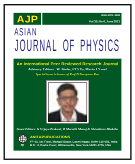

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Transverse optical current in off-axis vortex beams

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Dedicated to Professor D N Rao for his significant contributions and pioneering works in the fields of spectroscopy, optics, nonlinear optics and photonics

Optical beams nesting single or more off-axial distribution of point vortices is considered. The effective topological charge carried out by the beam is explored when the beam is nesting a fractional order vortex phase, beam containing an arbitrary distribution of point vortices as well as when two or more integer order vortex fields are superposed. The orbital angular momentum (OAM) carried out by this beam is generally a fractional number when average OAM per photon is considered. Depending on the distribution of the point vortices the beam may carry a net transverse linear momentum which dictates the nature of the OAM carried out by the beams. When more than one off-axis vortex is present within the beam, the transverse Poynting vector also exhibits saddle points which reveal intricate topological structure of such optical vortex beams.

Keywords: Optical vortex beams, Off-axis vortex beams, Optical current, Fractional vortex beams

1 Introduction

Optical beams that carry helicoidal phase fronts are popularly called as optical vortex (OV) beams. Optical vortices are composed of points of phase singularities which are formed when multiple coherent light fields are superposed [1]. The transverse optical current as given by the transverse component of the Poynting vector has a rotating nature around the dark core nesting the null intensity point gives the vortex like pattern with propagation [2]. The vortex beam can contain either a single or multiple, on-axis vortex or off-axis vortex. The effective topological charge and topological stability of the vortex beam as well as the orbital angular momentum (OAM) carried out by the beam depends explicitly on the distribution of the singular points across the beam [3,-5].

The Poynting vector distribution which is given by the intensity weighted gradient of phase of the optical field is an important measure of any structured light field [6]. This also enables one to describe the paraxial and non-paraxial light fields in a similar framework and thus can be employed to study the internal features of any light fields. Knowledge of the transverse Poynting vector is also essential for designing programmable optical tweezer for manipulation of sensitive biomolecules [7]. This study could be useful for structuring the optical gradient force distribution and manipulating the propagation characteristics of a light beam. The permanent redistribution of the energy inside the beam determines the evolution of the beam and useful in exploring the topological features of the light fields.

Off-axis vortex can be generated in multiple of ways namely off-axis illumination of cylindrically symmetric spiral phase function (SPF) [8], using cylindrically asymmetric SPF such as fractional order SPF and azimuthally non-linear SPF [9,10], superposing two or more symmetric on-axis vortex beams which are

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OAM eigen modes [4] or perturbing any non-zero order symmetric vortex modes [11]. As the OAM eigen modes are a set of orthonormal complete transverse mode, any spatial distribution of the phase singularities can be represented by their OAM spectrum [12]. The weight factors uniquely represent a particular transverse spatial mode. The effective topological charge of the resultant off-axis beams has an upper-bound and its maximum value is equal to the total number of point singularity [5].

The intricate transverse momentum structure even in paraxial scalar beams can be thought of as the superposition of multiple helical modes which give rise to the points of phase singularity. In vectorial beams the spin flow density needs to be included with the orbital flow density. In non-paraxial case things are more complex due to the spin-orbit coupling [13]. In general, any beam with inclined phase fronts carries OAM about the beam axis, which when integrated over the beam, can be an integer or non-integer multiple of $\hbar$ [14]. The internal energy across the beam profiles containing pair of fractional vortices [15] and dipolar fractional vortices [16] are studied recently. The OAM property of the optical beams has both similarities and differences with the spin angular momentum (SAM) counterpart which is associated with the polarization state of the electromagnetic field.

Though the OAM is solely due to the spatial structure of the light beam, the mechanical equivalence of the OAM and SAM has been investigated and concluded through the effective angular momentum imparted to matter interacting with light [17]. It is observed that they could be added or compensated in the total imparted angular momentum and thus confirming that the total optical angular momentum is sum of OAM and SAM. The SAM is known to be intrinsic to the beam as it is independent of the beam axis. The OAM is initially assumed to be mainly dictated by the choice of axis and OAM obtained about the beam axis gives the effective OAM. But later it was confirmed that the value of OAM remains same if the direction of axis is chosen such that the net transverse linear momentum vanishes about the chosen axis [18]. Thus the OAM shows both extrinsic and intrinsic behaviour and thus it is termed as quasi-intrinsic [19]. The underlying reason behind the emergence of an axis dependent extrinsic OAM component is the presence of a net transverse linear momentum (TLM) across the beam. The aim of the present study is to give a concise review of the internal energy flow landscape of the structured light that are nested by off-axis vortex points. The topological nature of the off-axis vortex beam has some unique characteristics as compared to their circular symmetric counterpart which have been extensively studied [3,4,9]. The propagation of the optical beams with OAM are intricately connected to the transverse energy flow. In the present paper, we discuss transverse optical current (energy flow) of the OAM beams with off-axis vortex.

2 Theoretical Details

The period averaged linear momentum density p and angular momentum density L of an electromagnetic field can be written in terms of the electric field E and magnetic field H, as

$$p = \epsilon_0 \langle E \times H \rangle = \epsilon_0 \, Re[E^* \times H]$$
$$L = r \times p = \epsilon_0 \, (r \times Re[E^* \times H]) \tag{1}$$

respectively. Here ϵ_0 is the free space permittivity, * represents the complex conjugate and $Re[..]$ implies the real part of the complex quantity inside the parenthesis. In Eq (1), both SAM and OAM densities are included. The linear momentum density takes the following form when paraxial approximation is considered [6,20],

$$p = \frac{i\omega\epsilon_0}{2} (u^* \nabla u - u \nabla u^*) + \omega k \epsilon_0 \, |u|^2 \, \hat{z} + \omega\sigma \, \frac{\epsilon_0}{2} \left(\frac{\partial |u|^2}{\partial r} \right) \hat{\varphi} \tag{2}$$

where u is the complex function representing the field and ω, k, σ are the angular frequency, propagation constant and polarization parameter ($\sigma = \pm 1$ implies the left and right circularly polarized field), respectively. In view of the cylindrical symmetry of the propagating optical field, the momentum is expressed in cylindrical coordinate system ($\hat{r}$, $\hat{\varphi}$, $\hat{z}$) and in the above Eq (2) the middle and the last terms indicate the longitudinal component of momentum and the SAM generating transverse component, respectively. The first term of Eq (2) is the transverse linear momentum responsible for the OAM in scalar paraxial electromagnetic field.

3 Experimental Details

The incoming coherent and spatially filtered light field from a He-Ne laser is linearly polarized and made to incident on the spatial light modulator (SLM). The SLM displays a typical SPF with a definite topological charge or a blazed computer generated hologram (CGH) for the same phase profile. The first order diffracted beam from the CGH results in the envisaged beam.

The resultant beam is then interfered with the expanded and collimated reference Gaussian beam with a non-collinear overlap with ensuring a resolvable straight fringes with fringe width of at least 3 pixels but fine enough to sample the object field in both horizontal and vertical directions in accordance with Nyquist theorem [21]. The fringes are the analysed using the Fourier fringe analysis technique [22]. The retrieved complex field has both the amplitude and phase information of the object beam.

4 Results and discussions

Transverse intensity and phase profile

The reconstructed intensity and phase profiles of the object beams as obtained from the Fourier fringe analysis of the recorded interferograms are shown in Fig 2. With increase of order from 0.0 in steps of 0.1 in the dislocation hologram one partial edge dislocation starts to form along the line of phase step discontinuity. As a result, the circular symmetry of the dislocation and consequently that of the generated beam is broken. But after half-integer order (i.e. 0.5) the edge dislocation transforms towards a new screw dislocation which has TC = 1.

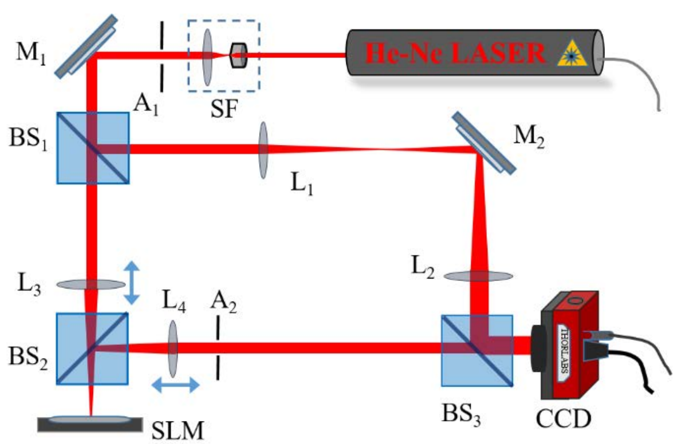

Fig 1. Schematic Experimental setup for generation and interferometric analysis of off-axis vortex beam. SF: spatial filter, BS: beam splitter, SLM: spatial light modulator, A: aperture, L: lens, M: mirror, CCD: charge coupled device camera.

It is observed that, the phase profiles of the optical beams with TC between 0.0 and 0.5 are deformed resulting the asymmetric intensity patterns (not shown in the figures). However, no phase singularity point is observed for these orders within the beam cross-section. At TC = 0.5, the phase singular point is observed in the cross-section of the beam and it moves towards the beam centre with the increase of TC in fractional steps (Fig 2). When the TC=1, the phase singularity reaches the centre and the intensity becomes circularly symmetric.

It has been pointed out that the dynamics of vortex motion depends on the initial symmetry of the dislocation hologram as well as the intensity and phase gradient of the beam [23]. Here, for the finite Gaussian beam diffraction, the input beam only has radial intensity gradient on which an azimuthal phase gradient is imposed by the fork hologram. And this azimuthal phase gradient has varying magnitude for the different fractional orders of the holograms which increases with increase in order. As a vortex moves perpendicular to the field gradient [23] for changing fractional order of dislocation, only the azimuthal phase gradient is effectively changed so that the vortex shifts perpendicular to the azimuthal direction i.e. in the radial direction. The trajectory of the point singularity as the TC is varied can be calculated from the phase profiles shown in Fig 2(d).

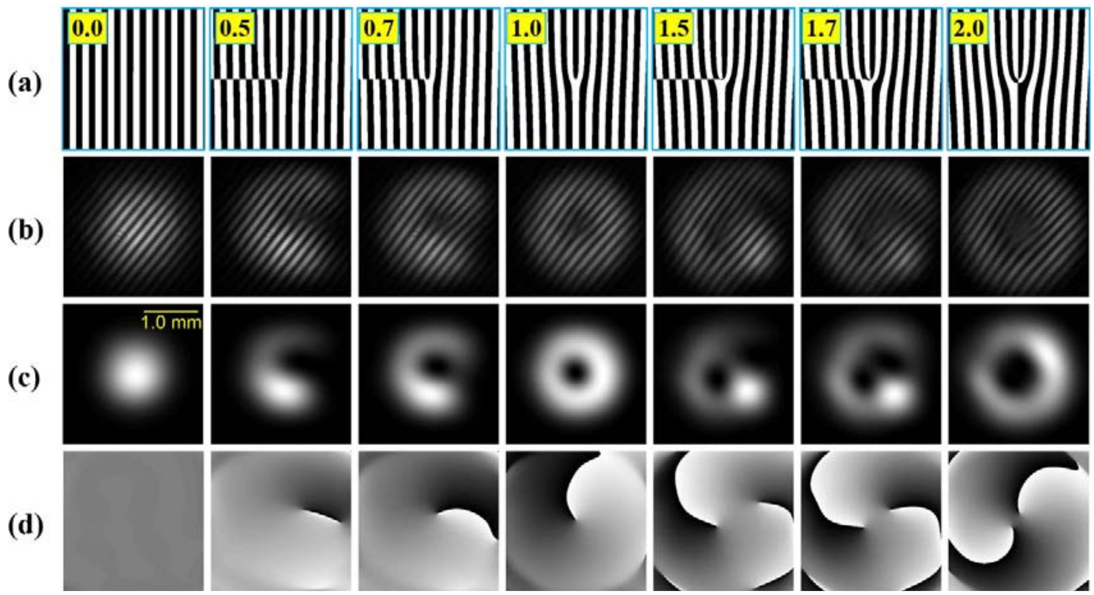

Fig 2. (a) Employed CGH of integer and fractional orders, (b) recorded interferograms, (c) reconstructed intensity and (d) reconstructed phase profile of the integer and fractional order vortex beams.

Transverse linear momentum density distribution

The linear momentum density has a complex structure with transverse (both radial and azimuthal) component in addition to the obvious longitudinal component (along propagation). The gradient of the helicoidal phase weighted by the intensity results in an intricate TLM density as is shown in Fig 3, where the first and second rows display the x and y components of the TLM. The y component of the TLM density shows a greater variation with changing order than the x component. This is due to the orientation of the dislocation along x axis which diffracts the light along y and the variation of the dislocation height changes the y component of TLM.

Transverse Poynting vector and net TLM

The main features of the internal energy flow across the beam is manifested by the vectorial distribution of Poynting vector associated with the field. In Fig 4, the transverse component of the Poynting vector is shown with the background intensity profile of the corresponding TC. It is seen from the Fig 4 that the energy flow is always curling around the phase singularity. It is obvious that the total Poynting vector has the axial component as well which is responsible for the forward energy flow resulting in propagation.

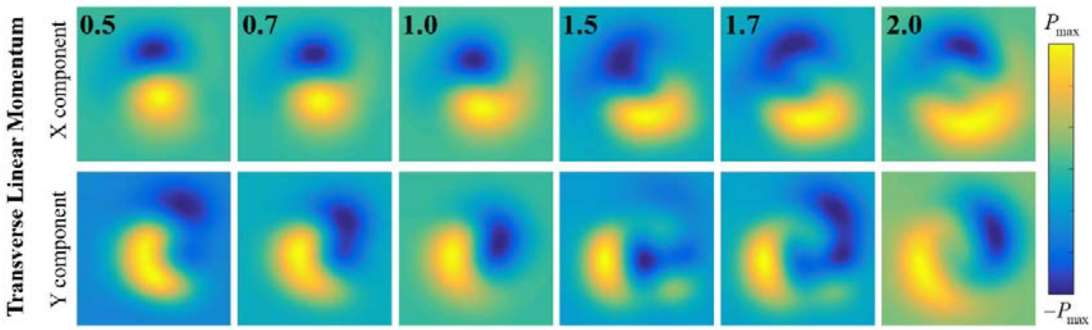

Fig 3. Transverse linear momentum of the vortex beams of different fractional orders.

The net TLM across the generated beams is also calculated after integrating the directional TLM distribution over the whole beam. The net TLM is also shown in the Fig 4 by the arrow in red colour. It is observed from the Fig 4 that the net TLM has magnitude and direction for the vortex beams with fractional TC. The net TLM is zero for the vortex beams with integer TC.

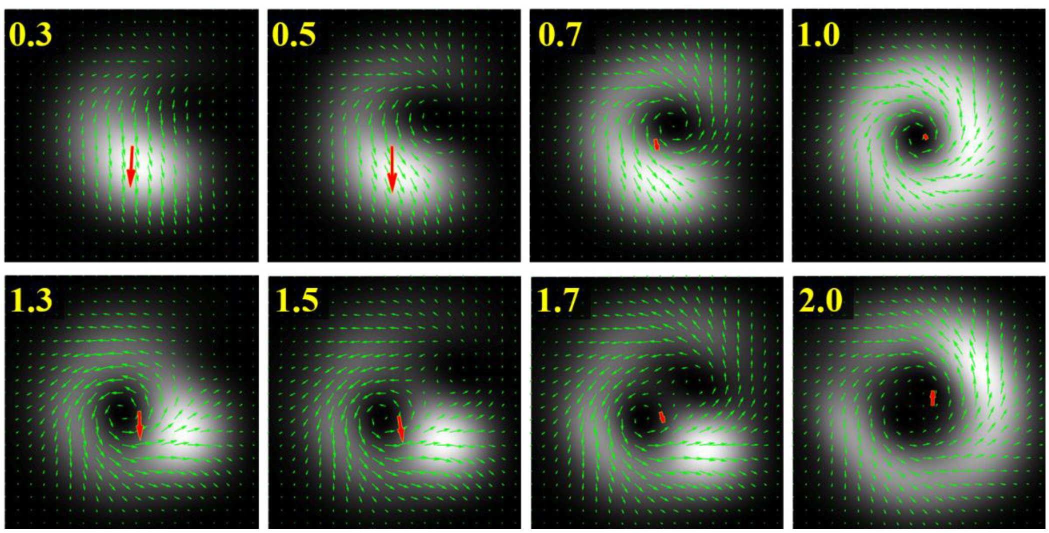

Fig 4. Poynting vector and net transverse linear momentum of the beams with the corresponding intensity profiles in the background.

The transverse energy flow of unit charge vortex beam has annular and toroidal symmetric distribution as is already concluded from the azimuthal linear momentum distribution in the previous section. The transverse component of the Poynting vector curls around the singular point if the vortex beam has unit TC. But for TC = 1.5, there occurs two point vortices across the beam profile and the Poynting vector has a complex pattern. The direction of rotation of the swirling flow lines is same around both the vortices, which confirms that both the vortices are of same magnitude and sign. As the direction of the transverse component of the Poynting vector at the singular point is undefined, the singular point is also treated as Poynting vector singularity.

Also, because of the countering action of the flow in between the two vortices, there arises another point where the flow due to two vortices exactly cancels and the transverse component of the Poynting vector has zero value in the transverse direction. This is called a saddle of the transverse Poynting vector which can also be treated as singularity but only of the transverse component of the Poynting vector. And so, this is a passive singularity in comparison to the other two singularities at the position of the vortices. This is shown in the Fig 5. In Fig 5(a) the energy flow pattern for the TC = 1.7 OV beam is shown and part of the region in between the two point vortices, as marked by the blue square has been magnified and is shown in Fig 5(b). In Fig 5(b) the saddle is marked by the dashed circle. The flow lines along one direction are clearly seen to be ending at the saddle whereas along the orthogonal direction they are diverging from the saddle. At this saddle point, the phase front has opposite curvature along any two perpendicular directions. The rate of change of phase is comparatively larger at the saddle point than any other non-singular point across the beam profile.

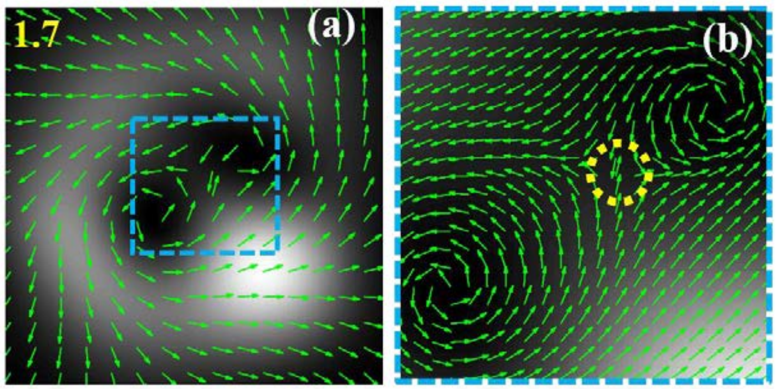

Fig 5. The presence of a saddle point in the Poynting vector distribution corresponding to the field of the OV beam of order 1.7: (a) shows the transverse energy flow where the region in between the point vortices is marked which is magnified and displayed in (b). The position of the saddle pont is marked with a yellow dotted circle in (b).

The importance of the saddle is that this gives a picture of the underlying field topology and the transformation of the topology with increase in TC. This enables us to connect the field topology of two integer order OV beams through the intermediate fractional OV field topology. The transverse energy flow pattern is also useful in the optical trapping applications using structured beams in which the momentum imparted to the trapped particle could be studied.

5 Conclusions

A mixed screw-edge dislocation hologram is prepared by numerically calculating the interference pattern of a fractional order OV beam with a plane wave. The topological evolution of the vortex beams with inter TC via intermediate fractional TCs is studied using a Gaussian beam with finite curvature and different CGHs with integer and fractional orders. A new vortex point occurs across the transverse profile of the vortex beam at the half-integer orders. With increase in fractional order, both the vortices move in the transverse plane. The initial vortex moves towards the axis up to order 1.0 and again moves away from the axis up to fractional order 1.5, afterwards it moves towards axis and thus cylindrical symmetry is restored at integer orders. Whereas the new vortex evolved from the edge dislocation starts from edge of the beam and always moves towards the axis. The linear momentum density and the Poynting vector distribution in the transverse plane are studied for these beams that enable us to visualize the internal energy flow associated with the intricate optical field of the fractional order vortex beams. Occurrence of saddle is also observed in the vectorial Poynting vector distribution. The presence of a net transverse linear momentum for any fractional order is linked to the partial extrinsic OAM component in the fractional OV beams. The magnitude of the net TLM depends on the fractional order and its direction depends on the Gouy phase acquired by the fractional OV beams.

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