



## Resolving power of optical instruments: A Tutorial

Rajpal Sirohi

Alabama A&M University, Huntsville AL 35802

Dedicated to Professor M S Sodha for his numerous contributions to  
Plasma Physics, Optics and Photonics, Energy Studies and Education Management

Optical instruments are used for observation and spectroscopy apart from host of many other applications. Both in observation and spectroscopy it is important to know how closely two objects, or two spectral lines can be resolved by the instruments. There had been a number of criteria of resolution: two most often used are Rayleigh criterion and Sparrow criterion. Rayleigh criterion of resolution has no mathematical support but is often used due to its simplicity. Sparrow criterion gives the lowest limit of resolution. Further in seeing, the nature of illumination of the object has profound effect on the limit of resolution. Rayleigh criterion tacitly assumes incoherent illumination, while the Sparrow criterion can be applied to incoherent, partially coherent and coherent illuminations. Applicability of Rayleigh criterion can be extended to partially coherent and incoherent illuminations by a simple modification. The paper through the tutorial approach describes the resolution of optical instruments used for seeing and spectroscopy. © Anita Publications. All rights reserved.

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### 1 Introduction

I chose this topic due to my early interest in this topic under the guidance of Professor Sodha. He himself has written many papers on the resolving power of spectroscopic instruments. That was the active period of research on this topic [1-17]. Resolution has been described in terms of either Rayleigh or Sparrow criteria of resolution. With the passage of time different criteria emerged for the assessment of the performance of optical instruments. There, however, been intermittent publications on resolution [18-21]. Most recent one is due to Cremer and Masters which is a survey of the field of research on resolution with emphasis on the resolution of microscope [22]. Ever since the path breaking research on the development of optical microscopes and theory of imaging by Abbe, the resolution of microscope has kept pace with the advances in biology and medicine. Alternately it may be argued that the advances in microscopy, and advances in biology and medicine are interlinked.

Optical instruments are used for both imaging and non-imaging applications. While discussing the resolving power of instruments, it would be assumed that the performance of the optical instruments is diffraction-limited. In principle, resolving power should depend on the performance of both the instrument and the detector. It would however, be assumed that the detector is an ideal detector capable of resolving details presented by the instrument. Therefore, the diffraction effect on the optical instruments will only be considered.

Ideally a point object should be imaged as a point. But this is not true, because only a part of a spherical wave emanating from the point source is captured by the instrument. The diffraction occurs when

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Corresponding author

e-mail: [rs\\_sirohi@yahoo.co.in](mailto:rs_sirohi@yahoo.co.in) (Rajpal Sirohi)

the wavefront is limited in transverse dimension by the mountings of the optics or the optics itself. Further, the effect of diffraction in far field is of importance, and hence only the Fraunhofer diffraction by the limiting aperture is considered. Assuming the limiting aperture to be circular, an image of a point source instead being a point turns into an irradiance distribution, which is called the Airy pattern. This comprises of a central disc surrounded by fringes of decreasing irradiance. The image of a point is thus approximated by the size of the central disc. However, the irradiance distribution as a whole plays a role in the resolving power of the instrument. If the limiting aperture is not of circular shape but is of rectangular shape, the irradiance distribution changes.

## 2 Image of a point source

Such a situation would arise when a telescope is used to view a star or binary stars. A telescope makes an image of each star, assumed to be a point. The irradiance distribution in the image of a point source formed by a diffraction-limited system is expressed as

$$I = I_0 \left[ \frac{2J_1(ka\omega)}{ka\omega} \right]^2$$

where  $k = \frac{2\pi}{\lambda}$  is the propagation parameter,  $2a$  is the diameter of the imaging system (exit pupil) and  $\omega = \sin^{-1}\left(\frac{r}{v}\right)$ ;  $r$  is the radial distance from the optical axis and  $v$  is the distance from the center of the optical system to the point where the image is formed.  $I_0$  is the irradiance at the center of the pattern. The function  $J_1(ka\omega)$  is the Bessel function of first order and first kind. This distribution was first obtained by Airy and is known as the Airy pattern. It has a maximum irradiance  $I_0$  at the center that drops to zero when  $ka\omega = 1.22\pi$ . The irradiance oscillates between zero and some finite decreasing irradiance values. In other words, the image of a point source formed by a diffraction-limited optical system consists of a bright disc, called the Airy disc, surrounded by rings of decreasing irradiances. Writing  $ka\omega = \delta$ , the irradiance distribution is expressed as

$$I = I_0 \left[ \frac{2J_1(\delta)}{\delta} \right]^2$$

The curve centered at  $\delta = 0$  in Fig 1 (a) shows the normalized irradiance distribution. The first zero corresponding to  $\delta = \delta_{\min}$  occurs for  $\delta_{\min} = 1.22\pi$ . The full width at half maximum (FWHM) of the distribution is  $\sim 1.03\pi$ . This distribution is also known as the point spread function of the system.

However, if the limiting aperture is rectangular of size  $2a \times 2b$  instead of circular aperture of diameter  $2a$ , the irradiance distribution is given by

$$I = I_0 \left( \frac{\sin\alpha}{\alpha} \right)^2 \left( \frac{\sin\beta}{\beta} \right)^2$$

where  $\alpha = kax/v$ ,  $\beta = kby/v$ , and  $(x, y)$  are the coordinates of a point in the image plane at which the irradiance is  $I$ . The first zero of irradiance distribution corresponds to  $\alpha = \beta = \pi$ .

## 3 Imaging of a pair of point sources under partially coherent illumination and with a constant background

It is assumed that the irradiances of the point sources are unequal ( $1:p$ ) and there is a background irradiance. Under such a situation, the irradiance distribution at the image plane can be expressed as

$$I = I_0 \left[ \frac{2J_1(\delta)}{\delta} \right]^2 + pI_0 \left[ \frac{2J_1(\delta - \Delta\delta)}{\delta - \Delta\delta} \right]^2 + 2\sqrt{p} I_0 \gamma \frac{2J_1(\delta)}{\delta} \frac{2J_1(\delta - \Delta\delta)}{\delta - \Delta\delta} + qI_0$$

where  $p$  and  $q$  are less than unity,  $qI_0$  is the background irradiance and  $\gamma$  is the spatial degree of coherence between the wavefields from the two-point sources. The normalized resultant irradiance distribution  $I_{\text{res}}$  is given by

$$I_{\text{res}} = \frac{I}{I_0} = \left[ \frac{2J_1(\delta)}{\delta} \right]^2 + p \left[ \frac{2J_1(\delta - \Delta\delta)}{\delta - \Delta\delta} \right]^2 + 2\sqrt{p} \gamma \frac{2J_1(\delta)}{\delta} \frac{2J_1(\delta - \Delta\delta)}{\delta - \Delta\delta} + q$$

This is a general expression. When  $\gamma = 1$ , both the wavefields are coherent and hence their amplitudes add, resulting in an irradiance distribution, which is given by

$$I_{\text{res}} = \left[ \frac{2J_1(\delta)}{\delta} + \sqrt{p} \frac{2J_1(\delta - \Delta\delta)}{\delta - \Delta\delta} \right]^2 + q$$

When  $\gamma = 0$ , both the wavefields are incoherent and hence their irradiances add, resulting in an irradiance distribution, which is given by

$$I_{\text{res}} = \left[ \frac{2J_1(\delta)}{\delta} \right]^2 + p \left[ \frac{2J_1(\delta - \Delta\delta)}{\delta - \Delta\delta} \right]^2 + q$$

Figure 1 (b) shows the variation of resultant irradiance distribution of two-point sources of equal irradiances ( $p = 1$ ) for various values of the spatial degree of coherence and in the absence of background ( $q = 0$ ):- the normalized resultant irradiance distributions due to two-point sources have been drawn for a fixed separation of  $\Delta\delta = 1.22\pi$ .

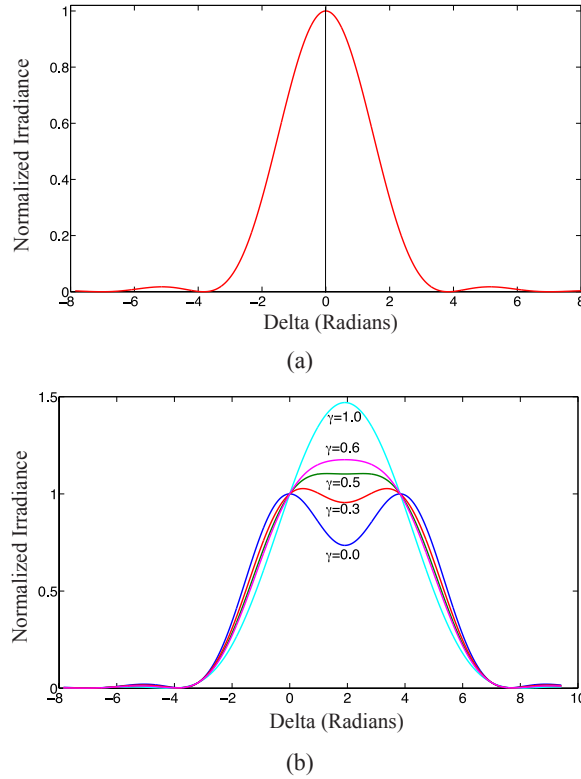


Fig 1. Normalized irradiance distribution in the image of (a) a point source, (b) two point sources angularly shifted by  $1.22\pi$  for different values of the coherence function.

#### 4 Criteria of resolution

There are several two-point source resolution criteria reported in the literature. According to Rayleigh criterion of resolution, the two-point sources are just resolved when the diffraction pattern due to one-point source falls on the first minimum of the diffraction pattern due to the second point source [23,24].

This results in a dip in the resultant diffraction pattern. There is no dip in the resultant irradiance distribution when Sparrow criterion is applied [7]. The Dawes criterion applies to the angular separation of closely spaced double stars using a telescope [25]. The angular separation in radians is given by  $1.02\lambda/2a$ , where  $a$  is the radius of the aperture and  $\lambda$  is the wavelength of light. It is close to the full width at half maximum ( $1.03\pi$ ) of the point-spread function and corresponds to a 5% dip between the maxima when the point-spread function is the Airy pattern. Schuster [26] proposed the criterion that two-point sources are resolved if the main lobes of their point-spread functions do not overlap. This is equivalent to twice the Rayleigh criterion spacing.

Houston [27] suggested comparing the distance between the central maxima of the resultant irradiance pattern with the full width at half maximum of the individual point-spread functions. If the former is greater than the latter, the sources are said to be resolved. This is thus very close to the Dawes criterion.

Abbe was the first to extend two-point resolution to fully coherent illumination with special reference to the microscope. Buxton's criterion [28] deals with the amplitude diffraction patterns and defines two-point sources to be resolved when the closest points of inflexion of the diffraction patterns coincide.

A modification of Rayleigh criterion of two-point source resolution applicable to both coherent and partially coherent illumination is described by Born and Wolf [29]. This modified version has been used by Bhatnagar, Sirohi and Sharma [14], and Nayyar and Verma [15].

There have been many other ways to look at the issue of resolution. The existence of an ultimate absolute limit for resolving power is investigated by Harris utilizing the ambiguous image concept, viz., different objects cannot be distinguished if they have identical images [30]. According to Toraldo di Francia, two-point resolution is impossible unless the observer has *a priori* an infinite amount of information about the object [31]. McCutchen mentions that in principle one can construct a super-resolving optical system that can resolve details finer than the diffraction limit [32]. Using Fourier transforms and coherence theory, Lukosz showed that the optical systems can far exceed the classical limit of resolution [33,34].

Most often used criteria of two-point source resolution are due to Rayleigh and Sparrow. Though not explicitly stated these are applicable to incoherent illumination. Both the criteria apply when the sources are of equal brightness and in the absence of background irradiance. These are described in detail in next sections and the Rayleigh criterion and its modified version will be used in the subsequent sections.

#### 4.1 Rayleigh criterion of resolution

According to Rayleigh criterion of resolution, the two-point sources of equal brightness are just resolved when the irradiance distribution due to the first point source falls on the first minimum of the irradiance distribution due to the second point source. Since the first minimum of irradiance distribution occurs at  $\delta = 1.22\pi$ , the normalized resultant irradiance  $I_{\text{res}}$ , at the condition of just resolution,  $\Delta\delta = ka\omega = 1.22\pi \rightarrow \omega = 0.61\lambda/a$ , is expressed as

$$I_{\text{res}} = \left[ \frac{2J_1(\delta)}{\delta} \right]^2 + \left[ \frac{2J_1(\delta - 1.22\pi)}{\delta - 1.22\pi} \right]^2$$

The resultant irradiance distribution exhibits a dip of about 26.5% of the maximum value. Therefore, the Rayleigh criterion can be stated in some-what different way, i.e., the two-point sources are just resolved when there is a dip of 26.5% in the resultant irradiance distribution. Rayleigh criterion in this form will be used to study resolution of point sources of unequal brightness and in the presence of background in incoherent, coherent and partially coherent illumination.

#### 4.2 Sparrow criterion of resolution

According to Sparrow criterion of resolution, the two-point sources of equal brightness are resolved when there is no dip in the resultant irradiance distribution, i. e.  $\left. \frac{d^2 I_{\text{res}}}{d\delta^2} \right|_{\delta = \frac{\delta_{\text{min}}}{2}} = 0$ . This certainly gives a

smaller value of the resolution, i.e.,  $\Delta\delta = 0.95\pi$ . The angular separation of two sources, according to Sparrow criterion, is given as  $\omega = 0.47\lambda/a$ . Figure 2 (a & b) show these two criteria. Notice that the resultant irradiance is higher than the maximum irradiance of any one of the sources when Sparrow criterion of resolution is satisfied.

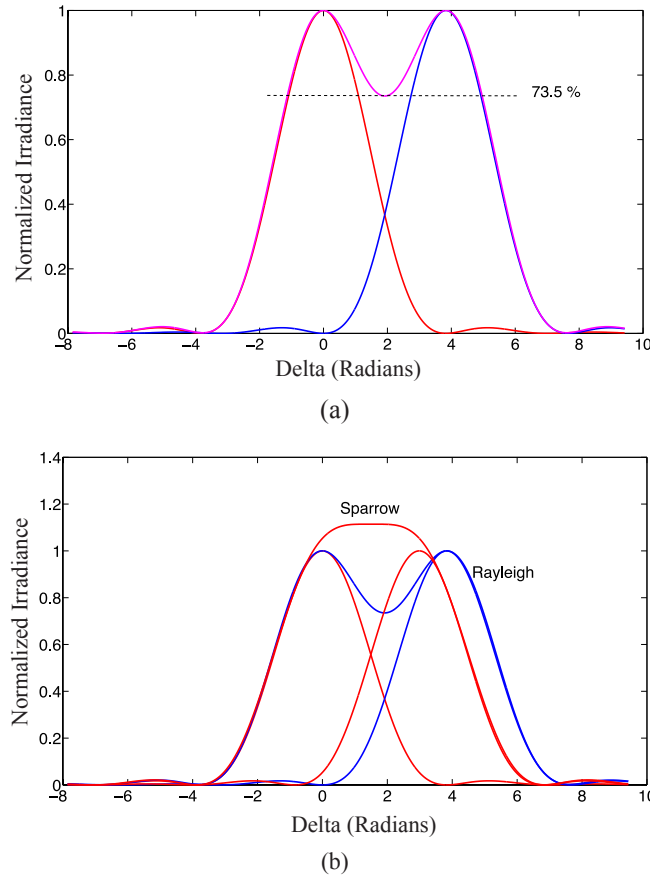


Fig 2. Resolution by (a) Rayleigh criterion, and (b) Sparrow criterion.

When a constant background is added to the irradiance distributions due to point sources, the relative magnitude of the dip decreases, requiring larger angular separation for just resolution according to Rayleigh criterion. This is due to loss of contrast.

Table 1 gives the limit of resolution of two point-objects under coherent and incoherent illumination for both the circular aperture and the slit aperture.

Table 1. Limit of resolution-Rayleigh vs Sparrow criterion

	Circular	Slit
Rayleigh - Incoherent	$\Delta\delta = 1.22\pi$	$\Delta\delta = \pi$
Rayleigh - Coherent	$\Delta\delta = 1.64\pi$	$\Delta\delta = 1.42\pi$
Sparrow - Incoherent	$\Delta\delta = 0.95\pi$	$\Delta\delta = 0.83\pi$
Sparrow - Coherent	$\Delta\delta = 1.46\pi$	$\Delta\delta = 1.33\pi$

### 5 Resolution of two-point sources of equal brightness under partially coherent illumination

The normalized resultant irradiance distribution can be written as

$$I_{\text{res}} = \left[ \frac{2J_1(\delta)}{\delta} \right]^2 + \left[ \frac{2J_1(\delta - \Delta\delta)}{\delta - \Delta\delta} \right]^2 + 2\gamma \frac{2J_1(\delta)}{\delta} \frac{2J_1(\delta - \Delta\delta)}{\delta - \Delta\delta}$$

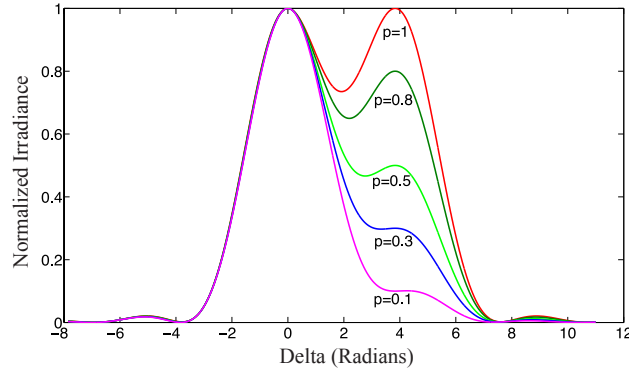
When Rayleigh criterion of resolution is invoked, we have

$$0.735 = \left\{ \left[ \frac{2J_1(\Delta\delta/2)}{\Delta\delta/2} \right]^2 + \left[ \frac{2J_1(\Delta\delta/2)}{\Delta\delta/2} \right]^2 + 2\gamma \frac{2J_1(\Delta\delta/2)}{\Delta\delta/2} \frac{2J_1(2\Delta\delta)}{\Delta\delta/2} \right\} = 2(1 + \gamma) \left[ \frac{2J_1(\Delta\delta/2)}{\Delta\delta/2} \right]^2$$

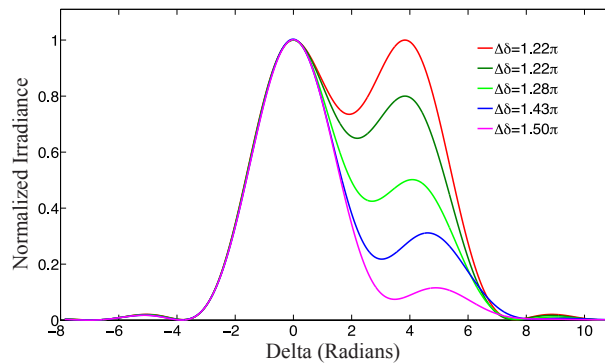
This equation can be solved iteratively to find the value of  $\Delta\delta$ .

### 6 Two incoherent sources of unequal brightness

When two incoherent sources of different brightness are considered, Rayleigh criterion can still be applied. The two sources will be just resolved if the dip is 73.5% of the maximum of the weaker source. **Figure 3 (a)** shows the resultant irradiance distributions for two incoherent sources which are angularly shifted such that  $k\alpha\omega = 1.22\pi$  but have brightness ratio of 1, 0.8, 0.5, 0.3 and 0.1. It is seen that the value of dip keeps on decreasing with the reduction of brightness ratio and at brightness ratio of 0.3 and lower, the two sources cannot be resolved. However, if the angular separation is increased, the sources are resolved as shown in **Fig 3 (b)**. Interestingly the two sources are just resolved when their brightness ratio is 0.8 and the angular separation  $\omega$  is such that  $k\alpha\omega = 1.22\pi$ , which is the case for the sources of equal brightness.



(a)



(b)

**Fig 3.** Normalized resultant irradiance distributions of sources with varying brightness ratio and angular separation such that  $k\alpha\omega = 1.22\pi$ , and (b) with varying angular separations.

It may be remarked that the stars (point sources) were resolvable according to Treanor [35] if the maximum of the diffraction pattern of a fainter star coincided with the first minimum of the diffraction pattern of the brighter one and the maximum of the fainter star was greater than the first sidelobe of the brighter star. This statement, however, does not seem to be supported if we carefully examine Fig 3 (a).

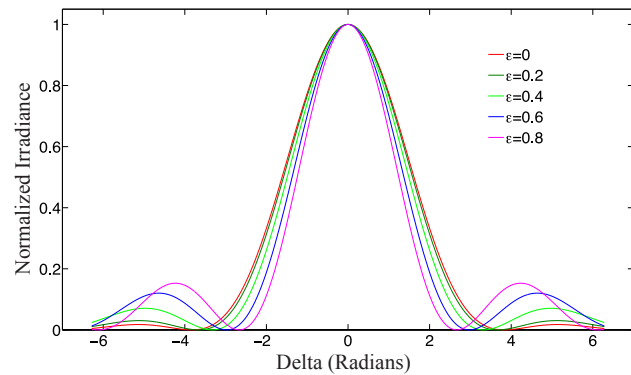
## 7 Improving the resolution

The resolution of an optical system can be improved if the dimensions of the diffraction pattern are decreased, i.e. the irradiance distribution is made narrower. Of course, it becomes narrower if the aperture of the system is increased. This is not always an acceptable solution. However, there are ways of achieving this without increasing the aperture size. For this to happen, the pupil transmittance instead of being unity is varied by using designed filters [20,21]. The simplest case is when the transmittance of the central portion of the pupil is taken as zero. The central obscuration of pupil is often due to the design of certain telescopes and microscopes.

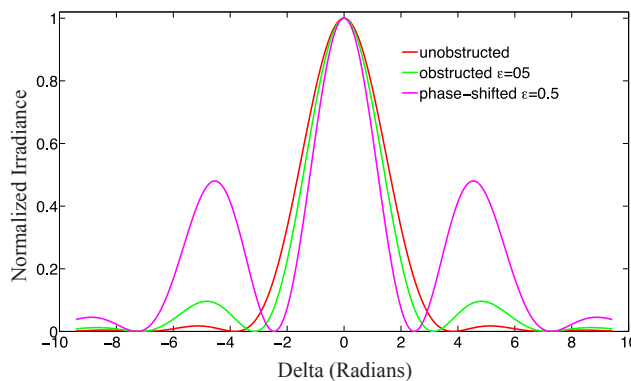
The normalized irradiance distribution in the Fraunhofer pattern of an obscured pupil (aperture) is given by

$$I_{\text{res}} = \frac{1}{(1-\varepsilon^2)^2} + \left[ \frac{2J_1(\delta)}{\delta} - \varepsilon^2 \frac{2J_1(\varepsilon\delta)}{\varepsilon\delta} \right]^2 \quad 0 \leq \varepsilon \leq 1$$

where  $\varepsilon$ , the obscuration ratio, is the ratio of the diameter of circular obstruction to the diameter of clear aperture  $2a$ .



(a)



(b)

Fig 4. Irradiance distributions for (a) various values of obstruction ratio, and (b) obstructed and obstructed phase-shifted aperture.

Figure 4 (a) shows the irradiance distributions for different obscuration ratios ( $\varepsilon = 0, 0.2, 0.4, 0.6, 0.8$ ). Due to the obscuration, there is a loss of light. Though the central lobe gets thinner with increasing obscuration, the irradiance also increases in second and higher lobes of the distribution.

The irradiance distribution can be modified by phase-shifting the transmittance of central portion of the aperture. As an example, circular portion of half the radius of the pupil has been phase-shifted by  $\pi$ . In this case there is no loss of light. Figure 4 (b) shows the irradiance distributions for an unobstructed aperture, an obstructed aperture with  $\varepsilon = 0.5$  and  $\pi$  phase-shifted aperture of radius  $0.5a$ . It is obvious that the central lobe is thinner but the irradiance of the first side lobe has considerably increased. As such, this kind of modification does not lead to any significant improvement in the resolution and has delirious effect when the sources are of unequal brightness.

## 8 Resolution of two coherent sources

When the sources are coherent or the two object points are illuminated by a coherent wave, then their amplitudes rather than their irradiances add. Figure (5) shows the resultant irradiance distribution when the two sources are angularly separated such that  $k\alpha\omega = 1.22\pi$ . Incoherent point sources were just resolved, according to Rayleigh criterion, for this angular separation. However, coherent sources are not resolved, and a larger angular separation is required to resolve them. When the angular separation is such that  $k\alpha\omega = 1.64\pi$ , the dip in the resultant irradiance distribution is about 26% and hence the two coherent points are just resolved according to Rayleigh criterion of resolution. This expression gives the separation between two points which are just resolved as  $0.82 \lambda_0 / \mu \sin\theta$ , where  $\mu \sin\theta$  is the numerical aperture of the object. This is slightly larger than that given in the book 'Principles of Optics' by Born and Wolf. Their value is  $0.77 \lambda_0 / \mu \sin\theta$ , which was obtained by the solution of a transcendental equation that did not take into account the negative values in the amplitude distribution. To make this matter clear, the amplitude distributions due to both the point objects are also shown in green in Fig 5. Note that the maximum irradiance in the resultant irradiance is not unity, and the minimum does not appear where the two amplitude distributions intersect. Further, the Sparrow criterion gives the condition  $k\alpha\omega = 1.46\pi$ . Young *et al* have carried out experiments in which test charts were illuminated by laser radiation and visual observation were made [12]. They estimated the angular separation at resolution to be about  $1.6\pi$ .

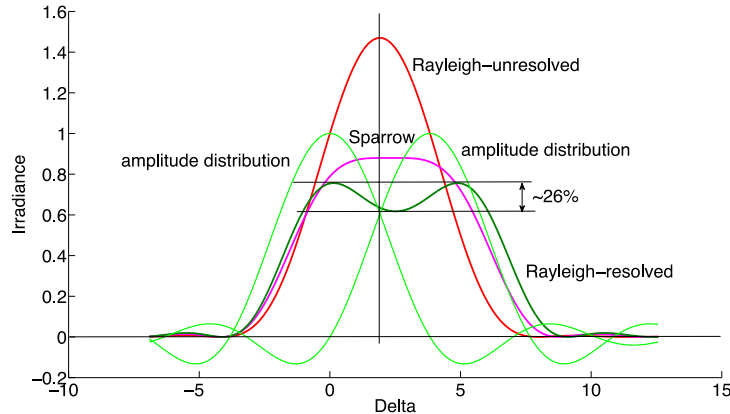


Fig 5. Rayleigh and Sparrow criteria under coherent illumination.

Figure 6 (a) shows the variation of limit of resolution,  $\Delta\delta$ , with degree of coherence,  $\gamma$ , using Rayleigh criterion  $I_{\min}/I_{\max} = 0.735$  for various of irradiance ratio,  $p$ , of two sources. Gamma equal to one ( $\gamma = 1$ ) corresponds to coherent sources and  $\gamma = 0$  to incoherent sources. Figure 6 (b) gives the variation



of limit of resolution with background for  $\gamma = 0$  and  $\gamma = 1$ : background 1 corresponds to the case when the background irradiance is equal to the irradiance of any one of the point sources: both the sources are assumed to have the same irradiance. It may be noted that the minimum irradiance of 0.735 of the maximum, could not be obtained for background equal to 3 when the sources are incoherent.

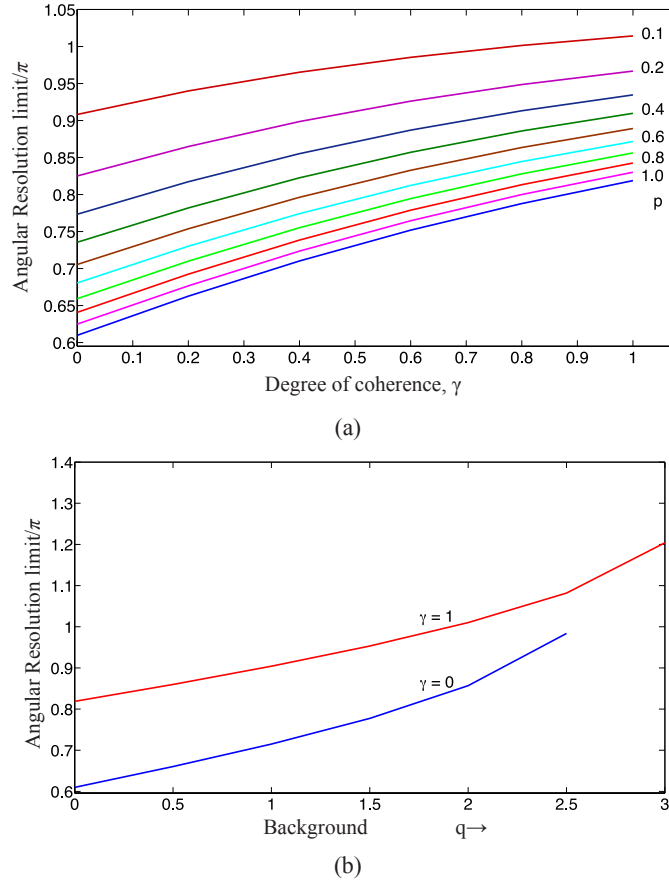


Fig 6. Variation of the limit of resolution (a) with  $\gamma$  for various values of irradiance ratio and (b) with background for  $\gamma = 0$  and 1.

## 9 Resolving power of spectroscopic instruments

Most of the instruments employ a narrow slit source (a line source) and the image of the slit, considering diffraction, is the spectral line. The instruments utilize dispersive elements to separate out the spectral components: the dispersive elements may include prism, grating, Fabry-Perot interferometer etc. The instrument should be able to resolve spectral lines corresponding to two closely separated wavelengths  $\lambda_1$  and  $\lambda_2$ . The resolution of the instrument is, therefore, defined as  $|\lambda_1 - \lambda_2|$ . However, it is customary to express the

resolving power as  $\frac{\bar{\lambda}}{|\lambda_1 - \lambda_2|} = \frac{\lambda}{\Delta\lambda}$ , where  $\bar{\lambda}$  is the mean wavelength.

If we consider a  $60^\circ$ -prism used at minimum deviation condition in a spectrometer, the irradiance distribution in the image of the slit is given by  $\text{sinc}^2(x)$  function: the beam is limited by the dimensions of the prism. Figure 7 shows the normalized irradiance distributions for wavelengths  $\lambda$  and  $\lambda + \Delta\lambda$ , and the

normalized resultant distribution when the irradiance distribution is given by  $\text{sinc}^2(x)$ . At the condition of just resolution, the dip in the normalized resultant distribution is  $\sim 19\%$ .

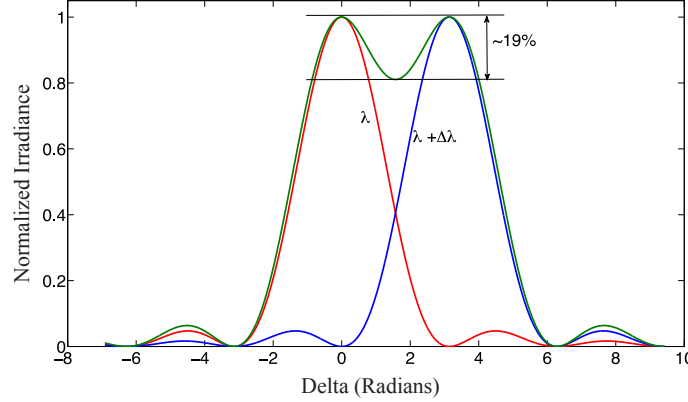


Fig 7. Irradiance distributions in the image when the incident radiation is dispersed by a  $60^\circ$  prism.

Using Rayleigh criterion, the resolving power  $\mathcal{R}$  of the  $60^\circ$ -prism spectrometer is given by

$$\mathcal{R} = \frac{\lambda}{\Delta\lambda} = t \frac{d\mu}{d\lambda}$$

where  $t$  is the prism base length and  $\frac{d\mu}{d\lambda}$  is the chromatic dispersion of the material of the prism. The resolving power of prism is different in different regions of the spectrum.

When a plane grating is used as a dispersive element, the incident collimated beam is diffracted by the grating elements. The diffracted beams are collected by a lens and the irradiance distribution at the focal plane, due to multiple beam interference, is given by

$$I(\delta) = I_0 \frac{\sin^2(N\delta/2)}{\sin^2(\delta/2)}$$

where  $N$  is the number of grating elements, the phase difference between any two adjacent beams is  $\delta = \frac{2\pi}{\lambda} d\sin\theta$ ;  $\theta$  is the angle of diffraction. Figure 8 (a) shows the diffraction patterns of gratings with different values of  $N$ . For smaller value of  $N$ , there are secondary maxima between the principal maxima. As  $N$  becomes very large, the diffraction pattern becomes very narrow. Different wavelengths in the incident beam are diffracted at different angles. The resolving power of the grating is given by

$$\mathcal{R} = \frac{\lambda}{\Delta\lambda} = m N$$

where  $m$  is an integer called the interference order, usually very small not exceeding 3. Resolving power of a grating spectrometer is the same over the whole spectrum.

A Fabry-Perot (F-P) interferometer consists of two high reflecting mirrors aligned parallel to each other and whose separation could be varied. A plane parallel plate of thickness  $t$  and refractive index  $n$  whose surfaces are coated to provide high reflectivity is called F-P etalon. Incident beam is multiply reflected: the amplitudes of multiply reflected beams decrease geometrically. Essentially an infinite number of beams participate in interference both in reflection and in transmission. The irradiance distribution in transmission of a F-P etalon is given by

$$I(\delta) = I_0 \frac{1}{1 + F \sin^2(\delta/2)}: \quad \delta = (2\pi/\lambda) 2nt \cos\theta_t = 2m\pi$$

where  $\theta_t$  is the angle of refraction inside the plate,  $m$  is the fringe order and the parameter  $F$  is defined as

$F = 4R/(1-R)^2$ :  $R$  is the reflectivity of the surfaces. This distribution is also called Airy distribution. Full width at half maximum (FWHM) of this distribution is  $4/\sqrt{F}$ . Two wavelengths are just resolved when the diffraction patterns of the two wavelengths are separated by FWHM. The resolving power is then expressed as

$$\mathcal{R} = \frac{\lambda}{\Delta\lambda} = m \frac{\pi}{2} \sqrt{F} = m\mathcal{F}$$

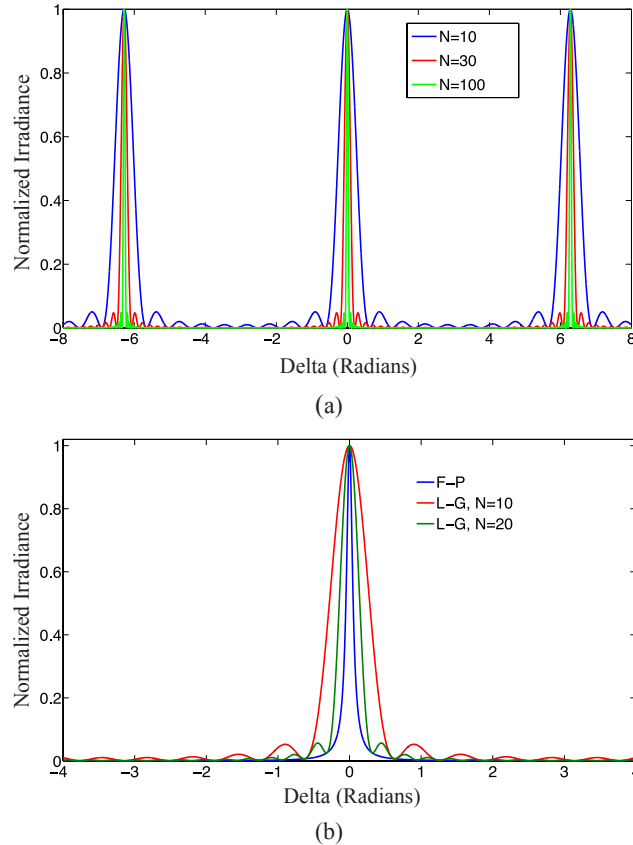
Prior to F-P interferometer or etalon, multiple beams were created by employing high reflectivity of a surface at larger angles of incidence such as in Lummer-Gehrcke (L-G) plate. Because of oblique incidence and finite length of the plate, only finite number of beams participate in interference. However, multiple beam interference in both transmission and reflection could be simultaneously seen or recorded. The irradiance distribution, both in reflection and transmission (when the first reflected beam is blocked) is of the type

$$I(\delta) = I_0 \frac{1 + F_N \sin^2(N \delta/2)}{1 + F \sin^2(\delta/2)}$$

where  $F_N = (4R^N)/(1-R^N)^2$  and  $N$  is the number of beams. The phase difference  $\delta$  is given by  $\delta = \frac{2\pi}{\lambda} 2nt \cos \theta$ . Using the same argument as used for F-P interferometer, the resolving power of L-G plate is given by

$$\mathcal{R} = \frac{\lambda}{\Delta\lambda} = m \frac{\pi}{2} \sqrt{F - 2N^2 F_N} = m\mathcal{F}'$$

where  $\mathcal{F}'$  is the finesse of the L-G plate.



**Fig 8.** Normalized irradiance distributions for (a) the grating (b) the Fabry-Perot interferometer and L-G plate in transmission.

Figure 8 (b) shows the normalized irradiance distributions for the Fabry-Perot interferometer as well as for L-G plate in transmission, both assumed to have a reflectivity of 0.95. The distributions are obtained when the surfaces of F-P interferometer and L-G plates are assumed free from any surface defects. There have been many studies on the effects of surface imperfections and misalignments [6,36,37].

In a Fourier transform spectrometer (FTS), an irradiance record is made when one of the mirrors of a Michelson interferometer is translated over a length  $L$  and the spectrum is obtained computationally from the record. Due to finite traverse of the mirror, there is an instrumental  $\text{sinc}$  function, which governs the resolving power. The resolving power of an FTS is,

$$\mathcal{R} = \frac{\lambda}{\Delta\lambda} = \frac{2L}{\lambda}.$$

## 10 Conclusions

This paper presents the study of resolution of optical instruments used for both seeing and spectroscopy.

## References

1. Sodha M S, Resolving power of Fabry Perot Etalon, *Curr Sci*, 22(1953)139-140.
2. Sodha M S, Effect of absorption by material of prism on its resolving power, *Am J Phys*, 21(1953)313; doi. 10.1119/1.1933431.
3. Sodha M S, Agrawal A K, On limit of resolution of telescope, *Optik*, 24(1966)189-196.
4. Sodha M S, Agrawal A K, Resolution of objects of unequal intensity in an annular aperture telescope, *Optik*, 24(1966)197-200.
5. Sodha M S, Resolving power of an instrument with circular aperture in white light, *Optik*, 16(1959)276.
6. Katti P K, Singh K, Resolving Power of the Fabry-Perot Interferometer and the Reflection Echelon Used for Visual Observations of Absorption Spectra, *Appl Opt*, 6(1967)1134-1136.
7. Sparrow C M, On Spectroscopic Resolving Power, *Astrophys J*, 44(1916)76-86.
8. Asakura T, On the Sparrow Resolution Criterion, *Oyo Buturi*, 31(1962)709-715.
9. Barakat Richard, Application of Apodization to Increase Two-Point Resolution by the Sparrow Criterion. I. Coherent Illumination, *J Opt Soc Am*, 52(1962)276-283.
10. Grimes D N, Thompson B J, Two-Point Resolution with Partially Coherent Light, *J Opt Soc Am*, 57(1967)1330-1334.
11. Falconi Oscar, Limits to which Double Lines, Double Stars, and Disks can be Resolved and Measured, *J Opt Soc Am*, 57(1967)987-993.
12. Young M, Faulkner B, Cole J, Resolution in Optical Systems Using Coherent Illumination, *J Opt Soc Am*, 60(1970)137-138.
13. Sirohi R S, Bhatnagar G S, Effect of partial coherence on the resolution of a microscope, *Optica Acta*, 17(1970) 839-842.
14. Bhatnagar G S, Sirohi R S, Sharma, S K, Two point resolution in partially coherent light, *Opt Commun*, 3(1971) 269-271.
15. Nayyar V P, Verma N K, Two-point resolution of Gaussian aperture operating in partially coherent light using various resolution criteria, *Appl Opt*, 17(1978)2176-2180.
16. Asakura T, Resolution of two unequally bright points with partially coherent light, *Nouv Rev Opt*, 5(1974)169-177.
17. Asakura T, Ueno T, Apodization for increasing two-point resolution by the sparrow criterion under the partially coherent illumination, *Nouv Rev Opt*, 5(1974)349-359.
18. den Dekker A J, van den Bos A, Resolution: a survey, *J Opt Soc Am*, 17(1997)547-555.
19. Heintzmann R, Sarafis V, Two point resolution in incoherent imaging, *Optik*, 112(2001)114-118.
20. Reddy A N K, Sagar D K, Two-point resolution of asymmetrically apodized optical systems, *Opt Pura Apl*, 46 (2013)215-222.

21. Reddy A N K, Khonina S N, Apodization for improving the two-point resolution of coherent optical systems with defect of focus, *Appl Phys B*, 124(2018)229; doi.org/10.1007/s00340-018-7101-z.
22. Cremer C, Masters B R, Resolution enhancement techniques in microscopy, *Eur Phys J H*, 38(2013)281-344.
23. Lord Rayleigh, Investigations in optics, with special reference to the spectroscope, *Philosophical Magazine, Ser V*, 8(1879)261-274.
24. Lord Rayleigh, On the resolving power of telescopes, *Philosophical Magazine*, 10(1880)116-119.
25. Dawes W R, Catalogue of micrometrical measurements of double stars, *Mem Roy Astron Soc*, 35(1867)137-502.
26. Schuster A, *Theory of Optics*, (E Arnold & Co, London, UK), 1924.
27. Houston W, A compound interferometer for fine structure work, *Phys Rev*, 29(1927)478-484.
28. Buxton A, Note on optical resolution, *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 23(1937)440-442.
29. Born M, Wolf E, *The Principles of Optics*, 4<sup>th</sup> edn, (Pergamon Press), 1970, pp 419-424.
30. Harris J L, Resolving power and decision theory, *J Opt Soc Am*, 54(1964)606-611.
31. Toraldo di Francia G, Resolving power and information, *J Opt Soc Am*, 45(1955)497-501.
32. McCutchen C W, Super resolution in microscopy and the Abbe resolution limit, *J Opt Soc Am*, 57(1967)1190-1192.
33. Lukosz W, Optical systems with resolving powers exceeding the classical limit: I, *J Opt Soc Am*, 56(1966)1463-1472.
34. Lukosz W, Optical systems with resolving powers exceeding the classical limit: II, *J Opt Soc Am*, 57(1967)932-941.
35. Treanor P J, On the telescopic resolution of unequal binaries, *The Observatory*, 66(1946)255-258
36. Mahapatra D P, Mattoo S K, Exact evaluation of the transmitted amplitude for a Fabry-Perot interferometer with surface defects, *Appl Opt*, 25(1986)1646-1649.
37. Palik E D, Boukari H, Gammon R W, Experimental study of the effect of surface defects on the finesse and contrast of a Fabry-Perot interferometer, *Appl Opt*, 35(1996)38-50.

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