



## Estimation of the Brillouin scattering threshold in few mode fibers

M Santagiustina<sup>1</sup> and I Abidi<sup>2</sup>

<sup>1</sup>Università di Padova, via Gradenigo 6b, 35131 Padova, Italy

<sup>2</sup>University of Carthage, Engineering School of Communication of Tunis, Tunis, Tunisia

Dedicated to Professor Bishnu P Pal for his enormous contributions to the advancement of research and education in science and technology through his unique vision and outstanding dedication

An accurate derivation of the power threshold of spontaneous Brillouin scattering in few mode fibers is presented. The key role played by the acousto-optic interactions in determining the threshold is demonstrated. © Anita Publications. All rights reserved.

**Keywords:** Few mode fibers, Brillouin scattering, Spatial division multiplexing.

### 1 Introduction

As single-mode fibers (SMFs) are rapidly approaching their fundamental capacity limit [1], spatial division multiplexing (SDM) [2] has been proposed as a solution for overcoming SMFs limitation. With multimode fibers mode coupling is a feature to consider in system design and two main strategies can be conceived: multiple-input-multiple-output processing when mode coupling is strong, or independent parallel mode selection if mode coupling is weak [3]. Few mode fibers (FMFs) have been manufactured [4] with large differences between mode effective indices to decouple the modes, and have been applied in both core [3] and access networks [5]. FMFs can be also considered for SDM high-power fiber amplifiers [6] including Raman ones [7,8]. In fiber access networks and fiber amplifiers, continuous waves (CWs) are launched at the fiber input (respectively to act as upstream data carriers and pump waves) and the spontaneous Brillouin scattering (SpBS) becomes a highly impairing process, causing wave depletion and unwanted reflections. SpBS is characterized by a power threshold which specifies the maximum input power such that SpBS signal remains below a prefixed value [9,10]. In [11] the SpBS threshold of the first four families of LP modes was evaluated based on the simplified formulation obtained in [9] which was valid only for the intra-mode scattering, i.e. when all optical waves are on the same mode, but it did not account for Brillouin scattering occurring between waves on different modes. However, as shown in [10], the SpBS threshold determination actually requires the accurate calculation of the Brillouin gain spectra, that highly depends on the interaction between optical and acoustic modes of the fiber structure [10]. In fact, the Brillouin gain spectra measured for both step- [12] and graded-index [13] FMFs, show the importance of the acousto-optic interaction, as scattering can occur also between different optical modes (inter-mode scattering). Finally, SpBS thresholds in FMFs have been also evaluated through numerical integrations of the equations governing the Brillouin interaction in short (8m) FMFs, so neglecting losses [14] however, this is a fundamental parameter that cannot be neglected in long fibers, because the loss coefficient also determines the SpBS threshold, as shown [10]. Here, by extending the SMF approach [10] to FMFs, accounting for the acousto-optic interaction, we estimate the intra- and inter-mode SpBS thresholds for the modes of a lossy FMF. This case is expected to be of practical interest when long fibers are used and losses cannot be neglected. This is the case of access

Corresponding author

e mail: [marco.santagiustina@unipd.it](mailto:marco.santagiustina@unipd.it) (Marco Santagiustina)

networks, in particular the fronthaul network segment that appears in centralized radio access network (C-RAN) architecture; in such systems unwanted reflections, like those due to SpBS, are the main source of signal distortions [15]. As well this effect must be accounted in Raman amplified long-haul systems, to avoid pump depletion [7,8].

## 2 Theoretical calculation of spontaneous Brillouin scattering

We assume to launch a CW at frequency  $\nu_p$  in one of the modes of the FMF from the fiber input ( $z = 0$ ) and that mode coupling is small enough to be neglected [4]. The SpBS threshold power  $P_{th}$  is defined by the condition  $P_{Stokes}(z = 0) = \mu P_{th}$ . We select  $\mu = 0.01$  [10], so that the undepleted pump approximation is well satisfied and the pump power in mode  $p$  simply decays exponentially along the fiber:  $P_p(z) = P_0 \exp(-\alpha z)$ . The loss coefficient is equal for all modes [4] ( $\alpha_{dB} = 0.2$  dB/km). Let us remark, however, that large mode dependent losses can affect the SpBS thresholds.

If the previous assumptions are satisfied, the calculation of the SpBS power that was carried out for a SMF [9] can be straightforwardly extended to FMFs; in fact, mode orthogonality, as shown in [9] enables to obtain the total spontaneous Stokes power as a sum of spontaneous scattering powers over all propagating modes. Therefore:

$$P_{Stokes}(z = 0) = KT \sum_s \int_{-\infty}^{+\infty} \nu G_{p,s}(\nu) d\nu \quad (1)$$

where  $K$  is the Boltzmann constant,  $T$  is the temperature, the summation is made over all propagating modes. In this study, we consider the first four families, i.e.  $s = [LP_{01}, LP_{11}, LP_{21}, LP_{02}]$ , and the functions  $G_{p,s}$  are then given by:

$$G_{p,s}(\nu) = \sum_{m=1}^{M_{p,s}} \nu_{p,s,m}^{-1} \{ \exp[L_{(p,s,m)}(\nu)(1-F)] [L_{p,s,m}^{-1}(\nu) + F] - 1 - L_{p,s,m}^{-1}(\nu) \} \quad (2)$$

where,  $F = \exp(-\alpha L)$  is the total fiber loss of the fiber of length  $L$ . The summation in Eq (2) is made over all the acoustic modes  $M_{p,s}$  that enable an effective interaction between a pump wave in the optical mode  $p$  and a Stokes wave in optical mode  $s$ . Each interaction term is modelled through Lorentzian functions:

$$L_{p,s,m}(\nu) = \frac{g_{p,s,m} P_0}{A_{p,s,m}^{aco} \alpha} \cdot \frac{w_{p,s,m}^2}{w_{p,s,m}^2 + 4(\nu - \nu_p + \nu_{p,s,m})^2} \quad (3)$$

The parameters needed to determine the Lorentzian functions are defined as follows. The peak Brillouin gain coefficients are defined by:

$$g_{p,s,m} = \frac{\pi n^6 p_{12}^2 (n_p + n_s)^2}{c \varepsilon_0 \lambda_p^3 + \nu_{p,s,m} + w_{p,s,m}} \quad (4)$$

where  $n$  is the core refractive index,  $n_p$  and  $n_s$  are the effective indices of the pump and Stokes modes at wavelengths  $\lambda_p$  and  $\lambda_s$ ,  $c$  is the speed of light,  $\varepsilon_0$  is the vacuum permittivity and  $p_{12}$  is a component of the electrostriction tensor. The acousto-optics area (AOA) is the fundamental parameter that defines the strength of the Brillouin scattering interaction [10]. In FMFs, it may enable the interaction of optical and acoustic waves among various combinations of modes and it can be defined by:

$$A_{p,s,m}^{aco} = \frac{\langle |f_p|^2 \rangle \langle |f_s|^2 \rangle \langle |\xi_m|^2 \rangle}{|\langle f_p f_s^* \xi_m^* \rangle|^2} \quad (5)$$

where,  $\langle \dots \rangle$  stands for integration on the transverse plane,  $f_p, f_s, \xi_m$  are the modal distributions respectively of the pump, the Stokes waves and of the  $m^{th}$  acoustic wave. The full-width at half maximum (FWHM) linewidth of each resonance is:

$$w_{p,s,m} = 2\pi\Gamma \left( \frac{n_p}{\lambda_p} + \frac{n_s}{\lambda_s} \right)^2 \quad (6)$$

where  $\Gamma$  is the acoustic wave damping factor. Finally, the resonant frequencies (Brillouin frequency shifts) are given by:

$$v_{p,s,m} = V_{p,s,m}^{ac} \left( \frac{n_p}{\lambda_p} + \frac{n_s}{\lambda_s} \right) \quad (7)$$

where,  $V_{p,s,m}^{ac}$  is the velocity of  $m^{\text{th}}$  longitudinal acoustic mode interacting with the optical modes  $p$  and  $s$ .

The FMF considered here as an example for the calculations is a Germanium doped core, pure silica-cladding, step-index fiber, with a core radius  $r = 5 \mu\text{m}$ , a core-cladding refractive index difference  $\Delta = 1.48 \%$  and an optical number  $V = 5.1$ . Refractive indices were calculated through Sellmeier formulas. Through a finite element method, the optical and acoustic modes were determined and the parameters of Eqs (4-7) were all calculated. This FMF guides four families of optical modes at  $\lambda = 1550 \text{ nm}$  and the computed values of the effective indices at that wavelength for the modes are  $n_{\text{eff}}^{01} = 1.4624$ ,  $n_{\text{eff}}^{11} = 1.4573$ ,  $n_{\text{eff}}^{21} = 1.4509$ , and  $n_{\text{eff}}^{02} = 1.4491$  (respectively, for  $LP_{01}$ ,  $LP_{11}$ ,  $LP_{21}$ ,  $LP_{02}$ ). The differences between effective indices of the modes are larger than those reported in ref [4] ( $1.3 \cdot 10^{-3}$ ), where a distributed mode coupling of 18.2 dB was measured for  $L = 500 \text{ m}$ . This makes valid the assumption of negligible mode coupling among modes. The computation yields very similar values of the gain coefficients, the frequency shifts and the FWHM linewidths for the interaction with all acoustic modes :  $g_{p,s,m} \simeq 1.96 \cdot 10^{-1} \text{ mW}^{-1}$ ,  $v_{p,s,m} \simeq 10.2 \text{ GHz}$ ,  $W_{p,s,m} \simeq 40 \text{ MHz}$ . The only parameters which highly depend on the specific interaction are the AOAs [6], as shown in Table 1.

**Table 1.** Calculated AOAs (in  $\mu\text{m}^2$ ) for all relevant interactions. Multiple values for the same couple of optical modes refer to different acoustic modes. The missing elements of the table are simply given by the symmetry relation  $A_{p,s,m}^{ao} = A_{s,p,m}^{ao}$

	$LP_{01}$	$LP_{11}$	$LP_{21}$	$LP_{02}$
$LP_{01}$	55.6			
$LP_{11}$	84.6	109.7		
		164.3		
		272.1		
$LP_{21}$	263.2	419.2	204.8	
	225.9	434.5	188.1	
		196.7	185.0	
		235.1	1663.4	
$LP_{02}$	344.2	4714.0	1176.6	111.7
	96.9	161.9	1011.0	630.1
			277.5	153.6

In Fig 1 an example of the computed gain spectra for all the relevant resonances of intramodal scattering for mode  $LP_{11}$  is shown.

As it can be observed in Table 1, there is a large difference among the values of the AOAs; this will imply, as it will better explained below, that there exists a large difference among the threshold powers of the various interactions. So, as typical of spontaneous phenomena, it might be expected that the generated Stokes power of the lowest threshold resonance will prevail. This remark is confirmed by calculating all the terms deriving from all relevant intra- and intermodal interactions that contribute to the Stokes power given by Eq (1). An example, calculated for mode  $LP_{11}$ , is presented in Fig 2. It is clear that the SpBS threshold in this case will be solely determined by the contribution of the inter-modal scattering between the pump

mode  $LP_{11}$ , and the Stokes mode  $LP_{01}$ , because when this contribution is at threshold ( $P_0 \simeq 7.5$  dBm) all other terms are at least one order of magnitude smaller. Let's remark again that this behavior is essentially dictated by the large difference between the AOAs.

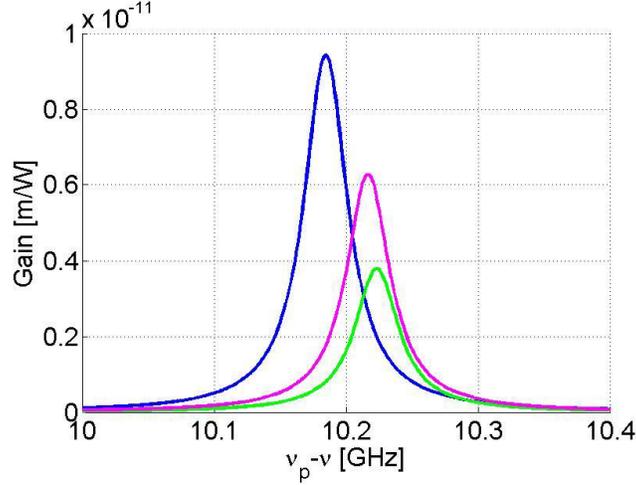


Fig 1. Intramodal Brillouin gain resonances for mode  $LP_{11}$ .

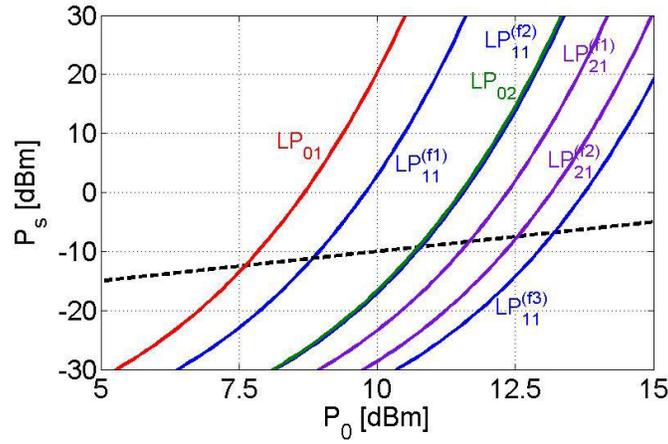


Fig 2. Stokes power at fiber input of all relevant terms of Brillouin interaction as a function of the power injected in mode  $LP_{11}$ . The dashed line is the threshold condition  $\mu P_0$ .

To calculate the SpBS threshold for each pumping mode, the term with smallest AOA must be selected from the column of Table 1. In fact, in these conditions, the threshold evaluation can be realized by using the same approach of SMF [10], that finally yields the following approximated solution for the selected interaction term  $\{p, s, m\}$  for a fully polarization scrambled pump:

$$P_{p,s,m}^{th} = \frac{A_{p,s,m}^{aco}}{g_{p,s,m}(1-F)} \frac{1 + 1.5 \ln \psi}{\psi - 1.5} \psi \quad (8)$$

where

$$\psi = -\ln \left[ \frac{\sqrt{\pi} KTF(1-F)}{\mu \alpha} \frac{(v_p w_{p,s,m}) g_{p,s,m}}{v_{p,s,m} A_{p,s,m}^{aco}} \right] \quad (9)$$

For the FMF under study the thresholds are presented in Fig 3 as a function of the fiber length  $L$ . Pumping in modes  $LP_{01}$ ,  $LP_{11}$ , and  $LP_{02}$  causes the spontaneous generation of a Stokes wave in mode  $LP_{01}$ , in fact, the interaction presenting the smallest AOA value is in all such cases that with mode  $LP_{01}$ . Therefore, the mode  $LP_{01}$  present the smallest threshold. Pumping in mode  $LP_{21}$  presents the highest threshold and moreover, the Stokes wave grows in the same mode. This might be caused by the particular mode profile of  $LP_{21}$ .

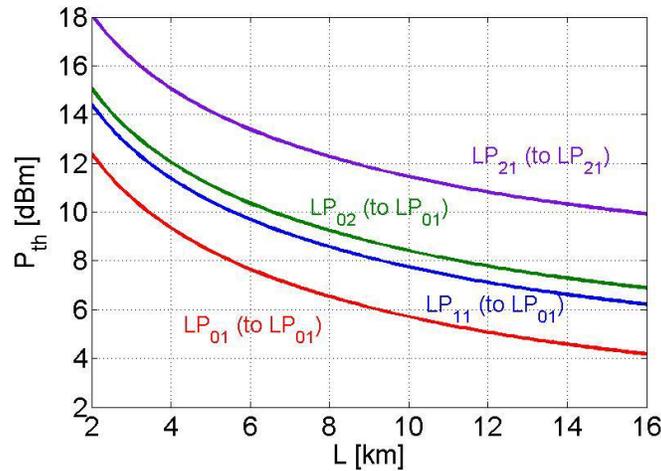


Fig 3. Estimated SpBS thresholds in different modes as a function of the fiber length  $L$ .

#### 4 Conclusion

An accurate estimation of the power threshold of spontaneous Brillouin scattering in few mode fibers is presented. Spontaneous Brillouin scattering can impact the design of access networks and fiber amplifiers based on spatial multiplexing. The key role played by the acousto-optic effective area in determining the threshold is remarked.

#### Acknowledgements

This research was supported by the Italian Ministry of University (project FIRST, PRIN 2017HP5KH7), by the University of Padova (project Optical spatial division multiplexing in 5G front-hauling, BIRD 2020). I Abidi acknowledges the support of University of Carthage.

#### References

1. Essiambre R J, Tkach R W, Capacity trends and limits of optical communication networks, *Proc IEEE*, 100(2012)1035–1055.
2. Richardson D J, Fini J, Nelson L, Space-division multiplexing in optical fibres, *Nat Phot*, 7(2013)354–362.
3. Yaman F, Bai N, Zhu B, Wang T, Li G, Long distance transmission in few-mode fibers, *Opt Express*, 18(2010)13250–13257.
4. Gruner-Nielsen L, Sun Y, Nicholson J W, Jakobsen D, Jespersen K G, Lingle R, Palsdottir B, Few Mode Transmission Fiber With Low DGD, Low Mode Coupling, and Low Loss, *J Light Technol*, 30(2012)3693–3698.
5. Xia C, Cen Xia, Chand N, Velázquez-Benítez A M, Yang Z, Liu X, Antonio-Lopez J E, Wen H, Zhu B, Zhao N, Effenberger F, Amezcuá-Correa R, Li G, Time-division-multiplexed few-mode passive optical network, *Opt Express*, 23(2015)1151–1158.
6. Mermelstein M D, Ramachandran S, Fini J M, Ghalmi S, SBS gain efficiency measurements and modelling in a 1714  $\mu\text{m}^2$  effective area  $LP_{08}$  higher-order mode optical fiber, *Opt Express*, 14(2007)15952–15963.

7. Esmaeelpour M, Ryf R, Fontaine N K, Chen H, Gnauck A H, Essiambre R-J, Toulouse J, Sun Y, Lingle R, Transmission over 1050-km few-mode fiber based on bidirectional distributed Raman amplification, *J Light Technol*, 34(2016)1864–1871.
8. Marcon G, Galtarossa A, Palmieri L, Santagiustina M, Model-aware Deep Learning Method for Raman Amplification in Few-Mode Fibers, *J Light Tech*, 39(2021)1371–1380.
9. Smith R G, Optical power handling capacity of low loss optical fibers as determined by stimulated Raman and Brillouin scattering, *Appl Opt*, 11(1972)2489–2494.
10. Kobayakov A, Sauer M, Chowdhury D, Stimulated Brillouin scattering in optical fibers, *Adv Opt Photon*, 2(2010)1–59.
11. Chen W, Hu G, Liu F, Wang F, Song C, Li X, Yong Y, Threshold for stimulated Brillouin scattering in few-mode fibers, *App Opt*, 58(2019)4105–4110.
12. Song K Y, Kim Y H, Characterization of stimulated Brillouin scattering in a few-mode fiber, *Opt Lett*, 38(2013)4841–4844.
13. Minardo A, Bernini R, Zeni L, Experimental and numerical study on stimulated Brillouin scattering in a graded-index multimode fiber, *Opt Exp*, 22(2014)17480–17489.
14. Wei W, Wang X, Tang X, Stimulated Brillouin Scattering Model in Multi-Mode Fiber Lasers, *J Sel Top Quant El*, 20(2014)0901610; doi. 10.1109/JSTQE.2014.2303256.
15. Pizzinat A, Chanclou P, Saliou F, Diallo T, Things You Should Know About Fronthaul, *J Light Tech*, 33(2015)1077–1083.

[Received: 20.02.2022; revised recd: 19.03.2022; accepted: 07.03.2022]



Marco Santagiustina (Ph D) is full professor in the Department of Information Engineering of the University of Padova (Italy); he is co-author of more than 200 papers published in international scientific journals and conference proceedings. His main research interests are in the field of fiber optics, guided photonics and nonlinear optics. He has been principal investigator and coordinator of various national and international funded research projects.



Imen Abidi received the Ph D degree in 2022 from the Higher School of Communication of Tunis (Sup'COM), University of Carthage, Tunisia. Since 2016, she has been working in the Laboratory research of Innovation of communication and cooperative mobiles (Innov'Com), Tunisia. Her research interests include non-orthogonal multiple access techniques, sparse code multiple access, polar codes, High Power Amplifier nonlinearity, fiber optics and machine learning.