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Polarization of light, entanglement, EPR paradox & Bell's inequality: A simple analysis[1]

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Invited Article as a Tribute to Professor Gallieno Denardo

In this article, we will discuss entangled photons and violation of Bell inequalities which are areas of great importance. In order to understand entangled photons, we will discuss polarization of photons which will be followed by a discussion on entangled photons and violation of Bell inequalities. A major portion of this article has been adapted from the 8th edition of my book on OPTICS.

Preamble: 1999, Professor Gallieno Denardo at ICTP (which is now known as The Abdus Salam International Centre for Theoretical Physics) had asked me to coordinate The Winter College on OPTICS & PHOTONICS which was held at ICTP, Trieste, during February2000. I fondly remember my frequent interactions with Professor Denardoand his ever positive attitude towards all issues. It was indeed a great pleasure to interact with him. We were shocked and extremely saddened when we came to know of the sudden passing away ofProfessor Denardo on July 23, 2007.It was indeed very nice of ICO and ICTP to institute the ICO-ICTP Gallieno Denardo Award which "*recognizes the work of* ***young researchers from developing countries*** *who are active in optics and photonics research*" – the citation is so consistent with the vision of Professor Denardo. I am very grateful to Professor Maria Yzuel and ProfessorJoe Niemela to invite me to write this article in memory of Professor Gallieno Denardo. © Anita Publications. All rights reserved.

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1 Introduction

The 2022 Nobel Prize in Physics was awarded to Alain Aspect, John Clauser and Anton Zeilinger and the citation read:

> *for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science*

In this Tutorial, meant primarily for young students, we will discuss entangled photons, Bell inequalities and their violation; these are areas of great importance and in order to understand entangled photons, we will first discuss polarization of photons.

[1]During 2025, many webinars were organized by IAPT (The Indian Association of Physics Teachers) celebrating "100 Years of Quantum Mechanics". This write-up is based on the webinar delivered on January 29, 2025 (organized by Regional Council 7 of IAPT); the theme of the webinar was "History of Quantum Science". If you go to youtube.com and search for "Curtain Raiser: IYQST-2025 By IAPT RC-07", you will be able to reach the website https://www.youtube.com/live/MjHzI8c9GWI, where you can hear the entire recording of the event. This particular lecture starts at the 17th minute.

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2 Polarization of the photon

An x- polarized electromagnetic wave (propagating in the + z-direction) is described by the equations,

$$E_x(z, t) = E_0\, e^{i(kz - \omega t)} \text{ and } E_y(z, t) = 0 \tag{1}$$

We will represent such a wave by the vector

$$|\mathbf{E}\rangle = E_0\, |x\rangle \tag{2}$$

where $$|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3}$$

is the normalized Jones vector representing the x-polarized wave; the symbol $|x\rangle$ is read as "ket x" and represents an x-polarized wave. Jones calculus was introduced by R C Jones in 1941 (Ref. 1; see also Section 21.14 of Ref 2). Similarly, a y-polarized electromagnetic wave (propagating in the + z-direction) is described by the equations

$$E_x(z, t) = 0 \qquad \text{and} \qquad E_y(z, t) = E_0\, e^{i(kz - \omega t)} \tag{4}$$

and would be represented by the vector

$$|\mathbf{E}\rangle = E_0\, |y\rangle \tag{5}$$

where $$|y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{6}$$

is the normalized Jones vector representing the y-polarized electromagnetic wave. In writing $|\mathbf{E}\rangle$, the phase factor $e^{i(kz - \omega t)}$ is suppressed. Now, for a Right Circularly Polarized wave (usually abbreviated as a RCP wave) propagating in the z- direction, we may write,

$$E_x(z, t) = E_0\, e^{i(kz - \omega t)} \text{ and } E_y(z, t) = E_0\, e^{i(kz - \omega t + \pi/2)} = iE_0\, e^{i(kz - \omega t)} \tag{7}$$

Thus neglecting the common phase factor, the normalized Jones vector representing the RCP wave will be,

$$|\text{RCP}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \tag{8}$$

where the factor$1/\sqrt{2}$ normalizes the vector. Similarly, the normalized Jones vector representing a Left Circularly Polarized wave (usually abbreviated as a LCP wave) propagating in the z- direction, will be

$$|\text{LCP}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \tag{9}$$

We can express $|RCP\rangle$ and $|LCP\rangle$ as superposition of x and y-polarized waves with a phase difference; thus

$$|\text{RCP}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{\sqrt{2}} \left[|x\rangle + i\, |y\rangle\right] \tag{10}$$

and $$|LCP\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}} \left[|x\rangle - i\, |y\rangle\right] \tag{11}$$

Also $$|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{i\sqrt{2}} \left[|RCP\rangle + |LCP\rangle\right] \tag{12}$$

and $$|y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \left[|RCP\rangle - |LCP\rangle\right] \tag{13}$$

A linearly polarized light oriented at θ with respect to the x-axis will be given by,

$$|\text{LP } \theta\rangle = \cos\theta\, |x\rangle + \sin\theta\, |y\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{14}$$

An ordinary light beam (like the one coming from a sodium lamp or from our Sun) is unpolarized; i.e., the electric vector (in a plane transverse to the direction of propagation) keeps changing its direction

in a random manner as shown on the extreme left of Fig 1. We assume such a beam falling on a Polaroid P_1; a Polaroid is a plastic-like material used for producing linearly polarized light and it consists of aligned long chain molecules. The component of the electric field **E** along the direction of the aligned long chain molecules gets absorbed by the Polaroid and the component at right angles to it passes through; the direction of the electric vector of the emergent wave is usually called the pass axis of the Polaroid. Thus when an unpolarized light beam is incident on a Polaroid, the emergent light is linearly polarized with its electric vector oscillating in a particular direction as shown above; this direction is known as the pass-axis of the Polaroid. Obviously, the direction of the electric vector of the emergent beam will depend on the orientation of the Polaroid. If the Polaroid is rotated about the z-axis we will produce light which is polarized at an angle to the x-axis; thus we can produce y-polarized light or light polarized at an angle (60º or 45º) to the x-axis. We may mention here that in 1929, the American physicist Edwin Land invented the Polaroid; later in 1937 he founded The Polaroid Corporation. The company initially manufactured Polaroid Sunglasses.

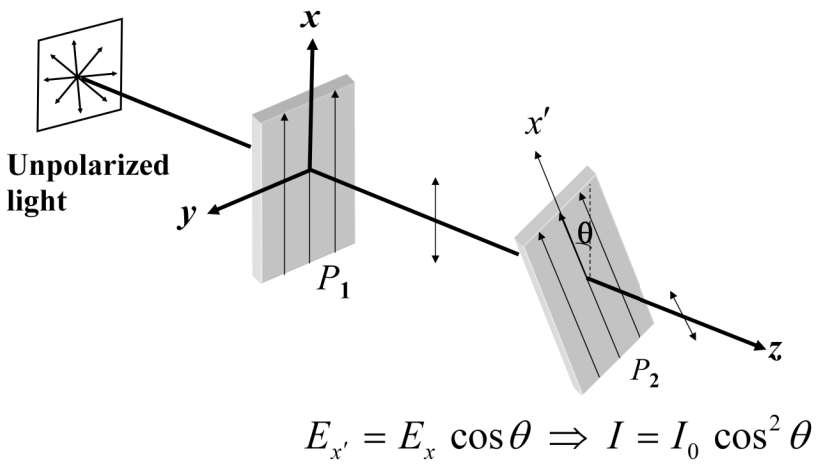


Fig 1. If an unpolarized light beam is allowed to fall on a Polaroid, then the emerging beam will be linearly polarized; i.e., the electric vector will oscillate along a particular direction. If we place another Polaroid P_2, then the intensity of the transmitted light will depend on the relative orientation of P_2 with respect to P_1; if the pass axis of the second Polaroid P_2makes an angle θ with the x-axisthe intensity of the emerging beam will vary as $\cos^2\theta$.

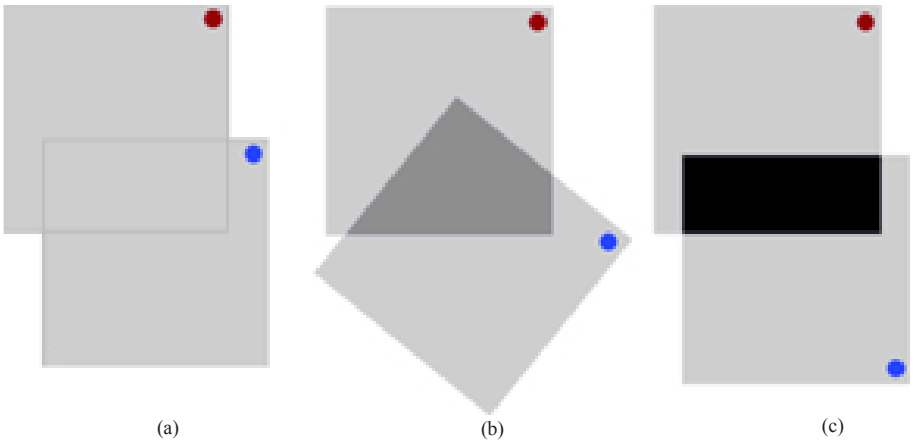
In Fig 1, if the Polaroid P_1 is rotated about the z-axis then one will observe no variation of intensity. However, if we place another Polaroid P_2 (as shown in Fig 1), then by rotating the Polaroid P_2 (about the z-axis) one will observe variation of intensity. We assume that the pass axis of the Polaroid P_2 makes an angle θ with the pass axis of the Polaroid P_1 (i.e., with the x-axis). Thus, if the amplitude of the incident electric field (emerging from the Polaroid P_1) is E_0, then the amplitude of the wave emerging from the Polaroid P_2 will be $E_0 \cos\theta$ and thus the intensity of the beam emerging from the Polaroid P_2 will be given by,

$$I = I_0 \cos^2\theta \tag{15}$$

where I_0 represents the intensity of the emergent beam when the pass axis of P_2 is also along the x-axis (i.e., when $\theta = 0$). The above equation represents the Malus' Law. Thus, if a linearly polarized beam is incident on a Polaroid and if the Polaroid is rotated about the z-axis, then the intensity of the emergent wave will vary according to the above law.

Figure 2(a) shows that if the two Polaroids are parallel to each other, almost the entire light passes through. Figure 2(b) shows that if the two Polaroids are oriented at 45º with respect to each other about 50% of the light passes through; this is because $I = I_0 \cos^2 45^\circ = I_0/2$. Figure 2(c) shows that if the two Polaroids

are at right angles to each other (notice the position of the blue dot) almost no light will pass through, as $I = I_0 \cos^2 90^o = 0$.



(a) (b) (c)

Fig 2. (a) If the two Polaroids are parallel to each other, almost the entire light passes through. (b) when the two Polaroids are oriented at 45° with respect to each other about 50% of the light passes through; this is because $\cos^2 45^o = 0.5$ (c) when the two Polaroids are at right angles to each other (notice the position of the blue dot) almost no light will pass through. Photographs adapted from ***A-level Physics Tutor***, www.a-levelphysicstutor.com/; used with kind permission from Dr Alan J Reed.

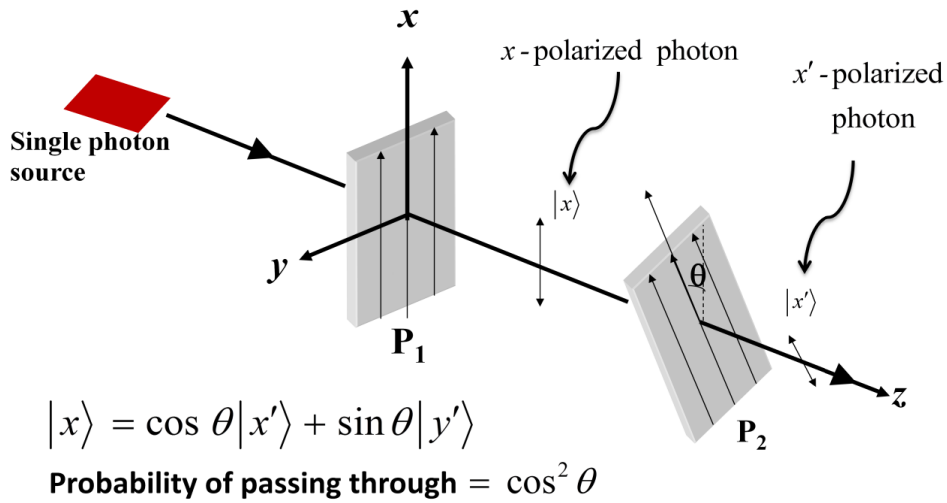


Fig 3. The probability for the photon to pass through the second polaroid is $\cos^2\theta$ and if the experiment is conducted with N photons (and if N is very large) then approximately $N \cos^2\theta$ photons will pass through; one cannot predict the fate of an individual photon.

We can associate a certain state of polarization with every photon. Consider a light beam from a single photon source (propagating in the z-direction) and incident on a Polaroid whose pass axis is along the x-axis (see Fig 3); a photon coming out of the Polaroid will be x-polarised as described by the vector,

$$|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{16}$$

If we pass x-polarised photons through a Polaroid whose pass axis makes an angle θ with the x-axis (see Fig 3), then each photon has probability $\cos^2\theta$ to pass through; we cannot say for sure whether the photon

will pass through or not. Thus the fact that the energy associated with electromagnetic radiation is quantized immediately leads to a probabilistic point of view. Similarly, a *y*-polarised photon is described by the vector,

$$|y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{17}$$

We next consider a light beam incident on an appropriately oriented calcite crystal whose optic axis is along a specific direction. The calcite crystal shown in Fig 4 is often referred to as a (*x*-*y*) device because no matter what the incident state of polarization may be, it will always split into a *x*-polarised beam and a *y*-polarised beam; the detailed theory of propagation in an anisotropic medium can be found in most text books in optics – see, for example, Refs [2, 3]. Figure 5 shows an actual photograph showing this split.

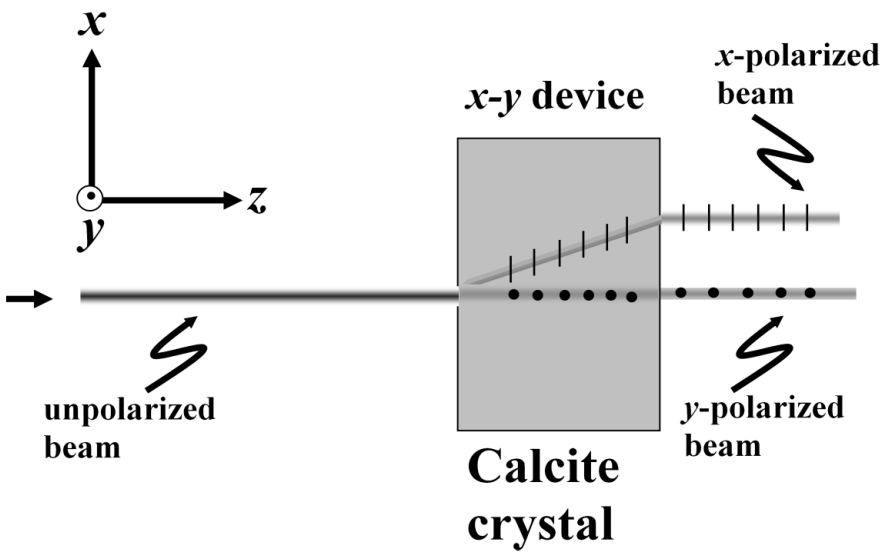


Fig 4. When an unpolarized light beam is incident on a properly oriented calcite crystal, it would split up into two linearly polarized beams.

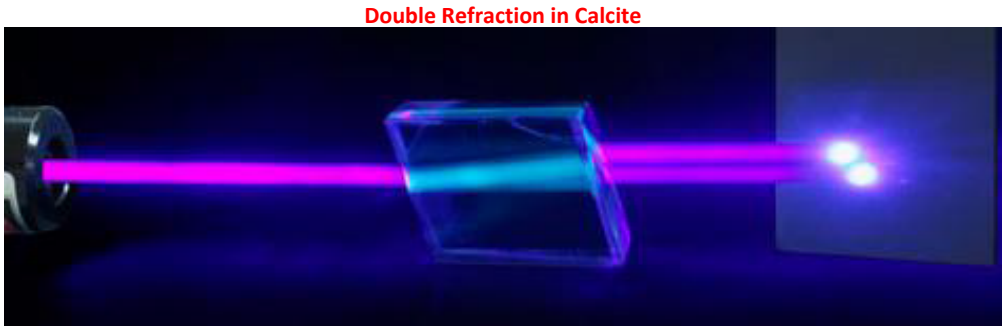


Fig 5. Photograph ofan unpolarized light beamincident normally on a calcite crystal resulting in its splitting into two linearly polarized beams. Photograph adapted from G Ropars, A Le Flocha and V Lakshminarayanan,*The sunstone and polarised skylight: ancient Viking navigational tools* ?, Contemporary Physics (2014). Photograph courtesy Professor V Lakshminarayanan.

Now, when a *x*-polarized photon is incident normally on a (suitably oriented) calcite crystal, it will propagate as an *x*-polarized photon [see Fig 6 (a)] and when a *y*-polarized photonis incident normally on the calcite crystal, it will propagate as an *y*-polarised photon [see Fig 6 (b)]. What will happen, if a 45º polarized photon is incident on the calcite crystalas shown in Fig 7 ? We can express a 45º polarized photon by the vector:

$$|\text{LP } 45^{\circ}\rangle = \begin{pmatrix}\cos 45^{\circ}\\ \sin 45^{\circ}\end{pmatrix} = \frac{1}{\sqrt{2}}|x\rangle + \frac{1}{\sqrt{2}}|y\rangle \tag{18}$$

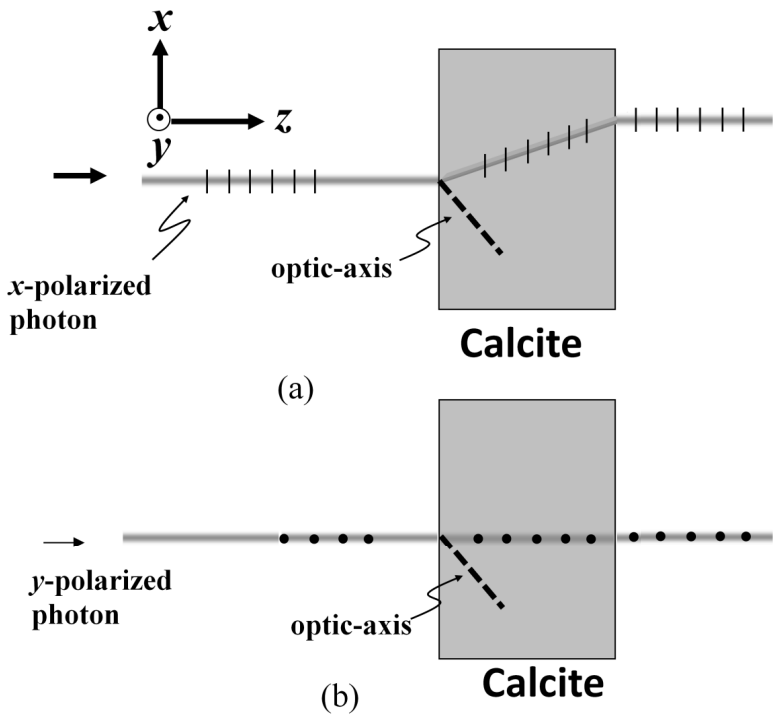
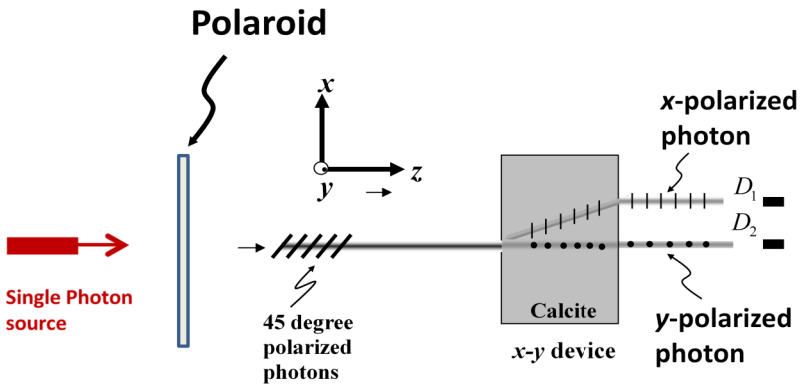


Fig 6. (a) When an x-polarized photon is incident normally on a (specifically oriented) calcite crystal, it propagates as an x-polarized photon. (b) When a y-polarized photon is incident normally on the calcite crystal, it propagates as a y-polarized photon.



$$\left|45^{\circ}\right\rangle = \frac{1}{\sqrt{2}}\left|x\right\rangle + \frac{1}{\sqrt{2}}\left|y\right\rangle$$

Fig 7. When a 45° polarized photon is incident on an x-y device, it is in both beams before it is detected. The process of measurement makes it collapse to either D_1 or D_2; there is a half probability of getting detected by D_1 and half probability of getting detected by D_2 .

Thus the photon is in a superposed state and there is half probability of it getting detected by the detector D_1 and half probability of it getting detected by the detector D_2 (see Fig 7). No one can predict for sure as to whether it will be detected by D_1 or by the detector D_2. In fact before detection, it is in both the beams; the process of measurement makes it collapse to either D_1 or to D_2.

$$|45^o\rangle = \frac{1}{\sqrt{2}}|x\rangle + \frac{1}{\sqrt{2}}|y\rangle$$

On the other hand, if the photon incident on the x-y device is polarized at an angle 60° with the x-axis, then

$$|\text{LP } 60^o\rangle = \begin{pmatrix}\cos 60^o \\ \sin 60^o\end{pmatrix} = \frac{1}{2}|x\rangle + \frac{\sqrt{3}}{2}|y\rangle \tag{19}$$

Thus the probability of it getting detected by the detector D_1 will be 0.25 and probability ofthe photon getting detected by the detector D_2 will be 0.75.

If we rotate the calcite crystal as shown in Fig 8, we will obtain an (u, v) device where the u-v axes are obtained by rotating the x-y axes as shown in Fig 8.

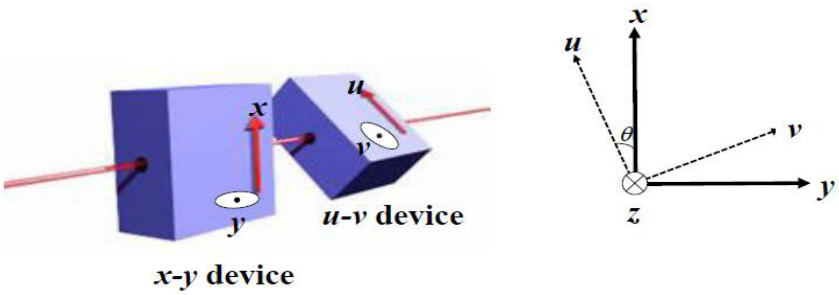


Fig 8. By rotating the (x-y) device by a certain angle about the z-axis, we obtain an (u-v) device. Adapted from http://www.upscale.utoronto.ca/PVB/Harrison/BellsTheorem/BellsTheorem.html

3 What is entanglement ?

There are atoms which emit two photons in quick succession and the two photons are of different frequencies propagating in opposite directions. There are actual transitions where the two photons are characterized by the property that the polarization of one is always orthogonal to the other. Thus, if the photon travelling to the left is passed through a x-y device and if the photon is found to be x-polarized then the photon travelling to the right will be$y$-polarized (see Fig 9); the two photons are said to be "entangled". This can be seen by making what is known as "coincidence" measurements. Thus in Fig 9, there are events when the detector D_1 on the left and the detector D_4on the right click simultaneously; such simultaneous measurements are called "coincidence" measurements. However, there are no events when the detector D_1 on the left and the detector D_3 on the right click simultaneously.

The word "entanglement" was introduced by Erwin Schrodinger in a paper published in 1935 (Ref. [4]); in that paper, he wrote "*I would call entanglement not one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought*."

In 1935, Einstein (along with Podolsky and Rosen) published a paper (Ref [5]) in which they argued that if quantum theory was correct, then two particles (which are millions of kilometers apart) can be entangled in the sense that by determining a property of one of the particles, the property of the second particle can be instantaneously changed. And special theory of relativity forbids the transmission of any signal faster than the speed of light. This came to be known as *The EPR Paradox*. About thirty years later,

experiments confirmed the predictions of quantum mechanics namely Einstein's impossible proposition was in fact correct: instantaneous changes in widely separated systems did occur.

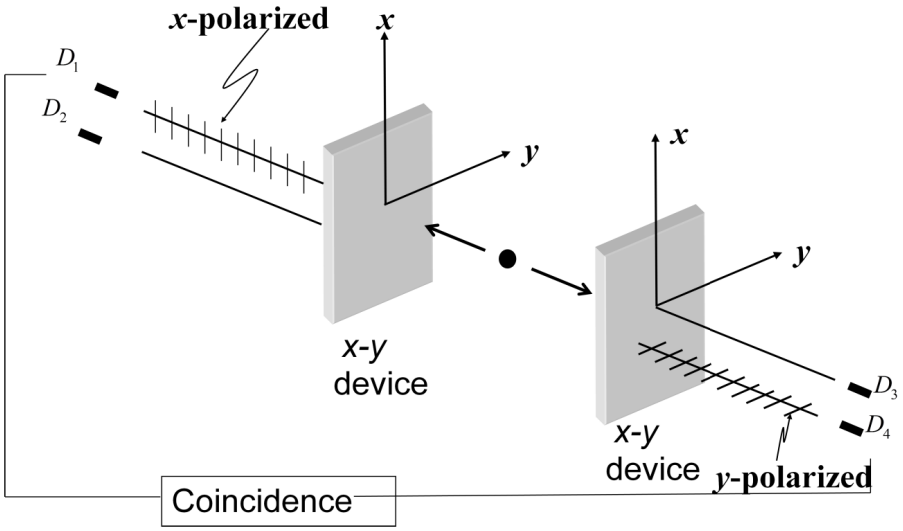


Fig 9. Two photons (emitted by the same atom) propagating in opposite directions are characterized by the property that the polarization of one is always orthogonal to the other; the polarization of the photon (traveling to the left – or to the right) is not known before one of them is measured. If the photon travelling to the left is passed through a *x*-*y* device and if the photon is found to be *x*-polarized then the photon travelling to the right will be *y*-polarized; we know the polarization of thephoton moving to the right without measuring it.

4 Non-locality of quantum theory

According to quantum theory, the polarization of the photon (traveling to the left – or to the right) is not known before one of them is measured; and, if the polarization of the photon going to the left is measured as *x*-polarised then the polarization of the photon going to the right collapses to state in which its polarization state is for sure *y*-polarised. Similarly, if the polarization of the photon going to the left is measured as *u*-polarised then the polarization of the photon going to the right collapses to state in which its polarization state is for sure *v*-polarised.

The two photons can be hundreds of kilometers apart. This collapse of the wave function is usually referred to as the "Copenhagen interpretation" of quantum mechanics. Thus quantum theory is "non-local" – i.e.,

measurement on one particle can instantaneously affect the state of another particle which is very far away.

Einstein could never accept this; he felt that this will contradict special theory of relativity according to which no signal can travel faster than light. Einstein called this "Spooky action at a distance". (In German, Einstein called this "Spukhafte Fernwirkung").

Einstein believed in "locality"; i.e., measuring the state of one particle will not affect the other. Einstein believed that if one could predict the polarization of the second photon by measuring the polarization of the first photon, then both photons had definite states of polarization all the time. Thus associated with the photons are "hidden variables" so that the polarizationof the photons are known before the measurements are carried out. In their 1935 paper (Ref [5]), EPR wrote:

> *if without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then the second particle musthave possessed the measured property before the measurement was carried out.*

Einstein had also written (p 85 in Ref [6]; see also Ref [7])

> *But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former.*

In his 1964 paper (Ref [7]), John Bell wrote

> *It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty*.

5 Simple analysis based on hidden variables

We construct a theory based on "hidden variables" so that we know the number of photon pairs which have their own independent attributes when passed through a (x, y) device or a (u, v) device or a (σ, η) device; the (x, y) axes, (u, v) axes and (σ, η) axes are defined in Fig 10.

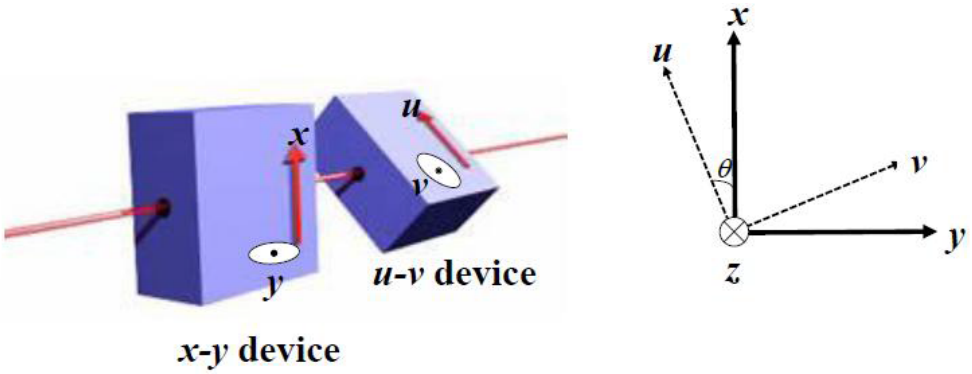


Fig 10. The (u,v) axes are obtained by rotating the (x,y)axes by an angle θ ; similarly the (σ,η)axes are obtained by rotating the (x,y)axes by an angle θ.

Thus we can assign values of all observables of a system *before* the measurement is carried out. For example, if a photon described by $\{x, u, \sigma\}$ is passed through a (x, y) device, *we will for sure* measure x-polarisation if it were to pass through a (u, v) device, *we will for sure* measure u-polarisation and if it were to pass through a (σ, η) device, *we will for sure* measure σ-polarisation Further, if a photon described by $\{x, u, \sigma\}$ is propagating to the left then thephoton propagating to the right will be described by $\{y, v, \eta\}$ (see Fig 11) so that if the photon propagating to the right is passed through a (x, y) device, *we will for sure* measure y-polarisation if it is passed through a (u, v) device, *we will for sure* measure v-polarisation and if it is passed through a (σ, η) device, *we will for sure* measure η-polarisation.

Let there be

N_1 photons characterized by $\{x, u, \sigma\}$ and equal number of photons characterized by $\{y,v, \eta\}$.

Similarly, we may have,

N_2 photons characterized by $\{x, u, \eta\}$ and equal number of photons characterized by $\{y,v, \sigma\}$.

Obviously, we can have 8 possible pairs as listed in Table 1.

If $P(x, u)$ represents the probability that the photon (going to the left) is x-polarised and the photon (going to the right) is u-polarised then from Table 1, it is obvious that,

$$P(x, u) = \frac{N_3 + N_4}{N}$$

where

$$N = N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7 + N_3$$

Similarly, if $P(x, \sigma)$ represents the probability that the photon (going to the left) is x-polarised and the photon (going to the right) is σ-polarised then (again from Table 1)

$$P(x, \sigma) = \frac{N_2 + N_4}{N}$$

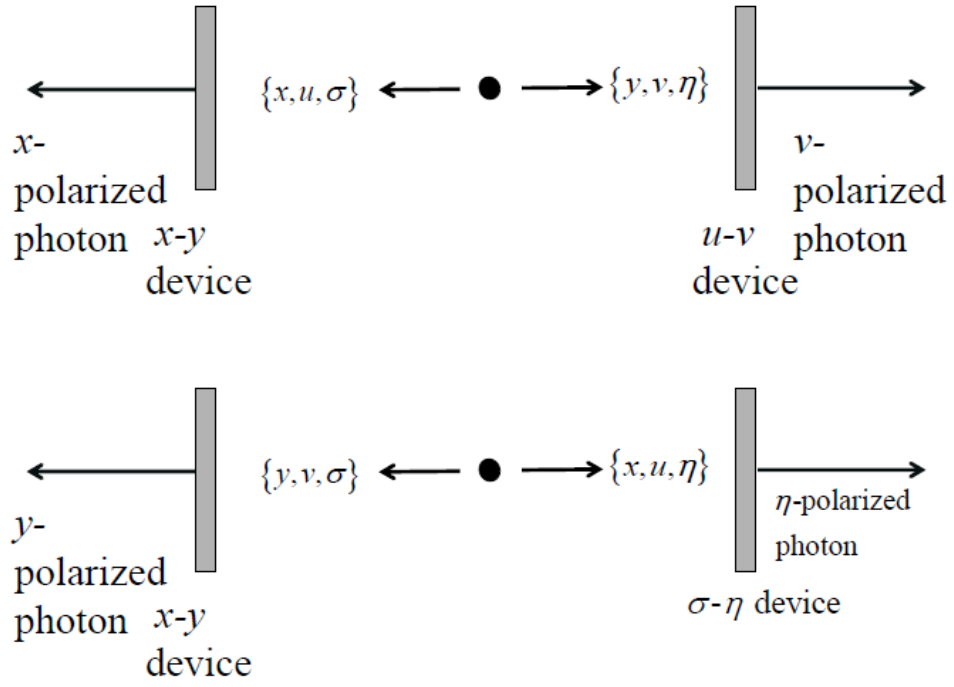


Fig 11. We assume that the photon must carry all the information;thus there must exist "hidden variables" so that we can assign values of all observables of a system.

Table 1

	Photons moving to the left	Photon moving to the right
N_1 pairs characterized by	$\{x, u, \sigma\}$	$\{y, v, \eta\}$
N_2 pairs characterized by	$\{x, u, \eta\}$	$\{y, v, \sigma\}$
N_3 pairs characterized by	$\{x, v, \sigma\}$	$\{y, u, \eta\}$
N_4 pairs characterized by	$\{x, v, \eta\}$	$\{y, u,\sigma\}$
N_5 pairs characterized by	$\{y, u,\sigma\}$	$\{x, v, \eta\}$
N_6 pairs characterized by	$\{y, u, \eta\}$	$\{x, v, \sigma\}$
N_7pairs characterized by	$\{y, v, \sigma\}$	$\{x\, u, \eta\}$
N_8 pairs characterized by	$\{y, v, \eta\}$	$\{x\, u, \sigma\}$

Similarly, if $P(\sigma, u)$ represents the probability that the photon (going to the left) is σ-polarised and the photon (going to the right) is u-polarised then (again from Table 1)

$$P(\sigma, u) = \frac{N_3 + N_7}{N}$$

Thus

$$P(x, \sigma) + P(\sigma, u) = \frac{N_2 + N_4 + N_3 + N_7}{N}$$

Since

$$N_3 + N_4 \leq N_2 + N_4 + N_3 + N_7$$

we will have

$$P(x, u) \leq P(x, \sigma) + P(\sigma, u) \tag{20}$$

This is a simple form of *Bell's inequality*. In 1964, John Bell published his famous theorem (now referred to as Bell's theorem) in a paper entitled "On the Einstein-Podolsky-Rosen Paradox" (Ref [7]). Many experiments have shown that above inequality is violated. The theorem was a revolutionary contribution to the understanding of quantum mechanics.

6 Results from quantum theory

We will now calculate expressions for $P(x, u)$, $P(x, \sigma)$ and $P(\sigma, u)$ using simple quantum theory. If N represents the total number of photons (going to the left) then $N/2$ photons will appear x-polarised. The corresponding photon going to the right will appear y-polarized. Thus, the probability that the photon (going to the left) is x-polarised and the photon (going to the right) is u-polarised (see Fig 10) will be given by:

$$P(x, u) = \frac{1}{2}\cos^2\left(\frac{\pi}{2} + \theta\right) = \frac{1}{2}\sin^2\theta$$

where the angle θ is defined in Fig 10. Similarly,

$$P(x, \sigma) = \frac{1}{2}\sin^2\phi$$

where the angle ϕ is defined in Fig 10, and

$$P(\sigma, u) = \frac{1}{2}\sin^2(\phi - \theta)$$

Thus, if Bell's inequality is consistent with quantum theory, we must have

$$\sin^2\theta \leq \sin^2\phi + \sin^2(\phi - \theta). \tag{21}$$

If weassume $\theta = 2\phi$, the above inequality takes the form,

$$\sin^2 2\phi \leq 2\sin^2\phi,$$

which is violated for $\phi = \pi/6$. Thus, quantum theory and Bell's inequality are not compatible which implies that either quantum theory is right or theory based on hidden variables, but not both. Aspect and his co-workers [8] have carried out experiments with photons and results were decisively in agreement with quantum theory and against the results obtained by using hidden variables. In Ref [8], Aspect *et al* write:

> "*The linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium has been measured. The new experimental scheme, using two-channel polarizers* (*i.e., optical analogs of Stern-Gerlach filters*), *is a straightforward transposition of Einstein-Podolsky-Rosen-Bohm gedanken experiment. The present results, in excellent agreement with the quantum mechanical predictions, lead to the greatest violation of generalized Bell's inequalities ever achieved*."

John Bell has written (p 90 in Ref 9):

> "*It seems to me then beyond dispute that there was at least one Einstein, that of the EPR paperand the Schilpp volume, who was fully committed to the view that quantum mechanicswas incomplete and should be completed – which is the hidden variable program.*"

Bell's theorem proved that the above view (of Einstein) is incorrect!

7 Correlation Function

We define a correlation function C

$$C = P(x, v) + P(y, u) - P(x, u) - P(y, v)$$

Now, according to quantum theory,

$$P(x, u) = \frac{1}{2}\sin^2\theta.$$

Similarly,

$$P(x, v) = \frac{1}{2}\cos^2\theta,\ P(y, u) = \frac{1}{2}\cos^2\theta, \text{ and } P(y, v) = \frac{1}{2}\sin^2\theta.$$

Thus according to quantum theory, the correlation function C is given by,

$$C = P(x, v) + P(y, u) - P(x, u) - P(y, v) = \cos^2\theta - \sin^2\theta = \cos 2\theta.$$

The above variation of the correlation function C with θ was verified very accurately by experiments carried out by Freedman and Clauser in 1972 (Ref [10]).

8 Conclusions

Bell's theorem is considered (to quote from Ref [11])

> *as one of the most profound discoveries since Copernicus… Bell delivered a death blow to the local realistic picture of the world... many experiments have demonstrated that the predictions of quantum mechanics for entangled particles are fully correct ... and the world is really as "crazy" as predicted by quantum mechanics.*

Nobel laureate Brian Josephson remarked that Bell's inequality is the most important recent advance in physics. John Bell's response was "*I would say that's probably a bit exaggerated. But if you are primarily concerned with the philosophy of physics, I can see the point*" (as quoted on page 30 in Ref 12).

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