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A note on the Haidinger Fringes

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In memory of Professor Gallieno Denardo, ICTP Italy

This note gives historical perspective of the Haidinger fringes. These fringes were first observed with thin mica sheet illuminated with light from a broad source. After the arrival of laser, it has been used for testing parallelism of surfaces, angle of wedge plate etc.

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1 Introduction

Wilhelm Karl von Haidinger observed a system of fringes in a thin mica sheet in 1849 [1]. These fringes came to be known as Haidinger fringes and are also called fringes of equal inclination. These fringes are formed, both in reflection and transmission, when a broad source of light illuminates a thin plate of dielectric material. These are localized at infinity and can be seen with relaxed eyes or photographed at the focal plane of a lens. Raman and Rajagopalan observed the Haidinger fringes in bent mica sheets [2]. They also described the methods to view these fringes in transmission as well as in reflection. Further, they also pointed out that these fringes can also be observed when the plane parallel plate is illuminated with a diverging wave from a pinhole illuminated by a mercury lamp. Pfund studied the formation of Haidinger fringes in transmission from partially silvered faces of a thin mica sheet (0.07mm thick) and observed a system of two intersecting fringe patterns; one of them could be eliminated using polarized light [3]. Obviously, the fringes were sharp due to multi-reflections inside the mica sheet. With a thicker mica sheet (0.13mm thick), moiré between these two patterns was also observed. He also demonstrated that the fringes in the two patterns were elliptical with their major axes perpendicular to each other. Billings provided theoretical treatment to the observations of Pfund [4].

Fabry and Perot in 1897 demonstrated that very narrow Haidinger fringes are formed in transmission from a partially silvered plane parallel plate [5]. This eventually came to be known as Fabry-Perot interferometer or Fabry-Perot etalon and has many applications in optics and spectroscopy. The Haidinger fringes were forgotten. After the advent of laser, a couple of papers were published to measure the wedge angle of optical flat or to measure the parallelism of two surfaces [6-9]. These made use of the fringes of equal inclination obtained by illuminating the object with diverging spherical wave. None of the papers mentioned these as Haidinger fringes though Raman and Rajagopalan mentioned in their paper that Haidinger fringes can be seen when the mica sheet is illuminated with a spherical wave. However, in the first edition of the book *Optical Shop Testing*, several configurations of the setup to measure wedge angle are described and

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the fringes thus obtained are called the Haidinger fringes [10]. We used the Haidinger fringes to measure the parallelism of Brewster windows in 70's and 80's.

Zhiwen *et al* described a method to measure angular error and pyramidal error of a scanning prism using Haidinger fringes [11] . Testing accuracy of 0.5" was claimed. Parks et al. used Haidinger fringes to measure thickness and total thickness variation of silicon wafer [12]. They used 2mW DFB diode laser operating at 1.55μm and IR vidicon as a detector. The silicon wafer is illuminated with convergent beam and Haidinger fringes in transmission are analyzed. Murty presented a paper in the Optical Society of America meeting in which he described the use of Haidinger fringes to measure the thickness of dichromated gelatin films before and after exposure [13].

An interesting paper reports the use of Haidinger fringes to change the spatial structure of an incoherent source for controlling the temporal coherence [14]. Since spatial coherence is controlled, it offers better possibility in optical tomography and profilometry. Using a cyclic interferometer Chatterjee and Kumar obtained Haidinger fringes in white light [15].

Rue *et al* obtained the refractive indices of lithium niobate crystal [16]. They used 1mm thick x-cut plate and recorded the Haidinger interferograms in transmitted light using a polarizer suitably oriented. Park *et al* [17] and Lee et al [18] used Haidinger fringes to measure thickness profile and wavelength of the light. It is very diffilcult to obtain white light Haidinger fringes in a Michelson interferometer. Azhari and Tagmouti obtained white light Haidinger fringes using a Michelson interferometer by inserting a plane parallel plate in one arm of the interferometer [19]..

2 Theory

Illumination with a broad source

Haidinger fringes are formed when a thin dielectric plate is illuminated with a broad quasimonochromatic source at near normal incidence. These are localized at infinite and can be seen either with a relaxed eye or at the focal plane of a lens. Hecht has suggested that they can be seen in window glasses when properly illuminated with neon lamp [20]. A schematic of observing the Haidinger fringes in reflection is shown in Fig [1]. A quasi-monochromatic broad source illuminates the film via a beam splitter. The rays reflected from the front and back surface of the film interfere and the interference pattern is observed at the focal plane of a lens. These are the Haidinger fringes being observed at near normal incidence on the film.

The path difference Δ between these two rays shown in Fig 1, can be expressed as,

$$\Delta = 2\mu t\cos\theta_r + \frac{\lambda}{2} \tag{1}$$

where μ is the refractive index and t is the thickness of the film, θ_r is the angle of refraction in the film and λ is the wavelength of light. Constructive interference will take place when,

$$2\mu t\cos\theta_r + \frac{\lambda}{2} = m\lambda \tag{2}$$

where m is the fringe order.

When we consider another point on the source, the path difference is governed by the same equation in which θ_r and t would be different. Since, it is an incoherent source, the irradiance at point P will be the sum of irradiances due to interference from different point sources on the broad source. The total irradiance will be maximum when the path difference Δ is the same for rays coming from different points on the source to the point P at the focal plane. The change in path difference $d\Delta$ can be expressed as

$$d\Delta = 2\mu t\cos\theta_r\, dt - 2\mu t\sin\theta_r\, d\theta_r = 2\mu t\cos\theta_r\, dt - 2t\tan\theta_r \sin\theta_i\, d\theta_i \tag{3}$$

where θ_i is the angle of incidence.

For a plane-parallel plate, $dt = 0$. Thus, the irradiance at point P will be maximum when $d\theta_i = 0$. This amounts to saying that the interference pattern is localized at infinity.

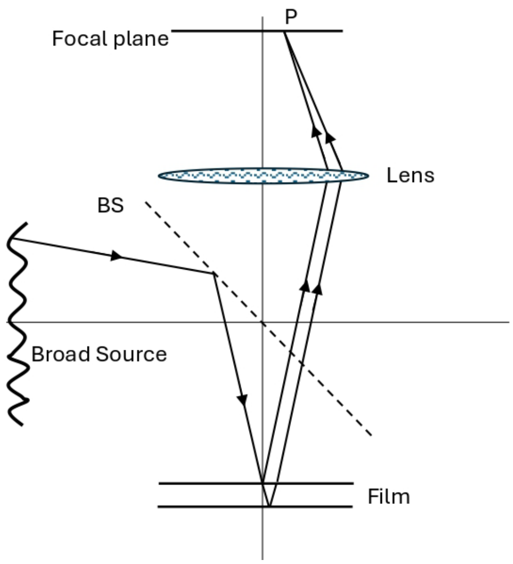

Fig 1. A schematic to observe Haidinger fringes.

From Eq (2),

$$\cos\theta_r = \frac{m'\lambda}{2\mu t} \rightarrow \rightarrow \sin\theta_r = \sqrt{1 - \left(\frac{m'\lambda}{2\mu t}\right)^2} \rightarrow \rightarrow \sin\theta_t = \sqrt{\mu^2 - \left(\frac{m'\lambda}{2t}\right)^2} \tag{4}$$

where m' is the fringe order for the dark fringes. This is the formula given in several papers [16,17]. It can, however, be recast in a simpler form but quite instructive when we consider near normal incidence as:

$$\sin\theta_i = \sqrt{\left(\frac{m_0\lambda}{2t}\right)^2 - \left(\frac{m'\lambda}{2t}\right)^2} = \frac{\lambda}{2t}\sqrt{m_0^2 + m'^2} \tag{5}$$

where m_0 is the fringe order for destructive interference for normal incidence. It should be noted that the fringe order is maximum for $\theta_i = 0$, and decreases for the increasing angles of incidence.

The angular width $d\theta_i$ of the dark fringes can be obtained from Eq (4) as:

$$d\theta_i = \frac{\mu\lambda}{t}\frac{\cos\theta_r}{\sin 2\theta_i} \tag{6}$$

The fringes are very broad at normal incidence and also at the grazing incidence (θ_i approaching 90°). The minimum width occurs when the term ($\cos\theta_r / \sin 2\theta_i$) takes the minimum value. It occurs when the angle of incidence is such that

$$\tan^4\theta_i = \frac{\mu^2}{\mu^2 - 1} \tag{7}$$

For a glass plate of refractive index 1.512, this occurs at an angle of incidence 49.11° and for mica (μ =1.56) at $\theta_i = 48.78°$. Angular width at this angle for a mica sheet of 0.1mm thickness at the wavelength of 633nm will be 873×10^{-5} rad.

When the Haidinger fringes are observed in transmission, the governing equation is,

$$2\mu t\cos\theta_r = m\lambda \tag{8}$$

Since the directly transmitted beam is much stronger than the beam reflected in the film and then transmitted, the fringe contrast is poor. However, the fringe contrast is very good when the film is coated [3,4].

An interesting case is when an air film is bounded by media of higher refractive indices. In that case, the condition for destructive interference is,

$$2t\cos\theta_r = m\lambda \tag{9}$$

Here, the refractive index of the air is taken as one. The angular width $d\theta_i$ is given by,

$$d\theta_i = \frac{\lambda}{t\mu^2}\frac{\cos\theta_r}{\sin 2\theta_i} \tag{10}$$

The fringes are broad at the normal incidence and their contrast keeps on decreasing with increase in the angle of incidence. This continues until the critical angle is reached.

Illumination by a wave from a point source

When a spherical wave illuminates a plane parallel plate at oblique incidence, the reflected waves from the top and bottom surfaces of the film are sheared and the interference takes place in the overlap region as shown in Fig 2. For the case when the film is thin and the radius of curvature of the illuminating spherical wave is large, straight line fringes are observed in the overlap region.

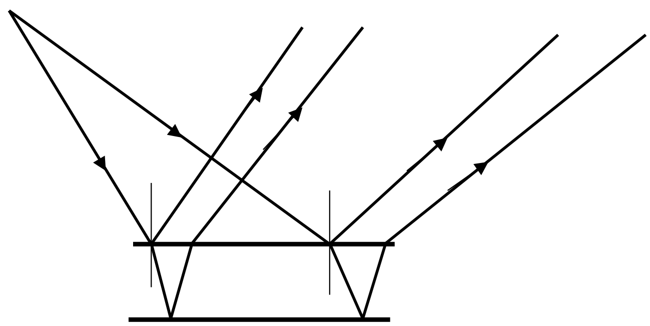

Fig 2. Oblique illumination by a spherical wave.

The shear will be zero when the angle of incidence is zero. Therefore, this configuration is chosen, and a laser light is used for illumination. A schematic of an experimental set up is shown in Fig 3.

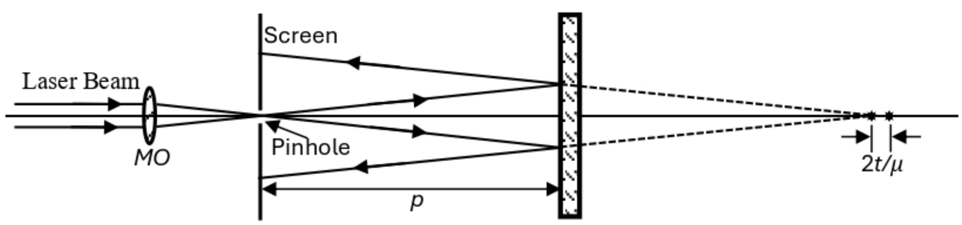

Fig 3. A schematic to observe Haidinger fringes in reflection when a plane parallel plate is illuminated by a spherical wave.

The laser beam is focused on a pin hole in the screen. The spherical wave is reflected from the front and back surfaces of the plane parallel plate, which is placed at a distance p from the screen. These two reflected spherical waves appear to originate from two virtual sources located at distances p and $p+2t/\mu$ behind the plane parallel plate. These distances are from the front surface of the plane parallel plate. The interference pattern between the two spherical waves of slightly different radii of curvature results in circular fringes centered on the pinhole.

Figure (4a &b) shows the ray paths when the observation is made in reflection and transmission respectively. In either case, the virtual sources are separated by $2t/\mu$.

The path difference between the rays from the virtual sources at any point $P\ (x, y, 0)$ on the screen is given by,

$$\Delta = \sqrt{(2p + 2t/\mu)^2 + x^2 + y^2} - \sqrt{(2p)^2 + x^2 + y^2}$$

$$= \left(2p + \frac{\theta}{2}\right)\left[1 + \frac{1}{2}\frac{x^2 + y^2}{(2p + 2t/\mu)^2}\right] - 2p\left[1 + \frac{1}{2}\frac{x^2 + y^2}{(2p)^2}\right]$$

$$\Delta \cong \frac{2t}{\mu} - \frac{t}{\mu}\frac{x^2 + y^2}{4p^2} \tag{11}$$

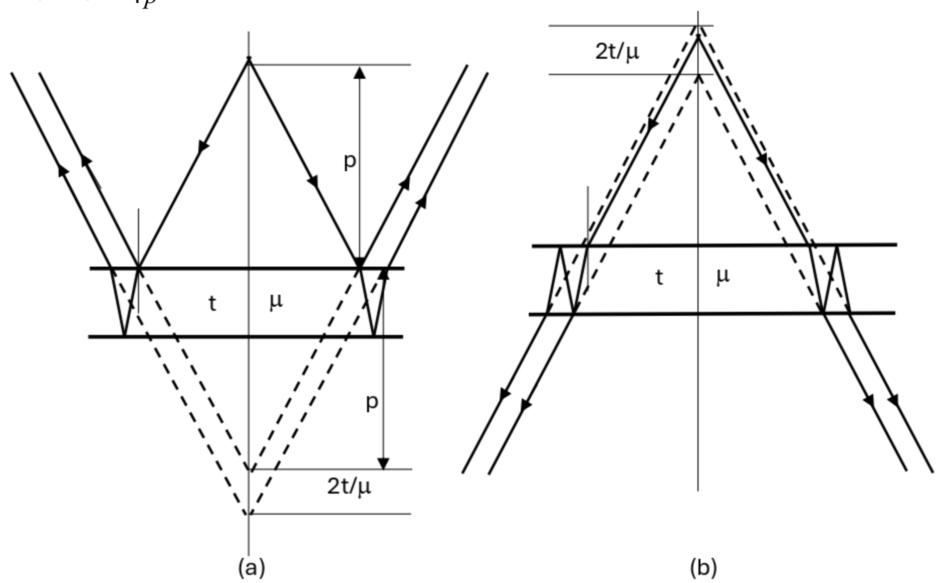

Fig 4. Formation of virtual sources (a) in reflection and (b) in transmission.

If m^{th} fringe is formed passing through point P, then,

$$\frac{2t}{\mu} - \frac{t}{\mu}\frac{x^2 + y^2}{4p^2} = m\lambda = m_0\lambda - m'\lambda \tag{12}$$

where m_0 is the order at the center and m' is the order of fringes measured from center, none of these are integer orders while $m_0 - m'$ is an integer order. If the diameter of the m^{th} fringe order is D_m, then [21]

$$\frac{2t}{\mu} - \frac{t}{\mu}\frac{D_m^2}{16p^2} = m\lambda \tag{13}$$

The fringes are circular and non-localized. This analysis is done under paraxial approximation.

Similar formulae apply when the observation is made in transmission.

There are several papers that have used these fringes to check the parallelism of surfaces [6,9], for aligning Fabry-Perot etalons [7], for measuring angle of a wedge plate [8]. Figure (5) shows a photograph of Haidinger fringes obtained from a 1 mm thick plane parallel plate of borosilicate glass.

However, if the plate has a small angle, that is, it is a wedge plate, then the fringe pattern is not centered on the pin hole but gets shifted. Figure (6) shows the formation of virtual sources when the wedge plate is illuminated with a spherical wave. Note that the virtual sources are no longer on the optical axis but one of them is displaced by $2\mu\alpha p$ from the optical axis. The fringe pattern center will lie on the line joining these virtual sources and hence is displaced towards the thicker end of the wedge plate. We have used the Haidinger fringes to check the parallelism of Brewster windows in early 70s when we were building our own lasers. Figure [7] shows a photograph of Haidinger fringe pattern when a wedge plate is illuminated by a spherical wave.

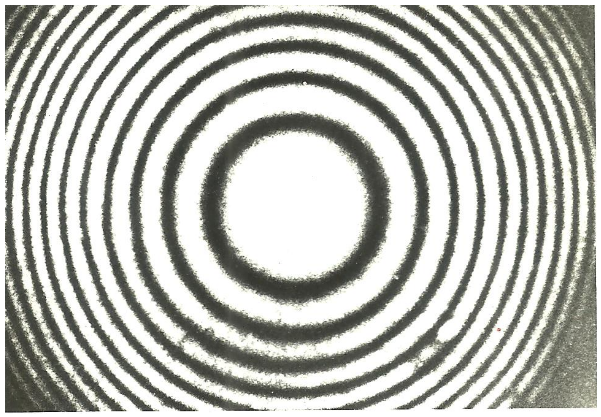

Fig 5. Haidinger fringes from plane parallel plate in reflection.

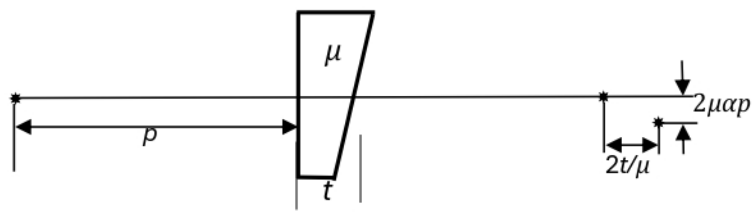

Fig 6. Formation of virtual sources by a wedge plate when illuminated by a spherical wave.

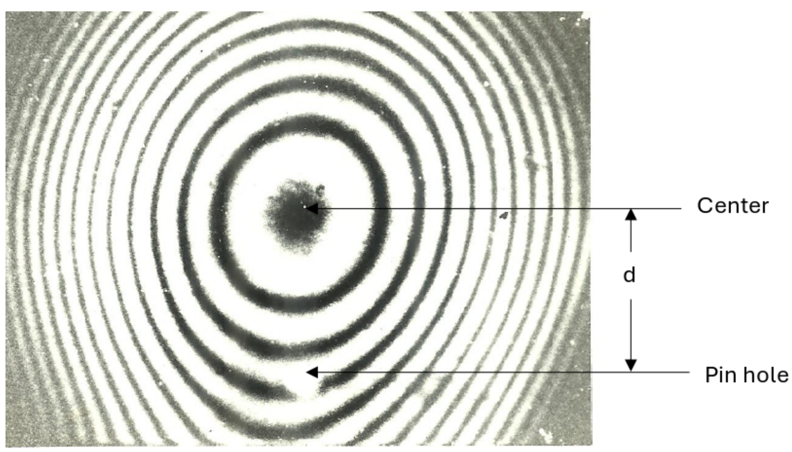

Fig 7. Haidinger fringes from a wedge plate in reflection.

If the distance between the pin hole and the center of Haidinger fringe pattern is d, then the angle α of the wedge is calculated from an approximate formula [21]

$$\alpha = \frac{D_m^2}{2\mu^2 p^2} \tag{14}$$

where t is the nominal thickness of the wedge. In the experiment carried out by us to measure the angle of a wedge plate made of borosilicate glass (μ=1.512), the nominal thickness of the wedge plate was 1.0mm, the distance p was 1000mm and d was 50mm. Inserting these values, we obtain the angle of the wedge plate α = 10.94μrad (= 2.26 arcsec).

3 Conclusion

Haidinger observed the fringes in mica in 1849. It is, however, not clear which light source he used for observation. Later studies have used mercury arc lamp or mercury lamp. The fringes are localized at infinity. It was also mentioned in an early paper that the fringes can also be observed when illuminated by a divergent beam from a point source of mercury light. Until the advent of laser, Haidinger fringes formation was just an optical phenomenon and no application was reported. However, with laser as source, it found several applications.

References

1. Haidinger W, *Pogg Ann*, 77(1849)219.
2. Raman C V, Rajagopalan V S, Haidinger's Rings in Curved Plates, *J Opt Soc Am*, 29(1939)413–416.
3. Pfund A H, Haidinger Interference in Silvered Mica, *J Opt Soc Am*, 32(1942)383–387.
4. Billings Bruce H, Haidinger Interference Fringes in Plane-Parallel Crystalline Plates, *J Opt Soc Am*, 35(1945), 570–573.
5. Fabry C, Pérot A, Sur les franges des lames minces argentées et leur application à la mesure de petites épaisseurs d'air, *Ann Chim Phys*, 12(1897)459–501.
6. Bergman T G, Thompson J L, An Interference Method for Determining the Parallelism of (Laser) Surfaces, *Appl Opt*, 7(1968)923–925.
7. Ford D L, Shaw J H, Rapid Method of Aligning Fabry-Perot Etalons, *Appl Opt*, 8(1969)2555–2556.
8. Leppelmeire G W, Mullenhof D J, A Technique to Measure the Wedge Angle of Optical Flats, *Appl Opt*, 9(1970) 509–510.
9. Wasilik J H, Blomquist T V, Willett C S, Measurement of Parallelism of the Surfaces of a Transparent Sample Using Two-Beam Nonlocalized Fringes Produced by a Laser, *Appl Opt*, 10(1971)2107–2112.
10. Malacara D (Editor), Optical Shop Testing, Chapter 1, (John Wiley & Sons, New Jersey), 1978.
11. Zhiwen Y, Xianglan X, Wei A, Measurement of scanning prism by Laser Haidinger interference, Proc SPIE 2899 (1996)432–437.
12. Parks R E, Shaob L, Davies A, Evans C J, Haidinger interferometer for silicon wafer TTV measurement, Proc SPIE 4344(2001)496–505.
13. Murty V M, Nondestructive thickness measurement of DCG films, Paper presented in OSA Annual Meeting FD3, (1992).
14. Wang W, Kozaki H, Rosen J, Takeda M, New principle for optical tomography and profilometry based on spatial coherence synthesis with spatially modulated extended light source, Proc SPIE 4596(2001)54–65.
15. Chatterjee S, Kumar Y P, Simple technique for generation of white-light Haidinger fringes with cyclic optical configuration, *Opt Lett*, 34(2009)1291–1293.
16. Ryu J Y, Choi H J, Lee C H, Jin J, Cha M, Estimation of Refractive Index of Crystal Plate from Haidinger Fringes, Conference on Lasers and Electro-Optics Pacific Rim, paper 28F2_2 (2015).
17. Park J E, Kim J, Cha M, Measurement of thickness profiles of glass plates by analyzing Haidinger fringes, *Appl Opt*, 56(2017)1855–1860.
18. Lee C, Park J E, Choi H, Kim J, Cha M, Jin J, Optical interferometry with transmitted Haidinger fringes through a plane parallel plate, Conference on Lasers and Electro-Optics Pacific Rim, paper Tu3F.6, (2018)
19. Azhari Y E, Tagmouti M, Haidinger-Michelson rings in white light, *Am J Phys*, 89(2021)721–729.
20. Hecht E, Optics, 4th Edn, (Addition Wesley, New York), 2002, pp 403.
21. Sirohi R S, Introduction to Optical Metrology, (2nd edn), Chapter 10, (CRC Press Boca Raton), 2025.

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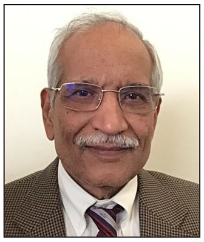

Having served as Professor of Physics at IIT Madras for more than two decades, Director of IIT Delhi and Vice Chancellor to several Universities in India, Distinguished Scholar to Rose-Hulman Institute of Technology, Terre Haute, USA, Chair Professor Tezpur University, Assam India, Faculty at Alabama A&M University, Huntsville, Alabama USA, Prof Rajpal Singh Sirohi is now retired. He spends his time reading books on history of science and spends mornings and evenings with his grandchildren. He is the recipient of many (21) awards such as Vikram and Gabor awards from SPIE, Padmashri from Govt of India etc. He is the Fellow of two academies and many societies including SPIE, OSA, OSI etc. Prof Sirohi's research areas are Optical Metrology, Optical Instrumentation, Laser Instrumentation, Holography and Speckle Phenomenon.