



## Optimized off axis ellipse design

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*Dedicated to Prof (Dr) Daniel Malacara-Hernández*

An off-axis ellipse reflector is a surface that generates a perfect image of one focus onto the other. By adapting the properties of ellipse we tailored the optical design to obtain an aberrated free imaging system using the combination of conical constant, illuminating angle and ellipse section to meet specific design requirements. © Anita Publications. All rights reserved.

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### 1 Introduction

Parabolic and ellipsoidal mirrors are key components in various optical systems, such as telescopes, microscopes, and lighting devices, due to their ability to manage light with great precision [1]. Their main difference lies in their focusing characteristics: parabolic mirrors concentrate parallel rays to a single focal point, making them ideal for telescopes, satellite antennas, and solar concentrators. In contrast, ellipsoidal mirrors, with their two focal points, are particularly suitable for transferring light precisely between two locations such as optical fiber coupling, stage lighting, and certain types of projectors. These mirrors stand out for their precision and efficiency in light collection and direction. Compared to a parabolic mirror-based system, an ellipsoidal mirror-based system offers promising potential for controlling aberrations of the incident beam.

In modern optical systems, there is also the possibility of using off-axis elements. An off-axis optical system is an optical system in which light propagates through a path that does not coincide with the optical axis defined as the line passing through its foci. This path allows oblique illumination and light blocking by other components among others. In general, an off-axis optical system has the inconvenience that in general increases image aberrations.

Off-axis parabolic (OAP) mirrors produce perfect images for an image at infinite, or in other words, for a perfectly collimated light directed along the optical axis of the parabola it produces an aberration free image at its focus. For finite conjugate points, an off-axis ellipsoid (OAE) mirror finds similar aberration-free image formation [2].

OAE enable oblique illumination, converting a central illumination to an edge-focused pattern, as observed in dark-field microscopes or eye fundus cameras [3]. Reflective surfaces with such characteristics allows the use of multiple OAE in optical system design [4,5].

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Unlike a paraboloid or a hyperboloid, the ellipsoid allows point-to-point imaging optics and does not require extremely precise alignment to obtain images free of astigmatism. Additionally, alignment is simpler with other optical elements, such as an off-axis parabolic mirror (OAP), working as a relay element between focal points. This characteristic optimizes system efficiency and overall its optical performance.

In this paper, we present benefits and limitations of an OAE as a reflecting surface as point image formation element. As an image formation system, we pave the way for optimal OAE design using the combination of conical constant, illuminating angle and section of ellipse.

## 2 Reflecting properties of ellipses

The ellipse has unique reflective properties that make it highly valuable in optical designs, particularly in systems that require precise focusing of light. These properties arise from the geometric shape. It enables point-to-point imaging optics and does not require extremely precise alignment to produce an astigmatism-free image.

Parameters that define the ellipse are shown in Fig 1. Here,  $a$  and  $b$  represent the major and minor axes, respectively, and  $P$  is any point on its surface. Let,  $r_1$  be the distance between the first focus  $F$  of the ellipse and point  $P$ , and  $r_2$  the distance between the point  $P$  and the secondary focus ( $F'$ ). An ellipse is defined by the property that the sum of distances of any point on its surface to the foci is constant ( $r_1 + r_2 = 2a$ ).

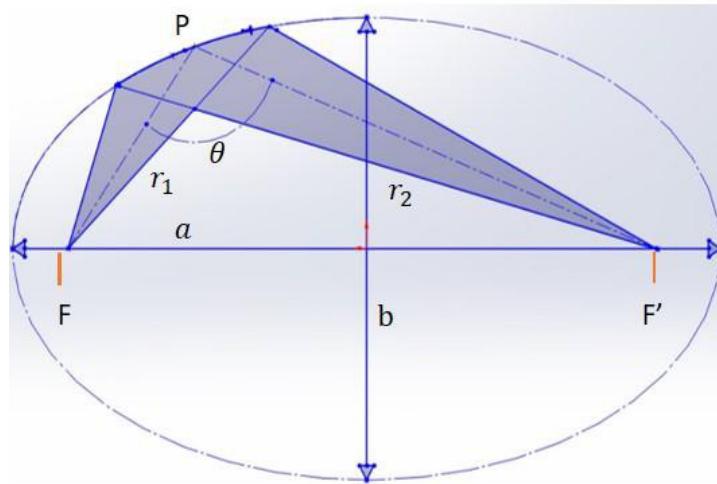


Fig 1. Ellipse construction

Without loss of generality, we position the foci symmetrically on the horizontal axis at  $(-c, 0)$  and  $(c, 0)$ . A more general treatment can be found in an analytical geometry book. Here, we are interested on the reflecting properties of an OAE. Its conic constant  $k$  and radius of curvature  $R$  are defined as:

$$k = -\varepsilon^2 = -\left(1 - \frac{b^2}{a^2}\right) \quad (1)$$

$$R = \frac{a^2}{b^2} \left( \frac{1 - \varepsilon^2(2 - \varepsilon^2)(\cos\theta)^2}{1 - \varepsilon^2(\cos\theta)^2} \right)^{3/2} \quad (2)$$

where  $\varepsilon$  is the eccentricity. The radius of curvature affects the dimensions of the ellipse and its ability to accurately focus light.

### 3 Off Axis design

With the basic ellipse elements, we can use an optical design program to calculate an OAE. An OAE is a surface that produces perfect point image and can be used to design an image formation systems [5]. We found that the OAE image formation capabilities are not ideal in most cases. There are some cases that the image formation system is optimized but in other cases it can produce an aberrated image.

First, we calculate the propagating capabilities of an optimized prolate OAE using the following parameters (Table 1).

Table 1. Optimized OAE parameters (Dimension in cm)

Oblate Curvature Radius	$r_1$	$r_2$	Semi-major axis (a)	Semi-minor axis (b)
-38.890873	20 cm	35 cm	2.355674665	27.5

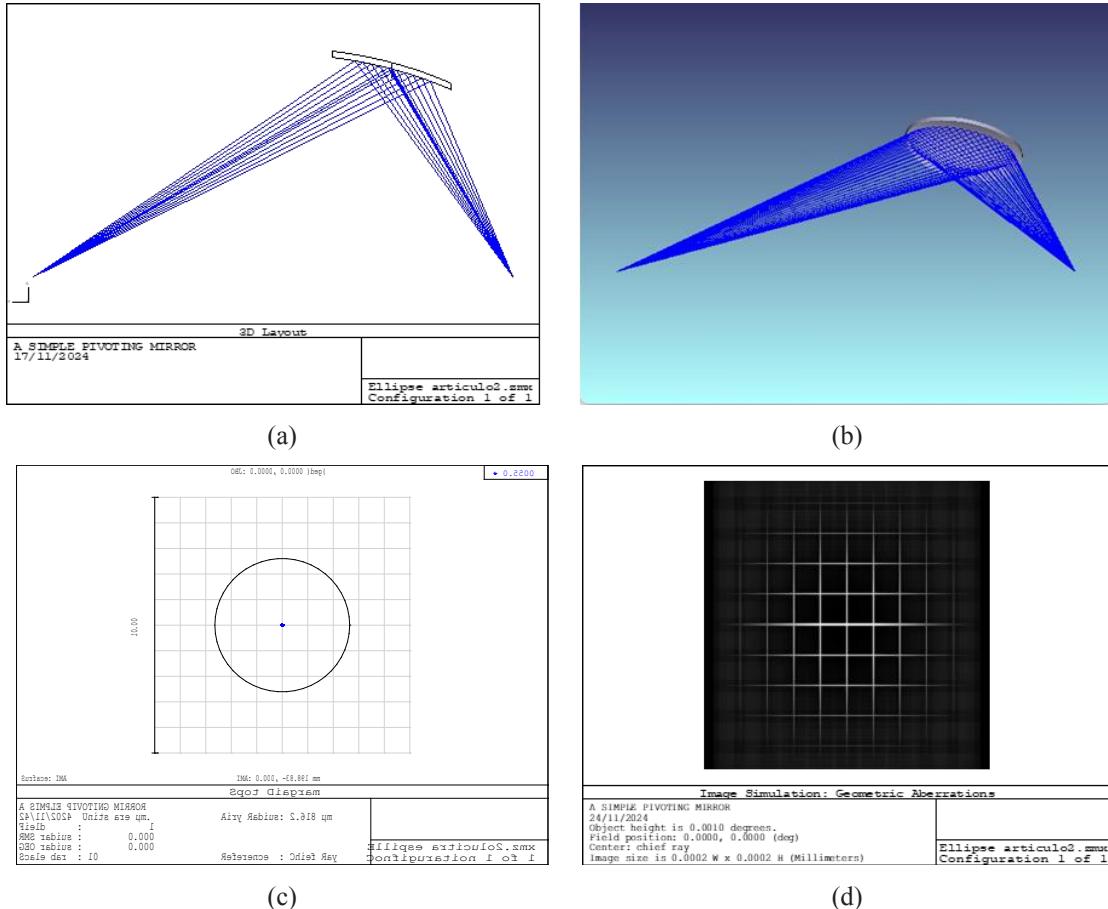


Fig 2. (a) Ray tracing on an OAE with the parameters of Table 1 at 25° over 10mm section, (b) volumetric propagation (c) Spot diagram showing the Airy disk (radius = 2.618 μm), and (d) Image propagation simulation.

Using these parameters (Table 1), we use Zemax® to obtain a complete hemisphere propagation. The resulting calculation is shown in Fig 2 in which we observe a perfect image of a point source at the center of Fig 2c and no distortion (Fig 2d). In this case, we can see a perfect image smaller than the expected Airy disk using a 486 nm, 587 nm and 656 nm illumination.

Using the same parameters as given in Table 1, we use illumination at  $25^\circ$  on a 10 mm section, thus calculating an OAE. In this configuration the light is incident at  $60^\circ$ . This selection enhances system precision and minimize unwanted optical aberrations. This configuration produces not only a perfect point image (Fig 2c) but also a perfect distortion free image (Fig 2d).

Table 2. OAE parameters (Dimension in centimeters)

Oblate Curvature Radius	$r_1$	$r_2$	Semi-major axis (a)	Semi-minor axis (b)
-26.1081458	20 cm	20 cm	.704088191	20

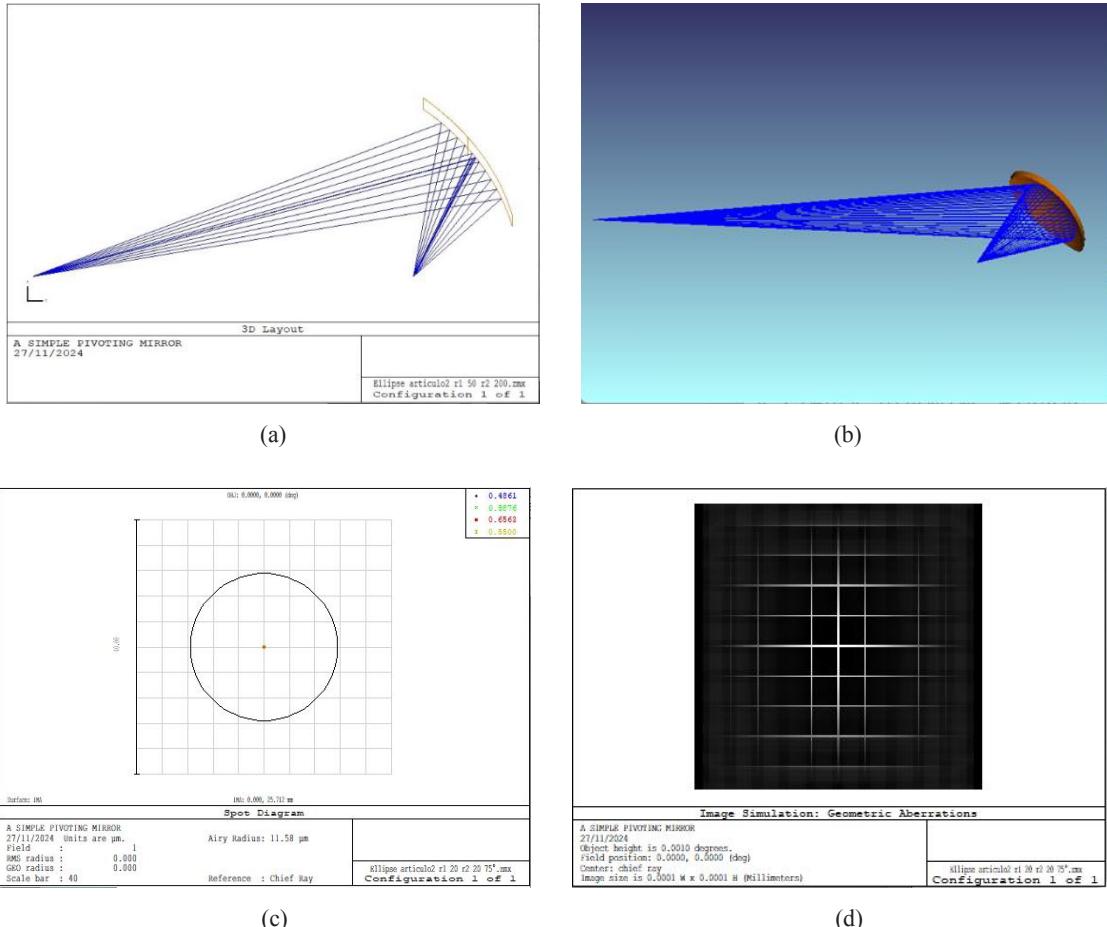


Fig 3. (a and b) Ray tracing on an OAE with the parameters of Table 2 at  $15^\circ$  over 10mm section, (b) volumetric propagation (c) Spot diagram showing the Airy disk (radius =  $11.580 \mu\text{m}$ ), and (d) Image propagation simulation.

Previous example is an OAE image optimized outcome. This is not the general case.

We can have an arbitrary OAE that produces fair imaging system. As an example we have an OAE calculated using the following design parameters (Table 3).

Table 3. OAE parameters (dimension in cm)

Oblate Curvature Radius	$r_1$	$r_2$	Semi-major axis (a)	Semi-minor axis (b)
-270.421225	50 cm	200 cm	3.6801698	27.5

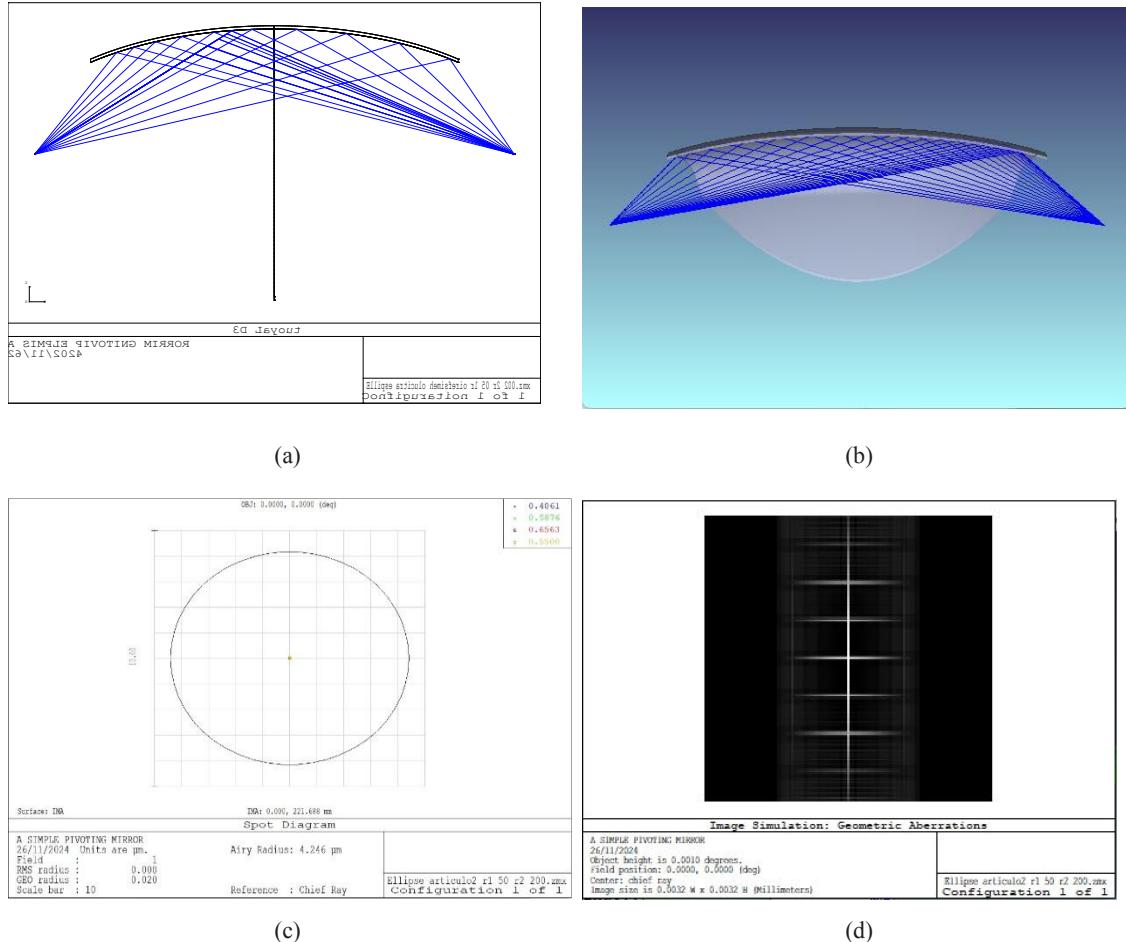


Fig 4. (a and b) Ray tracing on an OAE with the parameters of Table 1 at 25° over 10mm section, (b) Volumetric propagation (c) Spot diagram showing the Airy disk (radius = 4.426 μm), and (d) Image propagation simulation.

Using the parameters given in Fig 2, we use Zemax® to obtain a complete hemisphere propagation. The resulting image creates again a perfect point source image from a point source, as shown in Fig 4. In this case, we can see a perfect point image smaller than the expected Airy disk using a 486 nm, 587 nm and 656

nm illumination. However, the imaging capabilities of the OAE are not ideal and some distortion observed through the system (Fig 3d).

As in the first optimized example the point image is perfect but the image propagation presents a distorted outcome. In comparison to the optimized imaging system described before in Table 1, we now study the following OAE. In Table 2 we present an OAE operating in a non-optimal configuration.

Here, we use a 10 cm section at 45°. As in the previous cases, we obtain a perfect point image smaller than the expected Airy disk using a 486 nm, 587 nm and 656 nm illumination.

Contrary to the system depicted in Fig 3, here we obtain a large distortion on the optical system (Fig 4c) due to the combination of conical constant, illuminating angle and ellipse section used.

From the previous examples, we determine the existence of an optimal OAE reflector. There is only one incident angle onto an OAE reflector for which we can obtain both a perfect point image between its foci and an aberrated free image. This result has been obtained by numerical experimentation. We are currently working on an analytical expression to determine the optimal incident angle onto an OAE in which we observe an aberrated free image.

#### 4 Conclusions

OAE is a promising element for oblique illumination that produces ideal point image. In order to obtain an aberrated free image from an OAE the selection of the correct incident angle is significant. Nevertheless, the promising aberration free image is not as straightforward as expected due to the distortion created by an unoptimized OAE. We were able to obtain distortion free OAE by combining conical constant, illuminating angle and ellipse section used. Thus, the design of aberration free OAE opens a research avenue for developing feasible manufacturing techniques tailoring new requirements, reducing the costs and complexity while maintaining the desired optical functionality. By considering a specific segment of the ellipse as the system under study, the analysis focuses on eliminating unwanted reflections to meet design requirements. This approach optimizes optical performance and simplifies manufacturing by reducing the area of interest to a manageable region.

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