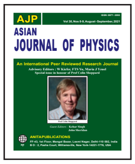

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On three-dimensional polarization states of light

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Dedicated to Prof CJR Sheppard

As the electric field vector of a monochromatic optical field oscillates in a plane, truly three-dimensional polarization states of light necessitate polychromatic fields. We consider random three-dimensional polarization states, and employing the characteristic decomposition of the spectral polarization matrix we assess various physical properties of such light states. These properties include polarimetric purity (degree of polarization), the concept and measure of nonregularity, apparent dimensionality, spin angular momentum, and various anisotropies of the state. Polarization states endowed with these features are typically encountered in connection with fluctuating vectorial evanescent waves and highly focused random fields.

Keywords: Polarized light, Degree of polarization, Nonregularity, Dimensionality.

1 Introduction

Complex structured states of coherence and polarization are at the core of modern near-field optics and nanophotonics. Whereas the coherence theory of two-dimensional (2D, beamlike) electromagnetic fields is relatively well understood both in the space–time and space–frequency domains [1,2], the corresponding theory of random three-dimensional (3D) vectorial electric fields is still under development. As regards 3D polarization of random fields, which is the topic of the current paper, considerable progress has taken place within the last two decades [3–6], including contributions by C J R Sheppard [7–9]. In particular, generalized Stokes parameters have been introduced in terms of the Gell-Mann matrices.

It is known that the electric field vector of a monochromatic electromagnetic field oscillates in a plane [10], but the oscillation ellipse's shape, size, and orientation may vary from point to point. The polarization state of monochromatic fields is thus intrinsically 2D in nature, although by spatial averaging 3D polarization conditions may arise. To achieve a genuinely 3D polarization state, i.e., one which at a given point in any laboratory coordinate frame contains three orthogonal electric field components, we consider polychromatic, statistically stationary, random electric fields that are partially polarized [3]. We adopt the spectral representation and analyze general 3D polarization states and their associated properties locally at a single frequency. The properties in which we are interested include polarimetric purity, the novel concept of nonregularity, effective dimensionality of the state, and the spin (angular momentum) associated with the electric field. We show that polarization states of totally symmetric intensity may nonetheless contain a high amount of spin and that the state's nonregularity influences the magnitude and direction of the spin vector. The properties addressed here are commonly found with random evanescent electromagnetic waves in nanophotonics and with high-numerical-aperture electric fields in advanced optical instrumentation.

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2 Representation and purity of 3D polarization states

A cornerstone in random 2D vectorial fields is that their polarization is unambiguously expressible as a sum of a fully polarized and a completely unpolarized state [1]. This result does not, however, hold for arbitrary 3D polarization states, but is replaced by the so-called characteristic decomposition. We let $\mathbf{R}(\mathbf{r}, \omega) = \langle \mathbf{E}^*(\mathbf{r}, \omega)\, \mathbf{E}^{\mathrm{T}}(\mathbf{r}, \omega)\rangle$ be the spectral polarization matrix of a random 3D field at a position $\mathbf{r}$ and (angular) frequency ω. Here the three-component column vector $\mathbf{E}(\mathbf{r}, \omega)$ is a realization representing the electric field and the angular brackets stand for the ensemble average (the asterisk and superscript T denote complex conjugate and transpose, respectively). The 3×3 matrix $\mathbf{R}$ is Hermitian and nonnegative definite [11]. Thus, it can be diagonalized by means of a unitary matrix $\mathbf{U}$ such that $\mathbf{R} = \mathbf{U}\,\mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)\mathbf{U}^\dagger$, where the dagger denotes Hermitian conjugate and $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0$ are the eigenvalues of $\mathbf{R}$. The characteristic decomposition of $\mathbf{R}$ now readily follows in the form [12]

$$\mathbf{R} = I\,[P_1\mathbf{R}_p + (P_2 - P_1)\mathbf{R}_m + (1 - P_2)\mathbf{R}_u], \tag{1}$$

where $I = \lambda_1 + \lambda_2 + \lambda_3 = \mathrm{tr}\,\mathbf{R}$ is the optical intensity. Additionally, $\mathbf{R}_p = \mathbf{U}\,\mathrm{diag}\,(1, 0, 0)\,\mathbf{U}^\dagger$, $\mathbf{R}_m = \mathbf{U}\,\mathrm{diag}\,(1, 1, 0)\,\mathbf{U}^\dagger/2$, and $\mathbf{R}_u = \mathbf{U}\,\mathrm{diag}\,(1, 1, 1)\,\mathbf{U}^\dagger/3 = \mathbf{I}/3$, and $\mathbf{I}$ is the 3×3 identity matrix. Furthermore,

$$P_1 = (\lambda_1 - \lambda_2)/I, \tag{2}$$

$$P_2 = (\lambda_1 + \lambda_2 - 2\lambda_3)/I = (I - 3\lambda_3)/I. \tag{3}$$

The quantities P_1 and P_2, which obey $0 \leq P_1 \leq P_2 \leq 1$, are known as the indices of polarimetric purity [13]. Equation (1) expresses $\mathbf{R}$ generally as an incoherent superposition of three matrices, of which $\mathbf{R}_p$ represents a pure state (only 1 nonzero eigenvalue) and $\mathbf{R}_u$ is a maximally mixed 3D state (3 equal eigenvalues). Provided the two smallest eigenvalues are not equal (i.e., if $\lambda_2 \neq \lambda_3$), a third matrix $\mathbf{R}_m$ also appears in Eq (1). This matrix is called the discriminating component of $\mathbf{R}$. It is central to our subsequent analysis and leads to several important consequences.

An immediate consequence of the characteristic decomposition of $\mathbf{R}$ in Eq (1) is the physical difficulty of assessing the 3D state's degree of polarization. Obviously the pure state $\mathbf{R}_p$ must be fully polarized, while the maximally mixed state $\mathbf{R}_u$ should be fully unpolarized. But what about the discriminating state $\mathbf{R}_m$ – as it is not completely polarized or completely unpolarized, then what is it [3]? Because of the term $\mathbf{R}_m$, the degree of polarization for fields in 3D polarization states can no longer be unambiguously defined, unlike with beamlike electromagnetic fields [1], as the ratio of the fully polarized intensity to the total intensity. Instead, the degree of polarization might be specified by the two indices of polarimetric purity, P_1 and P_2 in Eqs (2) and (3) [12]. If a single number is desired, general 3D polarization states appear best characterized by means of a quantity P_{3D}, defined as

$$P_{3D} = \sqrt{\frac{1}{2}\left[\frac{3\mathrm{tr}(\mathbf{R}^2)}{\mathrm{tr}^2(\mathbf{R})} - 1\right]}, \tag{4}$$

and called the degree of polarimetric purity [14] (earlier known as the degree of polarization [3]). This concept is a measure of how far the polarization state $\mathbf{R}$ in question is from the fully random 3D state (proportional to $\mathbf{I}$) [15]. For any pure state, $P_{3D} = 1$. A 3D maximally mixed state thus is characterized by $P_{3D} = 0$, whereas a 2D unpolarized state assumes $P_{3D} = 1/2$, representing 3D partial polarization [3]. It is of interest to note in passing that if $P_1 = P_2$ (i.e., if $\lambda_2 = \lambda_3$), the discriminating term $\mathbf{R}_m$ is absent in Eq (1) and the polarimetric purity of Eq (4) assumes the physical interpretation as the ratio between the intensity of the fully polarized part and the total intensity of the state [16].

On comparing 2D and 3D polarization formalisms, we can draw an interesting conclusion. Since the degree of polarimetric purity associated with the states of beamlike fields varies from $P_{3D} = 1$ (2D fully polarized beam) to $P_{3D} = 1/2$ (2D fully unpolarized beam), it follows at once that any value of polarimetric purity in the range $0 \leq P_{3D} < 1/2$ identifies a genuinely 3D polarization state. A field in such a state cannot be characterized by the conventional 2D polarization formalism but instead necessitates the comprehensive 3D representation.

3 Concept and degree of nonregularity

We saw that the discriminating component $\mathbf{R}_m$ in Eq (1) created confusion when we were trying to quantitatively identify a degree of polarization for the state. Another, quite remarkable consequence of the same middle term $\mathbf{R}_m$ is that it enables us to classify all 3D polarization states into two categories: (a) regular states, if $\mathbf{R}_m$ is real, and (b) nonregular states, if $\mathbf{R}_m$ is complex [12]. In the former case, $\mathbf{R}_m$ is a 2D unpolarized state (an equiprobable mixture of two orthogonal pure states with their polarization ellipses in the same plane). In the case of nonregular states, on the other hand, $\mathbf{R}_m$ is an equal mixture of two mutually orthogonal pure states whose polarization ellipses lie in different planes. Physically this means that even if the matrix $\mathbf{R}_m$ has two equal eigenvalues with the third eigenvalue being zero [see Eq (1)], it does not represent 2D unpolarized light, but rather the electric field oscillates in three Cartesian dimensions in the laboratory coordinate frame. An illustration of such an $\mathbf{R}_m$, composed of an equal incoherent superposition of a circular and a linear polarization state, is shown in Fig 1.

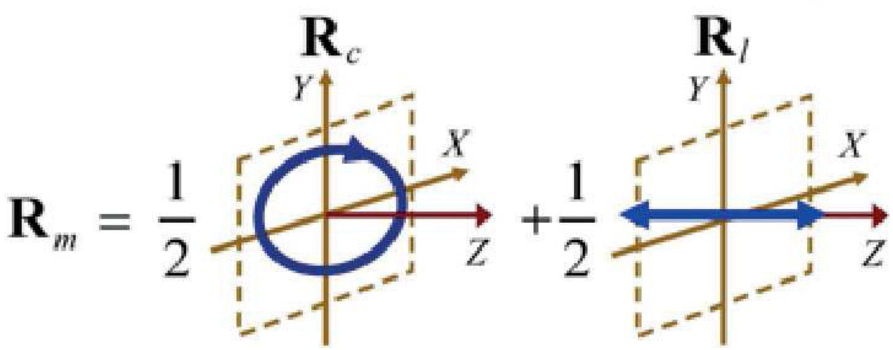

Fig 1. Perfect nonregular state. The matrix $\mathbf{R}_m$ consists of an equiprobable superposition of unit-intensity circularly ($\mathbf{R}_c$) and linearly ($\mathbf{R}_l$) polarized states, with their electric fields oscillating in perpendicular planes. The total intensity of the mixture is tr $\mathbf{R}_m = 1$ [14]. The state $\mathbf{R}_m$ is a maximally nonregular state, with its degree of nonregularity $P_N = 1$ [17].

After having distinguished nonregular states from the regular ones, i.e., from those for which the component $\mathbf{R}_m$ is a 2D unpolarized state (its polarization ellipse evolves fully randomly in a fixed plane), we can next identify the so-called perfect nonregular state and introduce the degree of nonregularity as a measure of the nonregular state's proximity to regularity. Regular states thus constitute the limiting case of nonregular states of polarization. To examine the nature of nonregularity in closer detail, we consider an arbitrary discriminating component $\mathbf{R}_m$ and especially its real part. One can show that Re($\mathbf{R}_m$) has three eigenvalues, of which the smallest ranges between $0 \leq m_3 \leq 1/4$. When m_3 takes on its maximum value, the discriminating component reads as [17]

$$\mathbf{R}_m = \frac{1}{2}\begin{pmatrix} 1/2 & \pm i/2 & 0 \\ \pm i/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

We observe that $\mathbf{R}_m$ is the polarization matrix of an equiprobable mixture of a circularly polarized state in the (X, Y) plane, with the upper (lower) sign corresponding to right-hand (left-hand) orientation, and a linearly polarized state in the Z direction (see Fig 1).

The constraint $0 \leq m_3 \leq 1/4$ above leads to a variety of situations with different proximities to regularity ($m_3 = 0$). Thereby, it is natural to introduce the degree of nonregularity of a general 3D polarization state $\mathbf{R}$, denoted by P_N, in terms of the smallest eigenvalue m_3 of $\mathrm{Re}(\mathbf{R}_m)$, appropriately scaled by the relative weight $P_2 - P_1$ of $\mathbf{R}_m$ in the characteristic decomposition [Eq (1)]. Hence, we set [17]

$$P_N^m = 4m_3, \tag{6}$$

$$P_N = (P_2 - P_1)\, P_N^m, \tag{7}$$

where P_N^m characterizes the nonregularity of $\mathbf{R}_m$. Since $P_2 - P_1 \leq 1$ and $P_N^m \leq 1$, the degree of nonregularity satisfies $0 \leq P_N \leq 1$, with the minimum $P_N = 0$ always, and only, taking place for regular states. The maximum value $P_N = 1$, associated with maximally nonregular polarization states, is saturated solely when $P_1 = 0$, $P_2 = 1$, and $m_3 = 1/4$. Thus the case $P_N = 1$ refers exclusively to states for which $\mathbf{R} = I\mathbf{R}_m$ with $m_3 = 1/4$. This implies that all maximally nonregular polarization states are perfect nonregular states, as indicated in Fig 1.

4 Dimensionality and polarimetric dimension

While a monochromatic electric field is strictly confined to a plane, the electric field associated with a totally random 3D polarization state, such as blackbody radiation, oscillates equally in all three Cartesian laboratory dimensions. Any random 3D polarization state $\mathbf{R}$, be it regular or nonregular, may be assigned a polarimetric dimension D, which is different from the actual dimensionality but instead characterizes the apparent physical dimensionality of the oscillating electric field vector. It is defined in terms of the principal intensities, which are the eigenvalues of the real part of the matrix $\mathbf{R}$. More specifically, we let $\mathbf{Q}$ be an orthogonal transformation (real-valued 3×3 matrix which can be viewed as a rotation of the Cartesian reference frame) that diagonalizes $\mathrm{Re}(\mathbf{R})$ such that [17,18]

$$\mathbf{R}_Q = \mathbf{Q}^{\mathrm{T}}\, \mathbf{R}\mathbf{Q} = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} + \frac{i}{2}\begin{pmatrix} 0 & n_3 & -n_2 \\ -n_3 & 0 & n_1 \\ n_2 & -n_1 & 0 \end{pmatrix}, \tag{8}$$

where the eigenvalues $a_1 \geq a_2 \geq a_3 \geq 0$ of $\mathrm{Re}(\mathbf{R})$ are the principal intensities and the vector $\mathbf{n} = (n_1, n_2, n_3)^{\mathrm{T}}$ is the spin (angular momentum) vector [19]. The matrix $\mathbf{R}_Q$ represents the same polarization state as $\mathbf{R}$, but now in this so-called intrinsic reference frame [20].

If all the eigenvalues a_j, $j \in \{1, 2, 3\}$, are positive, then the intensity of each Cartesian field component is nonzero for any orientation of the laboratory coordinate frame and the electric field vector fluctuates in all three dimensions. The actual physical dimensionality of the light field is thereby determined as follows [18]:

$$\text{1D light: } a_1 > 0,\ a_2 = 0,\ a_3 = 0,$$

$$\text{2D light: } a_1 > 0,\ a_2 > 0,\ a_3 = 0,$$

$$\text{3D light: } a_1 > 0,\ a_2 > 0,\ a_3 > 0.$$

It is clear from the categories above that isotropic 2D light has $a_1 = a_2$ and isotropic 3D light satisfies $a_1 = a_2 = a_3$. Another useful conclusion is that the condition $\det[\mathrm{Re}(\mathbf{R})] > 0$, identifies a genuinely 3D polarization state.

To characterize the dimensional nature of the polarization state more illustratively than the plain physical classification alone, we introduce the spectral polarimetric dimension D as a quantitative measure of the apparent state's dimensionality, by the expressions [18]

$$D = 3 - 2d, \tag{9}$$

$$d = \sqrt{\frac{1}{2}\left[\frac{3\mathrm{tr}[\mathrm{Re}(\mathbf{R})]^2}{\mathrm{tr}^2[\mathrm{Re}(\mathbf{R})]} - 1\right]}. \tag{10}$$

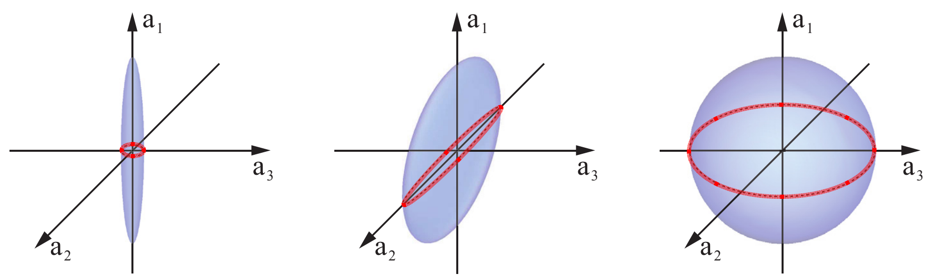

Fig 2. Illustrations of principal-intensity distributions for different types of genuine 3D polarization states: $D \approx 1$ (left), $D \approx 2$ (middle), $D \approx 3$ (right). The profiles appear cigar-shaped, disk-like, and nearly spherical, respectively, but nonetheless all correspond formally to random 3D electric fields [18]. The red lines represent the intersections of the distributions in the (a_2, a_3) plane.

The quantity d, called the dimensionality index [21], is defined as the distance between the matrix Re($\mathbf{R}$) and the 3×3 identity matrix $\mathbf{I}$. It varies continuously and monotonically in the interval $0 \leq d \leq 1$. Hence, the polarimetricric dimension is a real number, not necessarily an integer, in the range $1 \leq D \leq 3$. Values $D > 2$ are clear signatures of 3D light, and the maximum $D = 3$ is reached if, and only if, the polarization state is complely 3D isotropic. To emphasize the role of Re($\mathbf{R}$) in determining the light field's dimensionality, we note that, for example, the polarization state shown in Fig 1 has only 2 nonzero eigenvalues of $\mathbf{R}$, but all 3 eigenvalues of the real-valued matrix Re($\mathbf{R}$) are different from zero; therefore the superposition state is genuinely 3D in character. Physically truly 3D fields may effectively appear nearly 1D, 2D, or 3D, and this behavior is clearly reflected in the value of their polarimetric dimension D, as is demonstrated in Fig 2.

5 Spin

The notion of spin (intrinsic angular momentum) has in recent years gained interest in nanopho- tonics owing to various spin-orbit interactions and the observation that evanescent waves carry transverse spin [22,23]. The spin of an electric field is determined by the amount of circular polarization associated with its polarization state, obtained from the imaginary part of the corresponding polarization matrix $\mathbf{R}$ [19,24]. Indeed, we already saw in connection with Eq (8) that the spin vector may be represented as $\mathbf{n} = (n_1, n_2, n_3)^{\mathrm{T}}$, where n_1, n_2, and n_3 are given by the off-diagonal elements of Im($\mathbf{R}_Q$). We illustrate briefly two key consequences of the spin associated with 3D polarization states.

We consider first 3D polarization states that are totally isotropic as regards their intensity distribution. The maximally mixed state $\mathbf{R}_u$ in Eq (1) obviously is such a state. It is completely unpolarized with $P_{3D} = 0$. However, other intensity-isotropic states exist which contain varying levels of spin, characterized by the degree of circular polarization $P_c = |\mathbf{n}|/I$ [21,24]. It turns out that 3D polarization states, whose intensity is totally symmetric ($D = 3$), may be endowed with circular polarization (spin) in the range $0 \leq P_c \leq 2/3$ and, correspondingly, their overall polarimetric purity (degree of polarization) then is bounded between $0 \leq P_{3D} \leq 1/\sqrt{3}$ [21]. An intensity-isotropic 3D state is thereby not usually unpolarized, but may in fact be strongly spin-anisotropic and possess a relatively high degree of polarimetric purity. It can further be shown that a completely intensity-isotropic state ($D = 3$) with maximum spin ($P_c = 2/3$) can always be represented as an incoherent mixture of a circularly polarized state and an orthogonal linearly polarized state, with relative weights 2/3 and 1/3, respectively [21]. Remarkably, the same incoherent composition, but with equal weights of 1/2 each, represents the perfect nonregular state, shown in Fig 1.

Next, we consider the spin vector $\mathbf{n}$ associated with a 3D polarization state in terms of the characteristic decomposition of $\mathbf{R}$ given in Eq (1). As we have indicated, for regular states the discriminating component $\mathbf{R}_m$ takes the form of a 2D unpolarized state. Since unpolarized states, whether 2D or 3D such as $\mathbf{R}_u$, contain no spin, the spin vector of a regular 3D polarization state is that of the pure component $\mathbf{R}_p$ alone, i.e., $\mathbf{n} = P_1\mathbf{n}_p$. In the case of nonregular states, the discriminating component $\mathbf{R}_m$ exhibits both linear and circular polarization [17]. The spin vector $\mathbf{n}$ of the state $\mathbf{R}$ then is composed as

$$\mathbf{n} = P_1\mathbf{n}_p + (P_2 - P_1)\,\mathbf{n}_m, \tag{11}$$

where $\mathbf{n}_m$ is the spin vector of the discriminating component $\mathbf{R}_m$. This expression advances the main point, namely, that the total spin of a 3D polarization state, in general, has two contributions: one due to the pure polarization component and the other due to the nonregular discriminating component. Hence, a nonregular state always carries nonzero spin [21]. And both the magnitude and the direction of the spin vector $\mathbf{n}$ are regulated by the state's degree of nonregularity, as is analyzed and illustrated in detail in [24].

6 Examples

The concepts and properties we have elucidated bear relevance in near-field optics and nanophotonics, where fluctuating evanescent waves and tightly focused fields frequently appear. An electromagnetic evanescent wave is typically created by total internal reflection of an incident plane wave, and then the polarization matrix of the evanescent field above the surface can be explicitly evaluated [18,25]. It has been shown that $\det[\mathrm{Re}(\mathbf{R})] = 0$ only when the excitation beam is totally polarized, in which case the resulting evanescent wave is either 1D or 2D in character. However, when the incident beam is partially polarized, it turns out that $\det[\mathrm{Re}(\mathbf{R})] > 0$, corresponding to genuinely 3D light. This implies that electromagnetic evanescent waves are predominantly 3D light fields, whose polarization necessitates a rigorous 3D treatment. The evanescent wave can further be shown, under appropriate conditions, to come close to fully intensity-isotropic 3D light with the polarimetric dimension approaching $D = 3$ [18]. For instance, at SiO_2 and GaP interfaces with air the polarimetric dimension may reach $D \approx 2.67$ and $D \approx 2.96$, respectively.

From the 3×3 polarization matrix $\mathbf{R}$ of the evanescent electromagnetic wave created via total internal reflection by an incident, partially polarized or unpolarized plane wave one may analytically extract the discriminating component $\mathbf{R}_m$ [25]. It is an equiprobable mixture of an *s*-polarized linear state and a *p*-polarized elliptic state, and therefore corresponds to a 3D nonregular field. Much like the polarimetric dimension above, the degree of nonregularity of the evanescent electromagnetic wave can be shown, in suitable circumstances, to approach the maximum allowed value $P_N \approx 1$ [25]. At a typical SiO_2–air interface the maximum is around $P_N \approx 0.71$, while for a higher-index GaP–air interface the degree of nonregularity may become as large as $P_N \approx 0.97$, representing a nearly perfect nonregular state (see Fig 1).

The concepts of spectral polarimetric dimension and nonregularity associated with 3D polarization states also appear in the focal fields of tightly focused vectorial light beams, as has been explored numerically using the Richards–Wolf method for incident beams in certain polarization and coherence states, including the fully 2D unpolarized beam [26].

7 Conclusions

Not only quantum light, whose polarization states necessarily are 3D states because of non-commuting Stokes operators, also fluctuating classical electromagnetic fields admit genuinely 3D polarization states. We have analyzed the polarimetric purity (partial polarization), actual and apparent dimensionality, regularity and nonregularity, as well as the spin (angular momentum) and isotropy of statistically stationary, random 3D light fields in the spectral domain. We made extensive use of the characteristic decomposition of a 3D polarization state. Our results provide new foundational and practical insights into classical 3D mixed polarization states and their dimensionality, nonregularity features, and spin structure. Nonregular evanescent waves reveal numerous polarimetric aspects of electromagnetic near fields, with potential influences in nanoscale surface optics.

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