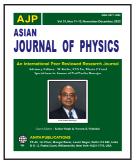
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Diffraction in a volume hologram made simple

Ching-Cherng Sun[1,2], Chih-Yuan Cheng[1] and Yeh-Wei Yu[1]

[1]*Department of Optical and Photonics, National Central University, Chung-Li, 32054 Taiwan*

[2]*Department of Electrophysics, National Yang Ming Chiao Tung University, Hsin-Chu, 30010 Taiwan*

Dedicated to Professor Partha Banerjee for his enormous contributions to the advancement of research
and education in holography through his unique vision and outstanding dedication

A volume hologram is different from a plane hologram owing to the non-negligible thickness along the direction of the diffracted light. The diffraction calculation from a volume hologram becomes complicated when the incident light is not a plane wave. In this paper, we introduce a calculation model, the VOHIL model, and make it simple to figure out the mechanism.

Keywords: Volume holography, Thick hologram, Diffraction efficiency, Bragg selectivity.

1 Introduction

Volume holography is a special class of holography when the thickness of the holographic recording medium is not negligible. The thickness makes the diffraction more complicated owing to the diffraction from different depths of the hologram [1-7]. The result is that the Fourier transform of the interference pattern fails to describe the far-field diffraction, which is the general way to calculate the diffraction field of a plane (thin) hologram. A hologram is used to serve as an optical component called a holographic optical element (HOE) [8-12], and a computer-aided hologram is called a diffractive optical element (DOE) [13-18]. Both HOE and DOE are usually called as thin holograms. One characteristic of a thin hologram is that there could be multiple diffractions, including 0^{th}, $\pm 1^{st}$, $\pm 2^{nd}$, $\pm 3^{rd}$,…..where the 0^{th}-order diffraction is the direct transmission of the incident light. Except for the 1st -order diffracted light, all others are the higher-order diffractions, and carry energy away from the 1st-order diffraction. This effect will degrade the diffraction efficiency of the 1st-order diffraction, which is always the most negative effect. Various schemes try to solve this problem and squeeze energy from the higher-order diffractions to the 1st-order diffraction. But its price is high, and sometimes it does not work well.

In contrast to the thin hologram, a volume hologram is equipped with non-negligible thickness. The diffracted light comes from all the recording volumes, so diffraction is an effect of interference from every scattering point across the whole volume of the hologram. The best way to calculate the true diffraction efficiency is to apply coupled mode equations by Kogelnik [19,20]. However, the coupling can be handled between two incident plane waves. The wavefront deviating from a plane wave is difficult to deal with using Kogelnik's approach. An alternative calculation model based on optical interference in a linear and invariant system was built in 1998 and was named the VOHIL model, which means that the volume hologram is an integrator of the lights emitted from the elementary light sources across the whole volume [21]. Based on the

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Corresponding author
e mail: ccsun@dop.ncu.edu.tw (Ching-Cherng Sun)

model, it is easy to calculate the Bragg selectivity of a volume hologram [22-24]. However, the diffraction inside the holographic volume differs from that in free space, so the true diffraction efficiency is not obtained using the VOHIL model. In fact, the simulation result of the VOHIL model is similar to that of Born's first approximation [25,26], but the calculation through the VOHIL model could be somewhat more efficient.

This paper is dedicated to honoring Prof Partha Banerjee because of his outstanding contributions to the holography and international optics community. The author (Sun C-C) collaborated with Prof Banerjee to launch a special issue of Optical Engineering, named '*Volume Holographic Optical Element*' in 2004, published by the SPIE [27]. Currently, VHOE plays an important role in the light coupling in AR/MR near-eye glasses [28-30]. The calculation model we introduce in this paper, is useful for simulating the optical performance of a VHOE.

2 VOHIL Model

The VOHIL model is a simple way to find effective diffraction of light by integrating the contributions of distributed point sources. The principle is based on the optical interference in free space, so it cannot determine the true diffraction efficiency during the propagation in the holographic volume. However, the VOHIL model is very useful for simulating/calculating the Bragg selectivity [22-24]. Figure 1 shows the geometry describing the optical mechanism of the VOHIL model [21].

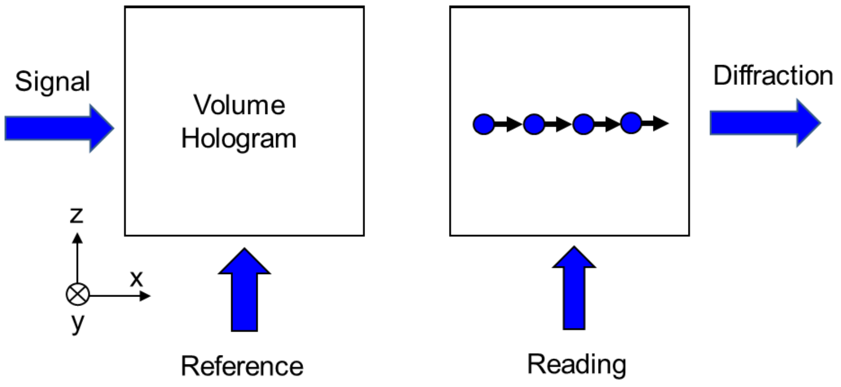

Fig 1. Schematic diagram of the mechanism of the VOHIL model. Left: the incidence, right: the reading and diffraction [21].

If there are two waves incident on a volume holographic recording medium, the phase difference at each location between the reference (A_1) and the signal (A_2) can be written [21] as

$$\Delta\varphi(x, y, z) = \varphi_2(x, y, z) - \varphi_1(x, y, z) \tag{1}$$

where φ_1 and φ_2 are respectively the phases of the reference and the signal. When a reading light is incident on the volume hologram, the scattering light at each elementary point source will bear an initial phase [21]

$$\varphi_i(x, y, z) = \varphi_r(x, y, z) - \Delta\varphi(x, y, z) \tag{2}$$

where φ_r is the phase of the reading light. Equation (2) determines the initial phase of each elementary point source which emits a spherical wave, and then the interference pattern outside the volume hologram will determine the effective diffraction. Thus, we can write the diffraction as

$$A_d(x_d, y_d, z_d) \propto \iiint e^{\varphi_r(x, y, z) - \Delta\varphi(x, y, z)}\, e^{ik\left(\sqrt{(x-x_d)^2 + (y-y_d)^2 + (z-z_d)^2}\right)}\, dx\, dy\, dz \tag{3}$$

where the range of the three-dimensional integral is limited by the volume of the hologram. When the diffraction is of a plane wave along a certain direction, e.g., the x-axis, Eq (3) can be further simplified [21] as,

$$A_d \propto \int_{-d/2}^{d/2} e^{\varphi_r(x,y,z) - \Delta\varphi(x,y,z)}\, e^{ik(d/2 - x)}\, dx \tag{4}$$

where the thickness of the volume hologram along the x-axis is d. In the case that the reading light is the same as that of the reference, Eq (4) can be rewritten [21] as,

$$A_d \propto \int_{-d/2}^{d/2} e^{\varphi_2(x,y,z)}\, e^{ik(d/2 - x)}\, dx \propto A_2. \tag{5}$$

Equation (5) shows the reconstruction of the signal, which is through the diffraction of a volume hologram [21].

The VOHIL model is especially useful in figuring out the Bragg selectivity. For example, assume that the reference light is a spherical wave 90° deviated from the signal [31-33], and the distance between the point source and volume hologram is z_o, as shown in Fig 2 [32,33]. In the reading condition, if the point source of the spherical wave is laterally shifted with an amount Δx, Eq (4) can be rewritten [21] as,

$$A_d \propto \int_{-d/2}^{d/2} e^{\varphi_2(x,y,z) + \Delta\alpha(x,z)}\, e^{ik(d/2 - x)}\, dx, \tag{6}$$

where $\Delta\alpha$ is the phase deviation as given by Eq (7)

$$\Delta\alpha(x,z) = k\{[(x+\Delta x)^2 + z_o^2]^{1/2} - [x^2 + z_o^2]^{1/2}\}. \tag{7}$$

Under the paraxial condition, Eq (7) can be expressed as,

$$\Delta\alpha(x,z) = k\{[(x+\Delta x)^2 + z_o^2]^{1/2} - [x^2 + z_o^2]^{1/2}\} \approx k\frac{x\Delta x}{z_o}. \tag{8}$$

As shown in Fig 2, the difference in the phase deviation at the two end points of the volume hologram is $kd\Delta x/z_o$. When $kd\Delta x/z_o$ reaches 2π, the integral in Eq (6) will approach to zero, which is the Bragg selectivity in horizontal shifting. This results in $\Delta x_s = \lambda z_o/d$ [32,34]. For example, if the wavelength λ is 0.5 μm, z_o = 5 cm, d = 2 mm, Bragg selectivity in horizontal shifting is 12.5 μm, or equivalently angular Bragg selectivity of 0.143°.

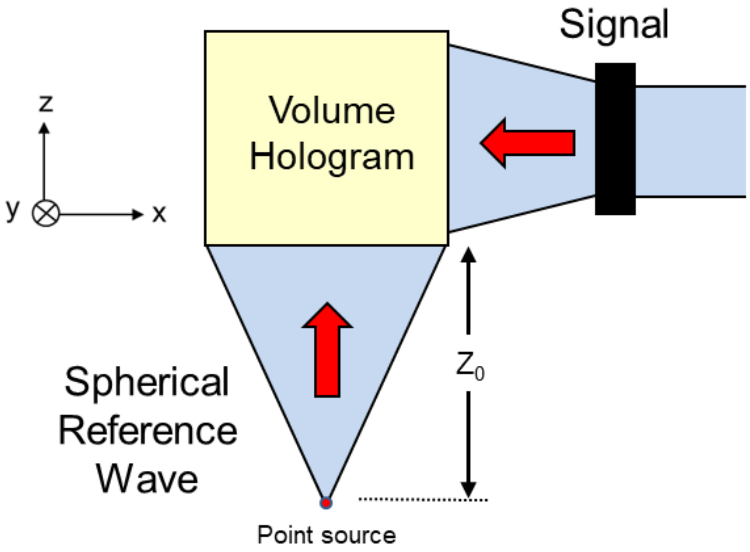

Fig 2. The volume hologram in a 90° geometry [32,33].

The calculation above is in the simplified case when the diffraction is along the x axis, and the reference and reading lights are identical. Thus, the analysis of the VOHIL model is one-dimensional. When the more complicated case occurs, we must go back to Eq (3) to calculate the Bragg selectivity in a three-dimensional version.

3 Application of VOHIL model for random phase encoding

In this section, we will introduce a successful case by applying the VOHIL model, which is to figure out the shifting Bragg condition of a ground glass for volume holographic multiplexing [34-42]. Figure 3 shows the geometry of the incidence [36].

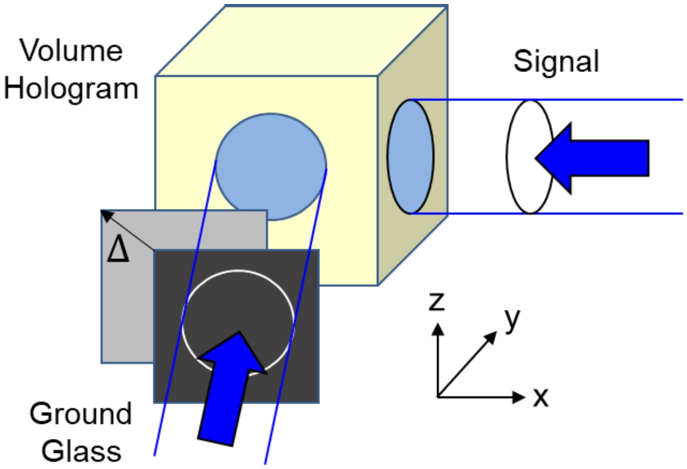

Fig 3. The scheme of incidence on a volume hologram when the reference light passes through a ground glass [36].

A ground glass is used to encode the incoming light with random phases and is then incident on a photorefractive crystal. The incident light with random phase encoding is the signal. The ground glass is regarded as composed of various point sources with the uncorrelated initial phase to each other, as shown in Fig 4 [34,36]. The signal is incident at 90° with respect to the reference. The VOHIL model was used to solve the problem of shifting selectivity of the ground glass. The diffraction of the signal when the ground glass is shifted laterally can be expressed [34] as,

$$|A_x|^2 \propto \mathrm{sinc}^2\left(\frac{\Delta x d}{\lambda z_o}\right)\mathrm{sinc}^2\left(\frac{\Delta x w_x}{\lambda z_o}\right), \tag{9}$$

where d is the thickness of the recording medium along the x-axis, and w_x is the ground glass width along the x axis. Equation (9) shows that the lateral shifting selectivity is a product of two factors of the ground glass [34] as,

$$\Delta x_{s1} = \frac{\lambda z_o}{d} = \frac{\lambda}{NA_d}, \tag{10}$$

$$\Delta x_{s2} = \frac{\lambda z_o}{w_x} = \frac{\lambda}{NA_{w_x}}, \tag{11}$$

where $NA_d = d/z_o$ and $NA_{w_x} = w_x/z_o$. The diffraction of the signal when the ground glass is shifted along the vertical direction is expressed [34] as,

$$|A_y|^2 \propto \mathrm{sinc}^2\left(\frac{\Delta y w_y}{\lambda z_o}\right), \tag{12}$$

where w_y is the ground glass width along the y-axis, and vertical shifting selectivity is written [34] as,

$$\Delta y_{s2} = \frac{\lambda z_o}{w_y} = \frac{\lambda}{NA_{w_y}}, \tag{13}$$

where $NA_{w_y} = w_y/z_o$.

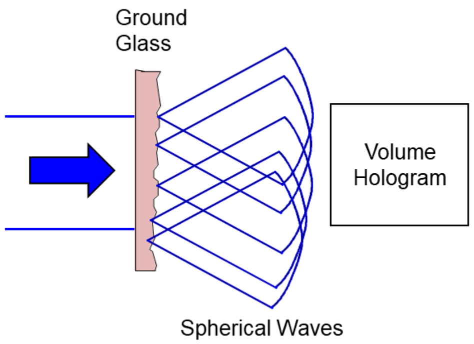

Fig 4. The schematic diagram of a ground glass serving a point-source generator with random initial phases [34,36].

The diffraction of the signal when the ground glass is shifted along the longitudinal direction is expressed [34] as,

$$A_z \propto \int_{-w_x/2}^{w_x/2}\int_{-w_y/2}^{w_y/2}\int_{-d/2}^{d/2} e^{ik\{(z_o+\Delta z)^2+(y_d-y_h)^2+(x_d-x_h)^2\}^{(1/2)}} e^{-ik\{(z_o{}^2+(y_d-y_h)^2+(x_d-x_h)^2\}^{(1/2)}}\, dx_d\, dy_d\, dx_h, \tag{14}$$

where x_d and y_d are the coordinates of the ground glass, and x_h is the coordinate of the hologram. Unfortunately, Eq (14) does not have an analytic solution as those in Eq (9) and Eq (12).

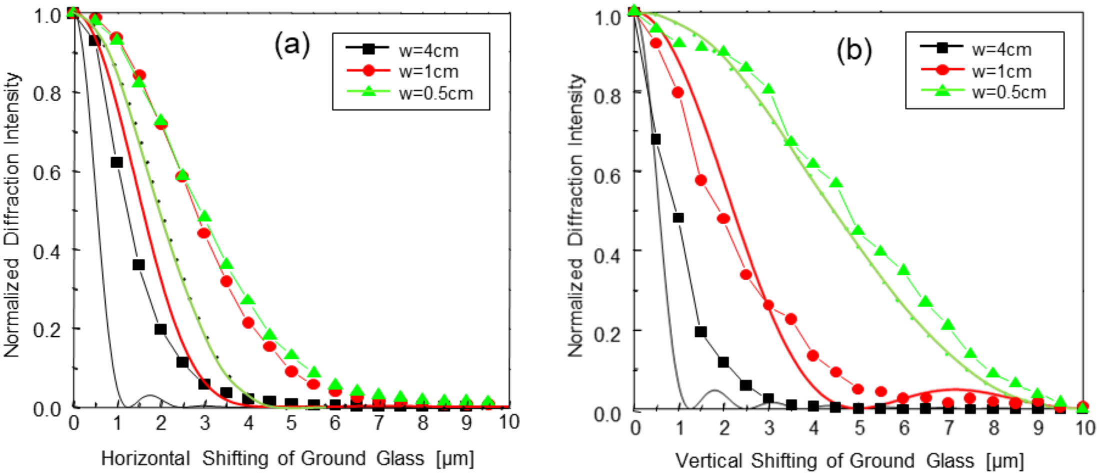

Fig 5. Comparisons between the simulation and the corresponding experimental measurements for (a) in the (a) horizontal and (b) vertical directions. The widths of the ground glass and the recording medium were all set 1 cm, and z_o 10 cm [34].

The simulation results and the corresponding experimental results are shown in Fig 5 [34], where we can find that the calculation fits well the experimental observation. The broader shifting tolerance observed in the experiment could be caused by imperfections in the incident wavefront of the signal, reference, and reading lights

4 Summary

In this paper, we first introduced the mechanism of the VOHIL model that we developed in 1998. The model is essentially to describe an interference phenomenon when a volume hologram is illuminated by a reading light. The assumption of the model is that the whole system is a linear and invariant system in free-space so that the concept of energy coupling inside the hologram volume is not applicable. This characteristic makes the model unable to calculate the true diffraction efficiency. However, it is very useful and helpful in figuring out the Bragg selectivity, which is one of the most important properties of a volume hologram. Finally, we introduced an application while using the VOHIL model in calculating the shifting selectivity of a ground glass which is a random phase generator for random phase encoding on the reference light for a volume hologram. Through the theoretical analysis based on the VOHIL model, the Bragg selectivity was successfully simulated and the corresponding experimental measurement was accurately predicted by the theoretical calculation.

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