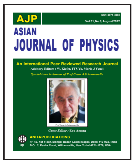

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Linear retardance model for a rotating polarizer-analyzer polarimeter

Geliztle A Parra-Escamilla[1,2], Francisco Joel Cervantes-Lozano[1] and David I Serrano-García[1]
[1]*Electro-Photonics Engineering Department, University Center of Exact Sciences and Engineering* (*CUCEI*),
University of Guadalajara, Av. Revolución No. 1500, CP. 44430 Guadalajara, Jalisco, México., México.
[2]*Faculty of Engineering, Universidad Panamericana. Álvaro del Portillo 49, Zapopan, Jalisco, 45010, Mexico*

This article is dedicated to Professor Cesar Sciammarella

We present a demodulation approach for a rotating polarizer-analyzer polarimeter dedicated to linear retardance measurements. Our rotating polarizer-analyzer polarimeter analysis is based on retrieving a transparent sample's partial Mueller matrix measurement to be later associated with its phase retardation properties. We present experimental results showing the feasibility of our proposal.

Keywords: Polarized Light, Mueller Matrix, Optics Instrumentation, Linear Retarder.

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1 Introduction

Polarized light can be used to determine optical properties of samples by analyzing the response of the polarized light that is reflected or transmitted by the sample under test. It has been widely used to develop new measurement systems for the atmospheric sensing field [1,2] and follow climate variations [3]. In remote sensing, polarimetry has also been used to analyze reflective objects [4], and even in the biomedical field, it has been proposed as a marker to identify cancerous tissue in its early stages [5].

Several polarimeters are described in the literature, for example, using a dual rotating retarder configuration [6,7], employing phase modulators [8], or by liquid crystal retarders [9,10]. After retrieving the Mueller matrix, the decomposition algorithms separate the information into three properties known as diattenuation, retardance, and depolarization [11-15], and each of these properties can be associated with the physical properties of the sample under test. More specifically, the retardance property is helpful for glucose measurement and also can be associated with stress analysis. One type of polarimeter commonly used employs a rotating polarizer and an analyzer that Azzam proposed in 1978 [6]. He showed theoretically the feasibility of the implementation and its capabilities to retrieve the partial Mueller matrix and Jones matrix coefficients through common transformations. Later, several authors followed the Azzam's approach [15-21].

In our proposal, we employed the rotating polarizer-analyzer system considering the sample under test as a linear retarder. With this representation, although we have a partial Mueller matrix polarimeter, we can retrieve the fast axis orientation and its linear retardance property to analyze transparent samples with retardance properties. The implementation presents several advantages compared with other systems that employ retarders with a strong dependence on the wavelength, and our approach has potential usage for large bandwidth spectroscopic measurements. The other advantage is the reduced cost of implementation by employing linear polarizers only.

Corresponding author
e mail: david.serrano@academicos.udg.mx (David I Serrano-García).

The paper is organized as follows: in section 2, we show our theoretical approach and the demodulation algorithm. Then, experimental results obtained with a layered retardance phantom are presented in section 3. Finally, conclusions and final remarks are given in section 4.

2 Rotating polarizer-analyzer polarimeter sensitive to linear retardation parameters

The approach for retrieving the linear retardance information consists of a polarization state generator with a light source at working wavelength λ, a linear polariser $LP(0°)$ oriented at angle 0° (employed as orientation reference) and a linear polarizer $LP(\theta)$ rotating at a rate θ. The polarization state detection unit consists of a linear polarizer $LP(4\theta)$, rotating at a rate 4θ and an intensity detector that could be a camera or a photodetector. The sample is considered as a linear retarder $LR(\theta_s, \delta_s)$ with properties of fast axis orientation θ_s and linear retardance δ_s. Figure 1 shows the diagram and the theoretical parameters involved in the system.

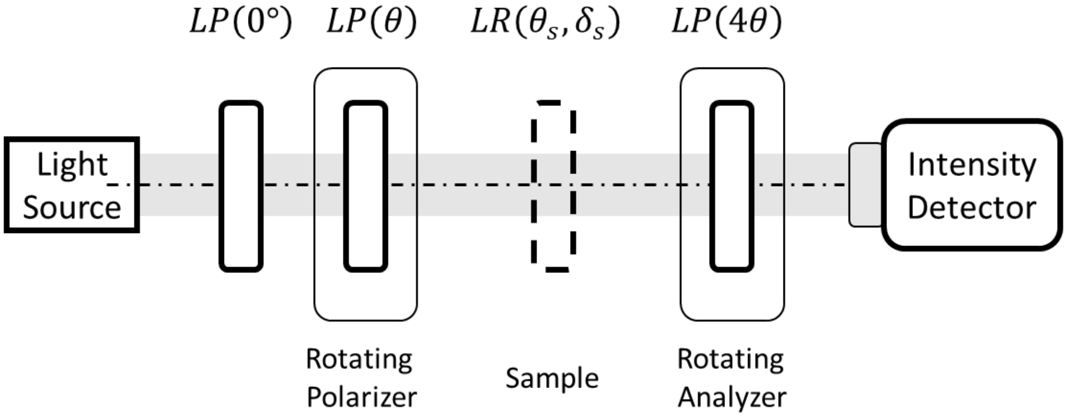

Fig 1. Rotating polarizer-analyzer polarimeter. LP: represents a Linear Polarizer, θ is the fast axis orientation of the linear polarizer; LR is a linear retarder, the angles δ_s and θ_s represent linear retardance and fast axis orientation, respectively.

The Mueller matrix of a linear polarizer $LP(\theta)$ for a given angle θ is [12,13]

$$\begin{bmatrix} 1 & \cos(2\theta) & \sin(2\theta) & 0 \\ \cos(2\theta) & \cos^2(2\theta) & \cos(2\theta)\sin(2\theta) & 0 \\ \sin(2\theta) & \cos(2\theta)\sin(2\theta) & \sin^2(2\theta) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{1}$$

while for a linear retarder $LR(\theta_s, \delta_s)$ is given by [12,13]

$$LR(\theta_s, \delta_s) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2(2\theta_s) + \sin^2(2\theta_s)\cos(\delta_s) & (1-\cos\delta_s)\sin(2\theta_s)\cos(2\theta_s) & -\sin(2\theta_s)\sin(\delta_s) \\ 0 & (1-\cos\delta_s)\sin(2\theta_s)\cos(2\theta_s) & \sin^2(2\theta_s) + \cos^2(2\theta_s)\cos(\delta_s) & \cos(2\theta_s)\sin(\delta_s) \\ 0 & \sin(2\theta_s)\sin\delta_s & -\cos(2\theta_s)\sin\delta_s & \cos(\delta_s) \end{bmatrix}, \tag{2}$$

where θ_s is the fast axis orientation and δ_s the linear retardance. Considering non-polarized light as input, $S_{in} = [S_0, 0, 0,0]^T$, and the output Stokes vector, S_{out}, is

$$S_{out} = LP(4\theta) \cdot LR(\theta_s, \delta_s) \cdot LP(\theta) \cdot LP(0) \cdot S_{in}, \tag{3}$$

moreover, the detected intensity, I_{out} is the first element of S_{out}, as

$$\begin{aligned} I_{out}(\theta) = \frac{S_0}{16}\Big[& 2 + 2\cos(2\theta) + \frac{1}{2}\left(1+\cos\delta_s\right)\cos(4\theta) + \left(1+\cos\delta_s\right)\cos(6\theta) \\ &+ (\cos^2(\delta_s/2) + \cos(4\theta_s)\sin^2(\delta_s/2))\cos(8\theta) + \sin(4\theta_s)\sin^2(\delta_s/2)\sin(8\theta) \\ &+ 2\cos(4\theta_s)\sin^2(\delta_s/2)\cos(10\theta) + \sin(4\theta_s)\sin^2(\delta_s/2)\sin(10\theta) \\ &+ \cos(4\theta_s)\sin^2(\delta_s/2)\cos(12\theta) + \sin(4\theta_s)\sin^2(\delta_s/2)\sin(12\theta)\Big] \end{aligned} \tag{4}$$

As the detected intensity is modulated by the rotation rates of the polarizer-analyzer, it can be modeled as a series of harmonics that can be analyzed as a Fourier Serie as

$$I_{out}(\theta) = a_0/2 + \sum_{k=1}^{6}\left[a_{2k}\cos(2k\theta) + b_{2k}\sin(2k\theta)\right]. \tag{5}$$

As the output signal is composed of sine-cosine functions with different amplitudes, each coefficient can be calculated from the experimental data to obtain the retardance information of the sample as

$$\tan(4\theta_s) = \frac{b_{10}}{a_{10}}$$

and

$$\cos(\delta_s/2) = \sqrt{2\frac{a_4}{a_2}} \tag{6}$$

By this procedure, we can retrieve the linear retardance parameters of the sample. The approach does not consider the initial angle of the rotating polarizer and analyzer; both elements need to be aligned with respect to the first polarizer.

3 Experimental implementation and Results

We employed a He-Ne laser with a wavelength of $\lambda = 632.8$ nm as a light source. The beam is spatially filtered and expanded to 25 mm in diameter. To acquire the images, we used an 8 bit CMOS camera (model acA2000-340km, Basler) with 2048×1088 pixels and an imaging system focusing on the sample location. The imaging system provides a spatial resolution of 14.25 cycles/mm corresponding to a line width of 35.08 microns by placing a standard USAF resolution chart at the sample plane. The polarizers used in the system are the standard polarizer sheets working in the visible range (Thorlabs-LPVISE2×2) with an extinction ratio of 1000:1, according to the provider.

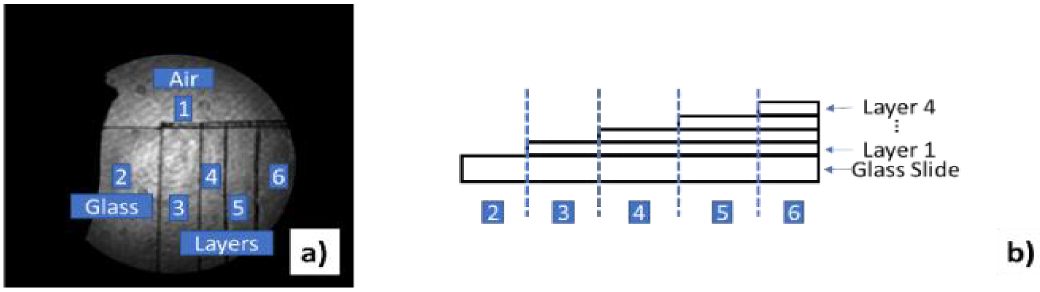

Fig 2. Retardance Layered Phantom. Fig (a) shows the distribution of different materials in the sample. Region 1 corresponds to air, region 2 the glass slide, and regions 3-6 correspond to the overlaps of different number of cellophane layers; Fig (b) shows a lateral view of the layer's distribution.

For experimental validation, we performed a measurement employing a layered retardance phantom presented in Fig (2). We made the phantom by overlapping transparent cellophane tapes as commonly used in photoelasticity experiments due to the induced birefringence [22]. Figure 2 (a) shows an acquired frame, where each region is labeled for reference. Region 1 corresponds to air, region 2 corresponds to the glass slide, and regions 3 to 6, the corresponding overlap of different number of layers of the cellophane tape as shown in Fig 2 (b).

Figure 3 (a) shows the intensity modulation obtained from region 1 corresponding to air (red line) and region 5 corresponding to cellophane tape (blue line). Figure 3 (b-c) presents the Fourier coefficients a_k and b_k used for the retardance calculation.

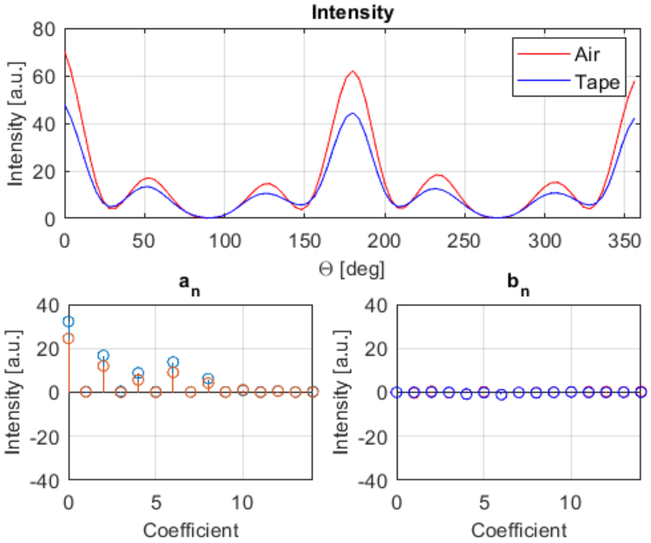

Fig 3. Intensity modulation obtained at region 1 – air (red line) and region 5 (blue line) and its corresponding Fourier series coefficients a_k, b_k.

Figures 4 (a) and (b) show the spatial distribution of the fast axis orientation $4\theta_s$ and linear retardance parameter $\delta_s/2$. It can be noted that the retardance in the air and glass remains minimum while in the layered parts vary depending on the analyzed region.

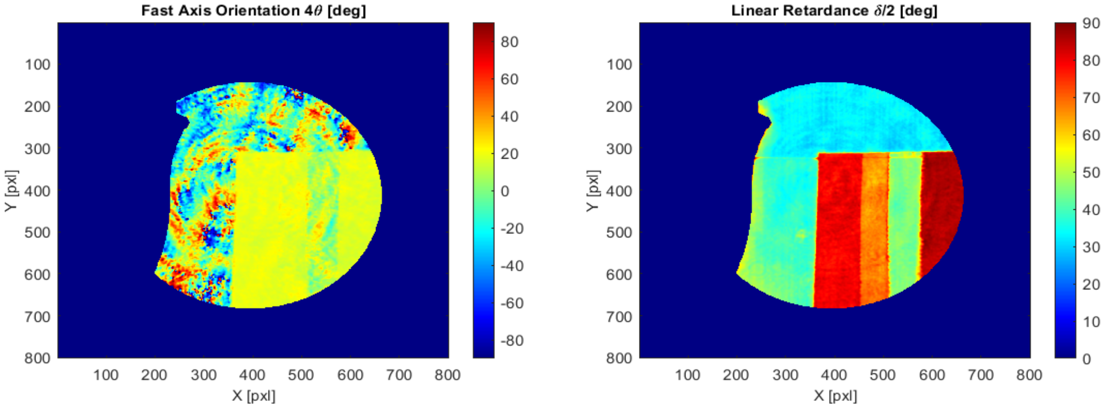

Fig 4. Fast axis orientation and linear retardance information of the sample.

4 Conclusion and Final Remarks

We developed a linear retardance-sensitive polarimeter based on a rotating polarizer-analyzer configuration. One of the main advantages of our proposal is that we acquire the retardance information of the sample without using retarders which are highly wavelength-dependent. As a result of this improvement, our proposal has a potential usage for a large bandwidth instrument implementation.

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Prof Geliztle Alejandra Parra-Escamilla obtained her Ph.D. in Innovation System Engineering focused on Optics from Utsunomiya University in Japan. She is member of the National System of Researchers (SNI). Prof Parra occupied a postdoctoral position at the University of Guadalajara in the Electronics and Computing Division, developing mathematical models and algorithms for 3D reconstruction techniques. She had the opportunity to collaborate on different projects with companies and the agriculture department at Utsunomiya University. Prof Geliztle has a number of peer-reviewed publications.

In 2008, Prof Joel obtained the degree of Engineer in Physics from the Institute of Physics of the University of Guanajuato (UG) in Leon Guanajuato, Mexico. In 2010 he obtained the degree of Master of Science and later in 2014 the degree of Doctor of Science, both with specialization in Optics by the Center for Research in Optics A.C. (CIO) in Leon city, Guanajuato, Mexico. Upon completing his studies, Prof Joel did a three-year postdoctoral stay at the Center Research in Optics and Education (CORE) of the University of Utsunomiya, Japan, where he worked for two different laboratories (Tomography Laboratory of Optical Coherence and Holography Laboratory e Instrumentation). He currently has international collaborations with the University of Western Australia (UWA) and the Center Optical Research Center and Education CORE (Japan), and national level with the University of Guanajuato (UG) and the Center for Research in Optics A.C (CIO). Research areas of his Interests are : • Optical coherence tomography sensitive to Polarization, Polarization, Diffraction and instrumentation, Flight time.

Prof David Serrano received his Ph D in Optical Sciences from the Optical Research Center (CIO) in Mexico in 2014. From 2014 to 2017, David occupied a postdoctoral position in the Center for Optical Research and Education (CORE) at Utsunomiya University in Japan. Since 2018 he has been a researcher professor at Guadalajara University in Guadalajara, Jalisco, Mexico. His research interests cover phase dynamics measurements by employing interferometric techniques and related polarization measurements based on Jones and Mueller matrix approaches. He has published a number of papers in National and International journals. Prof Serrano is a Senior Member of the OPTICA society, previously known as OSA (Optical Society of America). e mail: david.serrano@academicos.udg.mx