



## Digital refocusing of images recorded with white light using Lucy-Richardson-Rosen algorithm

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Dedicated to Professor Partha Banerjee for his enormous contributions to the advancement of research and education in holography through his unique vision and outstanding dedication

Lens-based imaging is one of the widely used scientific methods to record optical information. As long as the imaging conditions are satisfied, this method can be used to image an object faithfully. However, beyond the limit of the depth of focus of the optical element, the collected image appears blurred. Though shifting the location of the optical element or the sensor offers a solution to the above problem, it is not suitable for recording dynamic events. There are different deconvolution methods available for digital refocusing of blurred images. Recently, a new reconstruction method called Lucy-Richardson-Rosen algorithm (LR<sup>2</sup>A) was developed for deconvolution based 2D and 3D incoherent imaging applications. In the present work, we have demonstrated LR<sup>2</sup>A on blurred images recorded using white light for the first time. A simple, commonly available refractive lens along with an incoherent white light source was used to record the point spread functions (PSF) at different depths. Then, the object information in the corresponding planes were also recorded. Finally, the PSF library was used to digitally refocus the object information. The results were compared with standard algorithms such as Lucy-Richardson and nonlinear reconstruction methods. In all the cases, LR<sup>2</sup>A exhibited a superior performance. © Anita Publications. All rights reserved.

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### 1 Introduction

An incoherent source is a better choice for imaging due to many advantages such as higher imaging resolution, easy availability, eye-safe, low-cost, and minimal imaging noises in comparison to coherent sources [1]. Incoherent imaging methods have a wide applicability including imaging of astronomical and fluorescence objects [2]. Even though incoherent imaging in 2D is simple and well-established, much improvement and dedicated research is required for 3D incoherent imaging. Incoherent 3D imaging can be broadly classified into two types (i) holography and (ii) deconvolution. As is known, holography-based 3D imaging methods require two-wave interference, often leading to a complex optical setup followed by a computational reconstruction of the recorded hologram [3-6]. Few notable systems in this domain are

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rotational shearing interferometer [7], conoscopic holography [8], Fresnel incoherent correlation holography [9,10], Fourier incoherent single channel holography [11], and coded aperture correlation holography [12]. In case of deconvolution, the linearity conditions of incoherent imaging can be used along with the computational deconvolution methods to reconstruct the images in 3D. The setup is simple and does not require two-beam interference, vibration isolation systems, or any other complex accessories. Due to the above facts, deconvolution-based approaches are considered simpler, faster, and economical than the holography counterparts. In deconvolution-based imaging, it is necessary to prerecord point spread functions (PSFs) at all axial planes, which is one of the procedures not required in holography. The recorded object intensity is processed with the PSFs using computational deconvolution methods to reconstruct the 3D image of the object.

The deconvolution-based methods also have certain limitations and the major requirement is the availability of good computational algorithm with suitable computational power in addition to the requirement to pre-record PSFs. Due to the above reasons, even though the idea of 2D deconvolution imaging was reported in the 20th century [13,14], most significant literature appeared only in the last five years [15-17]. In the recent reports, a diffuser was often used as the optical modulator between the source and the sensor. The requirement of the diffuser arises mainly due to the requirement of the reconstruction algorithms which is a sharp autocorrelation function [18,19]. The scattered intensity distribution from the diffusers generates a sharp autocorrelation function as the average speckle size equals the diffraction limited spot size  $\sim \lambda/\text{NA}$ , where NA is the numerical aperture. Therefore, the autocorrelation function is twice that of the diffraction limited spot size  $\sim 2\lambda/\text{NA}$ . A recent computational reconstruction method called non-linear reconstruction (NLR) was able to obtain a computational resolution equal to the diffraction limited spot size. However, one of the challenges in using a diffuser is the low SNR, which prevents a broader applicability of this interesting 3D imaging method. So, it is necessary to find alternative optical fields that would have better SNR and focus light in a small area of the sensor.

Recently, the possibility of applying deterministic optical fields instead of scattered field for deconvolution-based imaging application was investigated [20]. Many deterministic fields were found to be suitable for indirect incoherent imaging applications when NLR was applied. In the present work, we have used a simple refractive lens along with a white light incoherent source to demonstrate the 3D imaging of test objects. For this purpose, a recently developed Lucy-Richardson-Rosen algorithm (LR<sup>2</sup>A) was used for the image reconstruction [21]. This algorithm is a combination of well-known Lucy-Richardson algorithm (LRA) and NLR [21,22]. This improved method has already proven efficient in the case of micro-spectroscopy and single wavelength optical deep deconvolution studies. In this work, we have shown that the algorithm is capable of reconstruction of object information with broadband incoherent source –with low spatial as well as low temporal coherence. The methods, results, and corresponding discussions are presented in the following sections.

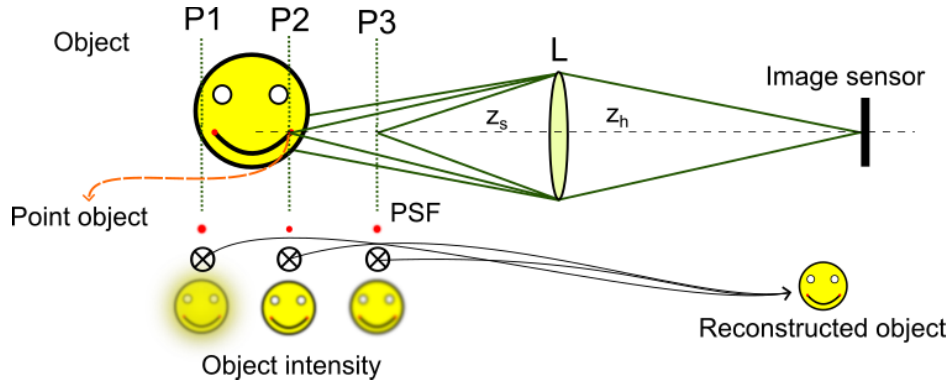
## 2 Method

A schematic of the optical configuration is given in Fig 1. A white LED light with negligible spatial or temporal coherence was used as the source. A point object generates a total amplitude  $\sqrt{\int_{\lambda_1}^{\lambda_2} I_o(\lambda) d\lambda}$  for a broadband source with wavelength limits of  $\lambda_1$  and  $\lambda_2$ . Considering a single wavelength from a point located at  $\vec{r}_o = (x_o, y_o)$ , a complex amplitude  $C_1 \sqrt{I_o(\lambda)} Q\left(\frac{1}{z_s}\right) L\left(\frac{\vec{r}_o}{z_s}\right)$  is generated at the plane of the refractive lens located at a distance  $z_s$  from it, where  $\lambda$  is the wavelength of the source used,  $Q(1/z_s) = \exp [j\pi R^2/\lambda z_s]$  and  $L(\vec{r}_o/z_s) = \exp [j2\pi (o_x x + o_y y)/\lambda z_s]$  are the quadratic and linear phase factors, respectively,  $R$  is the radial coordinate,  $R = \sqrt{x^2 + y^2}$ , and  $C_1$  is a complex constant. The focal length  $f$  of the refractive lens can be given by  $1/f = 1/z + 1/z_h$ , where

$z_h$  is the distance between the refractive lens and the sensor plane and  $z$  is the ideal distance required between the object plane and the lens in order to satisfy the imaging condition. The complex amplitude just after the lens is  $C_1 \sqrt{I_o(\lambda)} Q\left(\frac{1}{z_1}\right) L\left(\frac{\bar{r}_o}{z_s}\right)$ , where,  $z_1 = z_s f / (f - z_s)$ . The intensity distribution obtained at the sensor plane located at a distance  $z_h$  from the lens is given by,

$$I_{PSF}(\bar{r}_s; \bar{r}_o, z_s, \lambda) = |C_2 \sqrt{I_o(\lambda)} L(\bar{r}_o/z_s) Q(1/z_1) \otimes Q(1/z_h)|^2 \quad (1)$$

where,  $\otimes$  is a 2D convolutional operator,  $C_2$  is a complex constant, and  $\bar{r}_s$  is the location vector in the sensor plane along the transverse direction. In an ideal situation, the imaging condition is  $z_s = z$  to obtain a point image on the sensor. In such a case, the lateral and axial resolutions can be given as  $1.22 \lambda z_s / D$  and  $\sim 8\lambda(z_s/D)^2$ , respectively. Here,  $D$  is the diameter of the lens and the magnification of the system is given by  $M = z_h/z_s$ .



**Fig 1.** Schematic of the optical configuration used for the experiment. P1, P2, and P3 correspond to different object planes.

The total intensity recorded by the image sensor is given as

$$I_{PSF}(\bar{r}_s; \bar{r}_o, z_s) = \int_{\lambda_1}^{\lambda_2} I_{PSF}(\bar{r}_s; \bar{r}_o, z_s, \lambda) d\lambda. \quad (2)$$

The above equation can be modified as

$$I_{PSF}(\bar{r}_s; \bar{r}_o, z_s) = I_{PSF}\left(\bar{r}_s - \frac{z_h}{z_s} \bar{r}_o; 0, z_s\right). \quad (3)$$

A 2D object can be considered as a collection of points given as

$$O(\bar{r}_o) = \sum_i a_i \delta(\bar{r}_o - \bar{r}_i), \quad (4)$$

where  $a$  is the amplitude. The object intensity pattern at the sensor is the summation of shifted point spread functions which can be expressed as

$$I_O(\bar{r}_s, z_s) = \sum_i a_i I_{PSF}\left(\bar{r}_s - \frac{z_h}{z_s} \bar{r}_{o,i}; 0, z_s\right) \quad (5)$$

Using the linearity condition for the incoherent imaging and assuming insignificant chromatic aberration as the optical modulator is a refractive lens as opposed to a diffractive element, the object intensity distribution for an object  $O$  can be reconstructed using cross-correlation given as

$$\begin{aligned} P(\bar{r}_R) &= \iint I_O(\bar{r}_o; z_s) I_{PSF}^*(\bar{r}_o - \bar{r}_R; z_s) d\bar{r}_o \\ &= \iint \sum_i a_i I_{PSF}\left(\bar{r}_o - \frac{z_h}{z_s} \bar{r}_{s,i}; z_s\right) I_{PSF}^*(\bar{r}_o - \bar{r}_R; z_s) d\bar{r}_o \\ &= \sum_i a_i A\left(\bar{r}_R - \frac{z_h}{z_s} \bar{r}_{s,i}\right) \approx O\left(\frac{z_h}{z_s} \bar{r}_R\right) \end{aligned} \quad (6)$$

In direct imaging mode, the object information was sampled by the lateral resolution of the system, i.e.,  $I_O$  is obtained by sampling  $O$  with  $I_{PSF}$ . But when the imaging conditions are not satisfied  $I_{PSF}$  would be blurred and so is the object information. In the case of indirect imaging mode, the idea is to extract  $O$  from  $I_O$  using  $I_{PSF}$  as described above or simply  $I_R = I_O * I_{PSF}$ , where  $*$  is a 2D correlation operator. The above equation can be written as  $I_R = I_{PSF} \otimes O * I_{PSF}$  and can be rearranged as  $I_R = O \otimes I_{PSF} * I_{PSF}$ . By this way, the object information can be obtained by autocorrelating  $I_{PSF}$  and using it to sample  $O$ . The computational reconstruction method NLR can be given as,

$$I_R = \mathcal{F}^{-1} \{ |\tilde{I}_{PSF}|^\alpha \exp[j \cdot \arg(\tilde{I}_{PSF})] |\tilde{I}_O|^\beta \exp[-j \cdot \arg(\tilde{I}_O)] \}, \quad (7)$$

where  $\alpha$  and  $\beta$  are varied between  $-1$  and  $1$  until a minimum background noise is obtained. Essentially, the optimal NLR would reduce the background noise arising due to the positive nature of the  $I_{PSF}$ . The performance of NLR is sometimes affected by the nature of the intensity distribution especially when the  $I_{PSF}$  is a blurred spot. Our recent studies indicated that NLR is suitable when the intensity distribution has isolated points [20,22]. In the case of blurred images, NLR is not an ideal reconstruction method. In this study,  $LR^2A$  is used which can be described as follows. The  $(n+1)^{th}$  reconstructed image can be obtained by

$$I_R^{n+1} = I_R^n \left\{ \frac{I_O}{I_R^n \otimes I_{PSF}} \otimes I_{PSF}' \right\}, \quad (8)$$

where  $I_{PSF}'$  is the complex conjugate of the  $I_{PSF}$ . The loop begins with an initial guess solution and iterates the next solution by a forward convolution and backward correlation. As seen above, the ratio of  $I_O$  and forward convolution -  $I_R^n \otimes I_{PSF}$  is cross-correlated with  $I_{PSF}$  using an NLR is multiplied to the previous solution to obtain the next solution. The replacement of regular correlation by NLR improves the speed of convergence by more than 10 times in many cases and a better estimation of solution.

### 3 Results

The setup used for the experiment is shown in Fig 2. It consists of a spatially incoherent white LED light as the source. The object was critically illuminated by the source through an iris and a refractive lens (L1) of focal length 35 mm. A refractive lens (L2) with a focal length 50 mm was placed at  $2f$  configuration between the test object and the image sensor (Zelux CS165MU/M 1.6 MP monochrome CMOS camera,  $1440 \times 1080$  pixels with pixel size  $< 3.5 \mu m$ ) to achieve a magnification of 1.

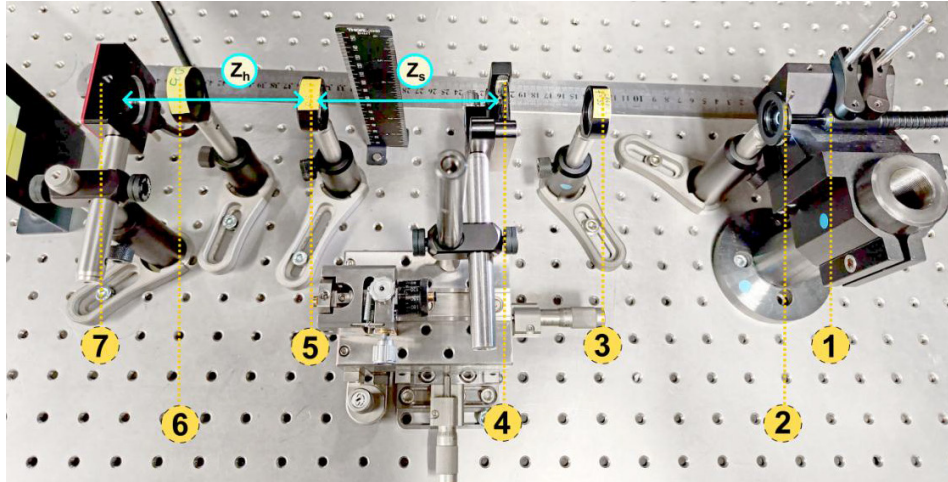


Fig 2. Experimental setup: (1) White LED, (2) Iris, (3) Lens L1 ( $f = 3.5$  mm), (4) Test object, (5) Lens L2 ( $f = 50$  mm), (6) ND filter (ND 1.5) and Image sensor.  $z_s$  is the distance between the test object and the lens and  $z_h$  is the distance between L2 and image sensor.

This aspect avoids the need for additional calibration as the physical space of the object is directly mapped to the sensor. The lateral and axial resolutions of the system are  $1.58 \mu\text{m}$  and  $10.4 \mu\text{m}$ , respectively. In order to reduce the light intensity on the image sensor a neutral density filter (ND 1.5) was placed between the sensor and the lens L2.

In order to create the PSF library a pinhole of diameter  $50 \mu\text{m}$  was used. The pinhole location was changed along  $+z$  to  $-z$  direction with a step size of  $0.25 \text{ mm}$ . Then the pinhole was replaced by the test objects one after the other to record the object intensity distributions.

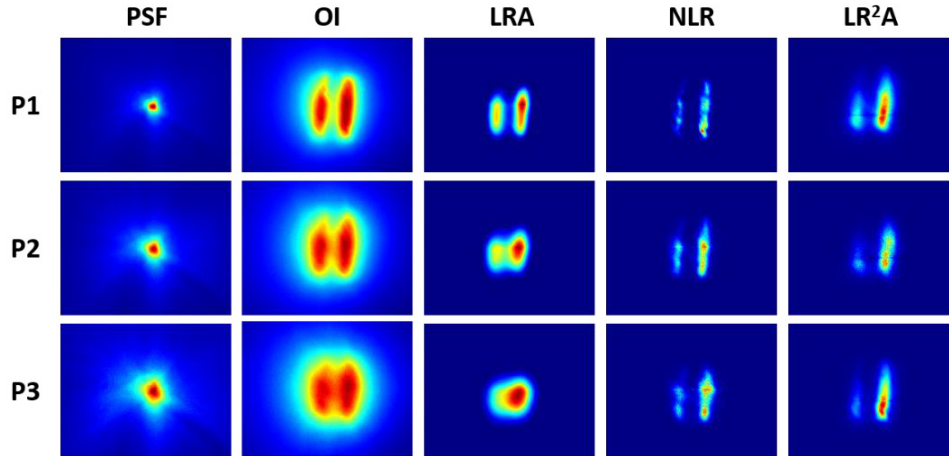


Fig 3. Images of PSF, object intensity of test object – 1, and the corresponding reconstruction results using LRA, NLR and LR<sup>2</sup>A for  $z_s = 5, 5.2$  and  $5.4 \text{ cm}$ .

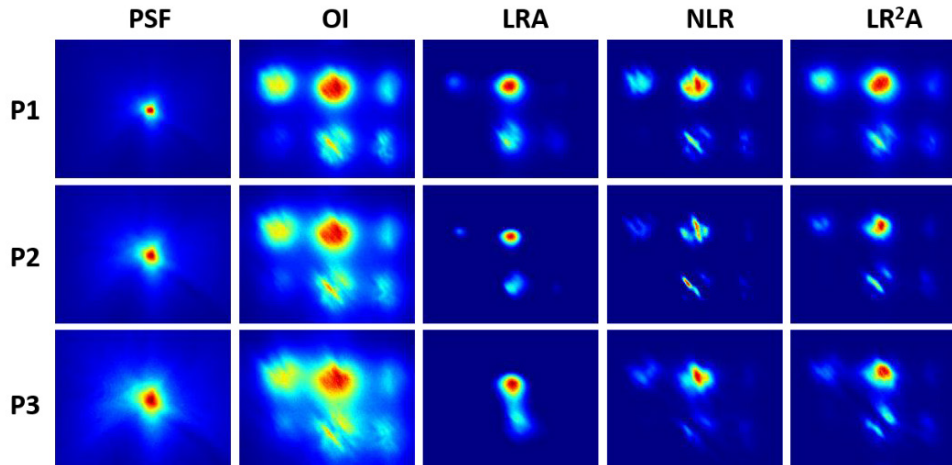


Fig 4. Images of PSF, object intensity of test object – 2, and the corresponding reconstruction results using LRA, NLR, and LR<sup>2</sup>A for  $z_s = 5, 5.2$  and  $5.4 \text{ cm}$ .

The objects were recorded at the same axial locations as the pinhole. These two data sets namely, recorded object intensities and PSF library, were then fed into the reconstruction algorithm and the images were reconstructed at different planes. Two kinds of test objects, one like a ‘=’ sign ( $1.5 \times 0.28 \text{ mm}$ ) and six concentric circles each of diameter  $0.3 \text{ mm}$  were used for the data collection. The object intensity and the PSF for both the objects were recorded at  $z_s = 5 \text{ cm}, 5.2 \text{ cm}$  and  $5.4 \text{ cm}$ . The recorded and reconstructed images of



test objects 1 and 2 are given in Figs 3 and 4, respectively and the parameters are as follows. For test object 1, LRA (40 iterations) for all the three planes, NLR ( $\alpha = -0.2, -0.2, -0.25$  and  $\beta = 0.8, 0.88, 0.88$ ), LR<sup>2</sup>A ( $\alpha = -0.1, -0.1, -0.2$  and  $\beta = 0.98$ ) with 3, 3, 4 iterations for the three planes. For test object 2, LRA (10, 40 and 30 iterations), NLR ( $\alpha = -0.2, -0.4, -0.15$  and  $\beta = 1$ ), LR<sup>2</sup>A ( $\alpha = -0.1$  and  $\beta = 1$ ) with 1, 3, 3 iterations for all the three planes. It can be seen that when the degree of blur increases, the performances of LRA and NLR are affected while LR<sup>2</sup>A yields a good quality reconstructed image matching with the direct imaging results.

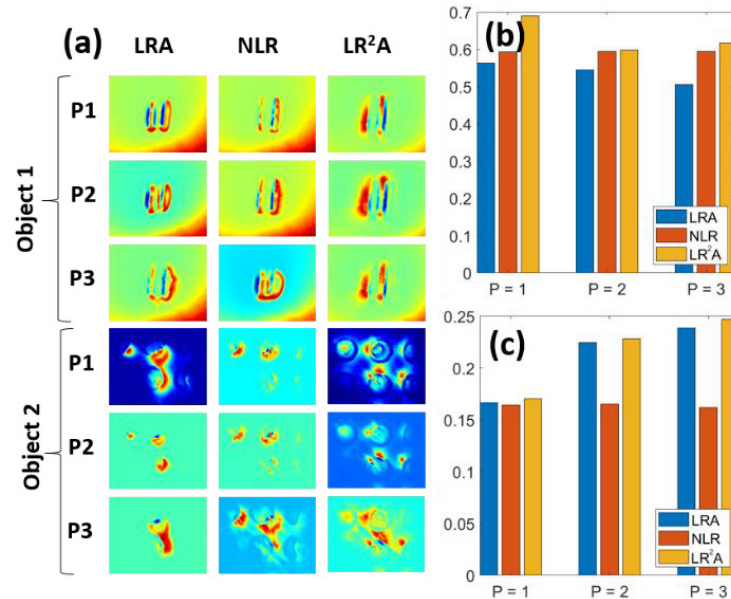


Fig 5. (a) SSIM maps of objects 1 and 2, SSIM values of the (b) object 1 and (c) object 2 for LRA, NLR and LR<sup>2</sup>A.

In order to quantify and compare the results, the SSIM values are calculated for the reconstructed images using the reference image at  $z_s = z$  position. The obtained values and the corresponding SSIM maps are given in Fig 5(a) and 5(b), respectively. It can be seen that in all the cases LR<sup>2</sup>A performs better than LRA and NLR.

#### 4 Conclusion

A refractive lens is one of the simplest optical components used for 2D imaging with an incoherent source. But the limitation is usually the depth of the focus which is  $\sim \lambda/NA^2$  and beyond that the object information is blurred [23]. There are several computational algorithms available to digitally refocus images but are limited to a small range of axial and spectral aberrations [24,25]. In this work, we have used a recently developed LR<sup>2</sup>A method for deconvolution of blurred images recorded using white light and compared its results with NLR and LRA methods. In all the cases, LR<sup>2</sup>A method performed better than other two methods and indicates the potential for higher dimensional imaging. If the same method was implemented using a diffractive lens instead of a refractive lens, then due to chromatic aberrations, the reconstruction results are affected. A recent study of white light imaging using a quasi-random lens with NLR showed a significantly low performance [26]. In that case, additional deep learning based denoising network was necessary to improve the reconstructed results. In this case, the same approach with a low-cost device-refractive lens and LR<sup>2</sup>A method has shown improved reconstruction results. We believe that this study will lead to the development of white light-based 3D microscopy.

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