the frequency domain for precise determination of thickness and refractive indices of normal dispersive materials, *J Opt Soc Am B*, 12(1995)1559–1563.
14. Viswanathan N K, Chandrasekhar Y, Rao D N, Measurement of optical constants of thin polymer films using spectrally resolved white light interferometry, *Pramana*, 47(1996)163–170.
15. Debnath S K, Viswanathan N K, Kothiyal M P, Spectrally resolved phase-shifting interferometry for accurate group-velocity dispersion measurements, *Opt Lett*, 31(2006)3098–3100.
16. Viswanathan N K, Rao D N, Two-beam interference experiments in the frequency domain to measure the complex degree of spectral coherence, *J Mod Opt*, 48(2001)1455–1465.
17. Veetil S P, Viswanathan N K, Vijayan C, Spectral and temporal evolutions of ultrashort pulses diffracted through a slit near phase singularities, *Appl Phys Lett*, 89(2006)041119; doi.org/10.1063/1.2236979
18. Brundavanam M M, Viswanathan N K, Rao D N, Spectral anomalies due to temporal correlation in a white-light interferometer, *Opt Lett*, 32(2007)2279–2281.
19. Brundavanam M M, Viswanathan N K, Rao D N, Nanodisplacement measurement using spectral shifts in a white-light interferometer, *Appl Opt*, 47(2008)6334–6339.
20. Kim D, Seo Y, Yoon Y, Dembele V, Yoon J W, Lee K J, Magnusson R, Robust snapshot interferometric spectropolarimetry, *Opt Lett*, 41(2016)2318–2321.
21. Stamm J, Benel J, Escoto E, Günter G, Dantus M, Milliradian precision ultrafast pulse control for spectral phase metrology, *Opt Express*, 29(2021)14314–14325.
22. Gbur G J, *Singular Optics*, (CRC Press, Boca Raton, FL), 2017.
23. Senthikumar P, *Singularities in Physics and Engineering-Properties, methods, and applications*, (IOP Publishing Ltd.), 2018.
24. Gbur G, Visser T D, Wolf E, Anomalous behavior of spectra near phase singularities of focused waves, *Phys Rev Lett*, 88(2002)013901; doi.org/10.1103/PhysRevLett.88.013901.

25. Popescu G, Dogariu A, Spectral anomalies at wave-front dislocations, *Phys Rev Lett*, 88(2002)183902; doi.org/10.1103/PhysRevLett.88.183902.

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